

JEE-MAIN

SOLUTIONS

TOPIC

GRAVITATION

GRAVITATION

Exercise-I

1. (A)

$$g = \frac{GM}{R^2} = \frac{G \frac{4}{3} \pi R^3 \rho}{R^2}$$

$$\rho = \frac{3g}{4\pi R G}$$

2. (B)

$$\frac{g}{4} = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

$$2 = 1 + \frac{h}{R_e}$$

$$h = R_e$$

3. (B)

$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{4g}{9}$$

$$\text{decrease} = g - g'$$

$$= g - \frac{4g}{9} = \frac{5g}{9}$$

4. (B)

$$\frac{g}{4} = \frac{GM_e}{(R_e + h)^2}$$

$$\frac{GM_e}{4R^2} = \frac{GM_e}{(R_e + h)^2}$$

$$R_e + h = 2R_e$$

$$R_e = h$$

5. (C)

Due to rotation of earth

$$g_{\text{eff}} = g - \omega^2 R \sin^2 \theta$$

$$g \downarrow$$

∴ weight ↓

6. (B)

$$F = \frac{Gm_1 m_2}{r^2}$$

$$F' = \frac{Gm_1 m_2}{(2r)^2} = \frac{F}{4}$$

7. (C)

$$g_h = g \left(1 - \frac{r}{R}\right)$$

$$g_h = \frac{GM}{R^2} \left(1 - \frac{r}{R}\right)$$

$$\frac{GMr}{R^3} = \text{constant} \Rightarrow d \propto \frac{1}{r}$$

8. (A)

According to given condition

$$g - \omega^2 R_e = g \left(1 - \frac{d}{R_e}\right)$$

$$\frac{\omega^2 R_e^2}{g} = d$$

9. (D)

$$g = \frac{GM_e}{R_e^2} = \frac{GM'_e}{(5R_e)^2}$$

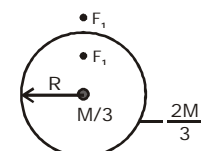
$$\frac{\frac{4}{3} \pi R_e^3 \rho}{R_e^2} = \frac{\frac{4}{3} \pi (5R_e)^3 \rho'}{(5R_e)^2}$$

$$\rho = 5\rho'$$

10. (B)

$$g' = \frac{G(2M_e)}{(2R_e)^2} = \frac{g}{2}$$

$$T = 2\pi \sqrt{\frac{l}{g}} = 2$$



New time period $T' = 2\pi \sqrt{\frac{l}{g/2}} = 2\sqrt{2}$

11. (D)
THEORY

12. (A)
 $\frac{1}{2}mv'^2 = 2 \times \frac{1}{2}mv^2$

$v' = \sqrt{2} v_0$

$v' = v_e$

∴ so escape.

13. (C)

$v_0 = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{2G\rho \frac{4}{3}\pi R_e^3}{R_e}}$

$= \sqrt{2G\rho \frac{4}{3}\pi R_e^2}$

$v' = \sqrt{2G\rho \frac{4}{3}\pi (2R_e)^2}$

$= 2v_0$

14. (A)

$g_A = \frac{G \frac{4}{3}\pi R_A^3 \rho}{R_A^2} ; g_B = \frac{G \frac{4}{3}\pi R_B^3 \rho}{R_B^2}$

$R_A = 2R_B$

⇒ $g_A = 2g_B$

$V_{es} = \sqrt{2gR}$

$(V_{es})_A = \sqrt{2g_A R_A} = 2\sqrt{2g_B R_B}$

$(V_{es})_B = \sqrt{2g_B R_B}$

$\frac{V_A}{V_B} = 2$

15. (B)

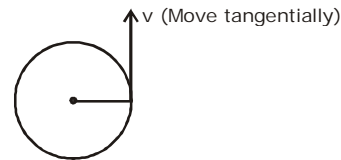
$V_p = -\frac{GM}{R}$ (-ve)

as $R \downarrow \frac{GM}{R} \uparrow$ but due to -ve it decreases.

16. (B)

$g = \frac{GM}{R^2}$

17. (B)

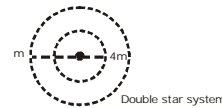


18. (C)

$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{2\omega} = \frac{2\pi \times 24 \times \text{hr.}}{2 \times 2\pi}$

$T = 12 \text{ hr.}$

19. (A)



$\frac{k_1}{k_2} = \frac{1/2I_1\omega^2}{1/2I_2\omega^2} = \frac{m_2}{m_1}$

20. (C)

(a) cavity at center, field is zero

(b) Arc of ellipse

(c) for escape T.E. = 0

(d) Notes.

21. (D)

P.E. = $-\frac{Gm_1m_2}{r}$

T.E. = $-\frac{Gm_1m_2}{2r}$

K.E. = $+\frac{Gm_1m_2}{2r}$

22. (D)

23. (C)

Kepler's second law

$\frac{dA}{dt} = \frac{L}{2M} = \text{constant}$

24. (B)

$U_f = u_i = \frac{-GmM}{\left(R + \frac{R}{5}\right)} + \frac{GmM}{R}$

$= \frac{-5GmM}{6R} + \frac{GmM}{R} = \frac{GmM}{6R} = \frac{msR}{6}$

25. (C)

$\frac{1}{2}mv^2 = mgh = \frac{mGM}{R^2} \times 90 \dots(1)$

$$\frac{1}{2}mv^2 = \frac{mG\left(\frac{1}{10}M\right)}{\left(\frac{R}{3}\right)^2} \times G_1 \quad \dots(2)$$

from (1) & (2)

$$m \frac{GM}{R^2} \times 90 = \frac{9}{10} \frac{mGM}{R^2} \times h_1 \Rightarrow h_1 = 10m$$

26. (C)

$$\begin{aligned} W &= \frac{GmM}{R} - \frac{GmM}{nR+R} \\ &= \frac{GmM}{R} \left[1 - \frac{1}{n+1} \right] = \frac{GmM}{R} \left[\frac{n}{n+1} \right] \\ &= mgR \left(\frac{n}{n+1} \right) \end{aligned}$$

27. (D)

$$\frac{GM_p m}{R_p} = 54 \Rightarrow \frac{GM_p}{R_p} \times 3 = 54$$

$$\frac{GM_p}{R_p} = 18$$

$$v_e = \sqrt{\frac{2GM_p}{R_p}} = \sqrt{2 \times 18} = 6 \text{ m/sec}$$

28. (C)

$$\frac{1}{2}mv^2 = \frac{GM_e m}{Re+h} = \frac{gRe^2 m}{Re+4Re}$$

$$\frac{1}{2}mv^2 = \frac{MgRe}{5}$$

29. (B)

Gravity of earth vanishes means there is no centripetal force that is gravitational force.

30. (A)

$$E = -\frac{Gm_1 m_2}{2r}$$

$$\frac{E}{E_A} = \frac{v_A}{v_B} = \frac{1.4}{1}$$

31. (C)

$$T \propto r^{3/2} \left[\omega = \frac{2\pi}{T} \right]; \quad \omega \propto \frac{1}{r^{3/2}}$$

$$\left(\frac{\omega}{2\omega} \right) = \left(\frac{R_1}{r} \right)^{3/2}$$

$$\frac{R_1}{r} = \frac{1}{(2)^{2/3}}$$

$$R_1 = \frac{r}{(4)^{1/3}}$$

32. (B)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

33. (C)

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{Gm}{R+h}}$$

$$v_1 = \sqrt{\frac{Gm}{R+\frac{R}{2}}} = \sqrt{\frac{2}{3}} v$$

34. (A)

$$v \propto \sqrt{\frac{1}{r}}$$

$$\frac{v_A}{v_B} = \sqrt{\frac{r_B+R}{r_A+R}}$$

35. (A)

$$\frac{A}{T} = \frac{L}{2m}$$

$$L = \frac{2mA}{T}$$

36. (B)

$$F = \frac{GM_s M_e}{R^2}; \quad F' = \frac{GM_s M_e}{(3R)^2} = \frac{F}{9}$$

37. (C)

38. (C)

$$r \propto T^{2/3}$$

$$\frac{r_1}{r_2} = \left(\frac{3}{24} \right)^{2/3} = \frac{1}{4}$$

$$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T}$$

$$\frac{v_1}{v_2} = \left(\frac{r_1}{r_2} \right) \left(\frac{T_1}{T_2} \right) = \left(\frac{1}{4} \right) \left(\frac{24}{3} \right) = \frac{2}{1}$$

39. B

$$F = \frac{Gm_1 m_2}{r^2}$$

Exercise-II

1. From E.C.

$$0 = -\frac{Gm_1m_2}{d} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

from M.C. $m_1v_1 - m_2v_2 = 0$

$$\text{Now, } \frac{Gm_1m_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1v_1}{m_2}\right)^2$$

$$v_1 = \sqrt{\frac{2Gm_2^2}{d(m_1+m_2)}}$$

$$v_2 = \sqrt{\frac{2Gm_1^2}{d(m_1+m_2)}}$$

Relative velocity of approach = $v_1 + v_2$

$$v = \sqrt{\frac{2G(m_1+m_2)}{d}}$$

2. $F_1 = \frac{GMm}{R^2}$

$$F_2 = \frac{GMm}{3R^2}$$

$$\therefore \text{Change} = \frac{2}{3} \frac{GMm}{R^2}$$

3. $G - \omega^2R = g/2$

$$\omega^2R = g/2$$

$$\frac{v^2}{R} = g/2$$

$$2v^2 = gR$$

$$v_{es} = \sqrt{2gR}$$

$$v_{es} = \sqrt{2(2v^2)} = 2v$$

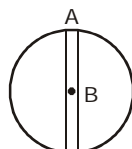
4. $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

from E.C.

$$-\frac{GMm}{R} = \frac{-3GMm}{2R} + \frac{1}{2}mv^2$$

$$\frac{GM}{2R} = \frac{1}{2}v^2$$

$$v = \frac{v_e}{\sqrt{2}}$$

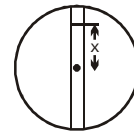


5. $F = \frac{GMmx}{R^3}$

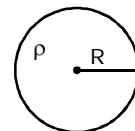
$$T = 2\pi \sqrt{\frac{R^3}{GM}}$$

$$T = 84.6 \text{ min}$$

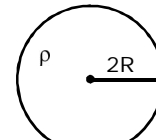
$$\text{time to each other end } t = \frac{T}{2} = 42.3 \text{ min}$$



6.



$$\rho \frac{4}{3} \pi R^3 = m$$



$$\rho \frac{4}{3} (2R)^3 = 8m$$

from E.C.

$$0 + 0 = -\frac{Gm8m}{3R}$$

$$+ \frac{1}{2}mv_1^2 + \frac{1}{2}8mv_2^2 \dots\dots (1)$$

from M.C.

$$mv_1 = 8mv_2$$

$$v_1 = 8v_2 \dots\dots (2)$$

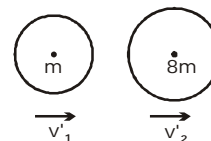
Put value from eq. (2) to eq. (1)

$$\frac{G8m^2}{3R} = \frac{1}{2}m64v_2^2 + \frac{1}{2}8m^2$$

$$v_2^2 = \frac{2Gm}{27R}$$

$$\Rightarrow v_1^2 = \frac{64 \times 2Gm}{27R}$$

After collision



$$u_1 = v_1$$

$$u_2 = -v_2$$

\therefore

$$v_1' = \frac{0 + \frac{8m}{2}(-v_2 - v_1)}{9m} = \frac{-4}{9}(v_1 + v_2)$$

$$v_2' = \frac{v_1 + v_2}{18m}$$

$$\text{Initial K.E.} = \frac{Gm8m}{3R}$$

$$\text{final K.E.} = \frac{1}{2}mv_1'^2 + \frac{1}{2}8mv_2'^2$$

$$= \frac{1}{2} m \frac{16}{81} (v_1 + v_2)^2 + \frac{1}{2} 8m (v_2 + v_1)^2 \frac{1}{(18)^2}$$

$$= \frac{m}{g} (v_1 + v_2)^2$$

$$= \frac{2}{3} \frac{Gm^2}{R}$$

7. After collision r is max. separation from M.C.

$$8mv'_2 + mv'_1 = 9mv$$

$$8m \left[\frac{v_1 + v_2}{18} \right] - m \left[\frac{4}{9} (v_1 + v_2) \right] = 9mv$$

v = 0
from E.C.

$$\frac{-G8m^2}{r} + \frac{1}{2} 8mv^2 + \frac{1}{2} mv^2 = \frac{2}{3} \frac{Gm^2}{R} - \frac{G8m^2}{3R}$$

$$\Rightarrow r = 4R$$

8. In (a) & (b)

$$\frac{2GM^2}{a^2}$$

In (c) & (d)

$$\frac{4GM^2}{a^2}$$

9.

$$\frac{-GM_e m}{R} = E_{\text{initial}}$$

$$E_{\text{final}} = \frac{-GM_e m}{2 \times 2R}$$

$$\therefore \text{difference} = -\frac{GM_e m}{R} + \frac{GM_e m}{4R}$$

$$= \frac{3}{4} mgR$$

10.

$$5M \xrightarrow{v_0 = \sqrt{\frac{GM_e}{r}}}$$

$$\xleftarrow{\sqrt{\frac{GM_e}{r}}} M \quad 4M \xrightarrow{v_1}$$

$$5Mv_0 = -Mv_0 + 4Mv_1$$

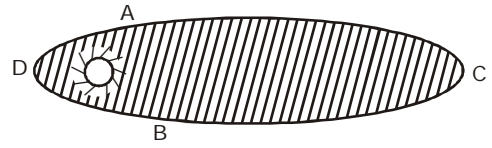
$$\frac{3v_0}{2} = v_1$$

$$v_1 = \frac{3}{2} \sqrt{\frac{GM_e}{r}}$$

$$\text{T.E.} = -\frac{GM_e 4m}{r} + \frac{1}{2} 4m \left(\frac{9}{4} \frac{GM_e}{r} \right)$$

$$\text{T.E.} > 0$$

11.



$t_1 > t_2$ between area is ACB is greater than ADB.

12.

Both D & C between
Total energy is always -ve

13.

$$\frac{dA}{dt} = \frac{m \sqrt{\frac{GM_e}{R}} \cdot R}{2m}$$

$$\frac{dA}{dt} \propto \sqrt{R}$$

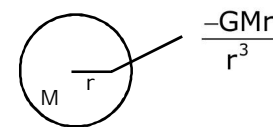
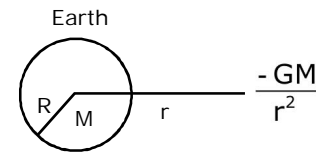
14.

$$\text{P.E.} = -\frac{Gm_1 m_2}{r}$$

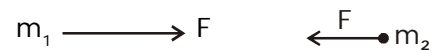
$$\text{T.E.} = -\frac{Gm_1 m_2}{2r}$$

$$\text{K.E.} = +\frac{Gm_1 m_2}{2r}$$

15.



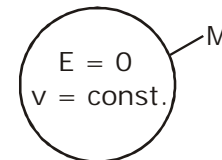
16.



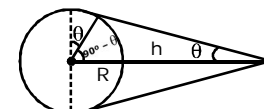
$$a_1 = \frac{F}{m_1} \quad a_2 = \frac{F}{m_2}$$

$$m_1 < m_2 \Rightarrow a_1 > a_2$$

17.



18.



$$\sin\theta = \frac{R}{R+h}$$

$$\begin{aligned} \text{Area covered} &= R^2 2\pi (1 - \cos(90-\theta)) \\ &= 2\pi R^2 (1 - \sin\theta) \end{aligned}$$

$$\begin{aligned} \text{Area escaped} &= 4\pi R^2 - 2\pi R^2 (1 - \sin\theta) \\ &= 2\pi R^2 (1 + \sin\theta) \end{aligned}$$

19. Radius decreases \Rightarrow Velocity increases due to which K.E. increases

$$\begin{aligned} m.v.r &\Rightarrow m \sqrt{\frac{GM}{r}} \cdot r \Rightarrow \alpha\sqrt{r} \\ T^2 &\propto r^3 \end{aligned}$$

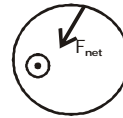
20. Theory

21. $\frac{-GM_e m}{r} = \text{P.E.}$

$$v = \sqrt{\frac{GM_e}{r}}$$

$$\omega = \frac{2\pi}{T} \quad r \uparrow \quad T \uparrow \quad \omega \downarrow$$

22. (A)



- (B) Both direction and Magnitude not change
 (C) Total Mechanical is constant
 (D) Linear momentum changes becoz v change as r changes
 but $rv = \text{constant}$

- 23.

$$v = \sqrt{\frac{GM_e}{R}} \quad (\text{Max.})$$

$$T^2 \propto r^3 \quad r \downarrow \quad T \downarrow$$

$$\text{T.E.} = \frac{-GMm}{2R}$$

- 24.

$$T = \frac{2\pi}{\sqrt{GM_e}} r^{3/2}$$

$$g = \frac{GM_e}{R_e^2} \quad \text{and } T_{\min.} \text{ at } r_{\min.} = R$$


$$\Rightarrow T_{\min.} = \frac{2\pi R^{3/2}}{\sqrt{gR_e^2}} = 2\pi\sqrt{\frac{R}{g}}$$



Exercise-III

Level - I

1. $TE = -\frac{Gm^2}{a} - \frac{2Gm^2}{a} - \frac{3Gm^2}{a}$
 $\Rightarrow T.E. = -\frac{6Gm^2}{a}$
 final = $-\frac{6Gm^2}{2a}$
 W.D. = $\frac{-6Gm^2}{a} + \frac{6Gm^2}{2a} = \frac{3Gm^2}{a}$

2. 
 $\lambda = \frac{m}{2L} \Rightarrow dF = \int_L^{3L} \frac{Gm\lambda dx}{x^2}$
 $F = Gm\lambda \left[\frac{1}{L} - \frac{1}{3L} \right]$
 $F = \frac{Gm^2}{3L}$

3. (i) $\frac{-GM_e m}{R} + \frac{1}{2}mv^2 = \frac{-GM_e m}{9R}$
 $v = \frac{4}{3} \sqrt{\frac{GM_e}{R}}$
 (ii) $\frac{-GM_e m}{R} + \frac{1}{2}mv^2 = \frac{-GM_e m}{5R} + \frac{1}{2}mv_1^2$
 $v_1 = \frac{2}{3} \sqrt{\frac{2GM_e}{R}}$

4. When sense of rotation of both earth and satellite is opposite

$$T_1 = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\frac{2\pi}{24} + \frac{2\pi}{1.5}}$$

When sense of rotation of both earth and satellite is same.

$$T_2 = \frac{2\pi}{\frac{2\pi}{24} - \frac{2\pi}{1.5}}$$

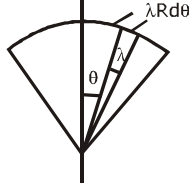
5. $\therefore |T.E.| = \left| \frac{P.E.}{2} \right|$

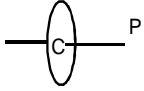
$$T.E._{final} = -1 \times 10^5 \text{ J}$$

$$T.E._{initial} = -2 \times 10^5 \text{ J}$$


So given energy = $1 \times 10^5 \text{ J}$

6. $E = \frac{G}{R^2} \int_{-\alpha}^{\alpha} \lambda R d\theta \cos \theta$
 $E = \frac{G\lambda}{R} [2 \sin \alpha]$
 Potential = $\frac{Gm}{R} = \frac{GR(2\alpha)\lambda}{R} = -G\lambda 2\alpha$



7. 
 $V_p = \frac{-GM}{\sqrt{2}a}$
 $V_p = \frac{-GM}{a}$
 $\frac{1}{2}mv^2 = \frac{GMm}{a} \left[1 - \frac{1}{\sqrt{2}} \right]$

8. $\frac{g}{\left[1 + \frac{h}{Re} \right]^2} = g \left[1 - \frac{h}{Re} \right]$

9. $-\frac{GM_e m}{2r} - \frac{GM_e m}{2r} = -\frac{GM_e m}{r}$
 By M.C. 
 final velocity = 0
 $\therefore T.E = \frac{-GM_e 2m}{r}$

Straight line.

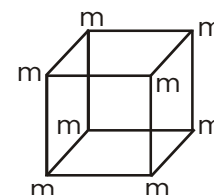
10. $\frac{-GM_e m}{R_e} + \frac{1}{2}m(k^2 v_e^2) = \frac{-GM_e m}{(R_e + h)}$

11. $\frac{-GMm}{2r} = T.E._{initial}$

$$\frac{-GMm}{2R_e} = T.E._{final}$$

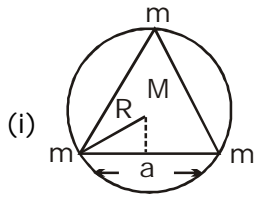
$$Ct = \frac{GMm}{2} \left(\frac{1}{R_e} - \frac{1}{r} \right)$$

12.



$$U = \frac{-8}{2} \left[\frac{3Gm^2}{a} + \frac{3Gm^2}{\sqrt{2}L} + \frac{Gm^2}{\sqrt{3}L} \right]$$

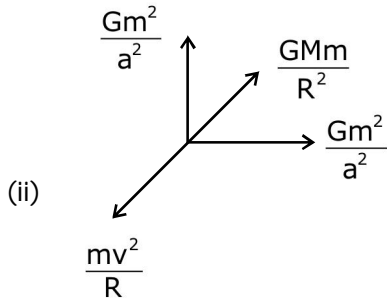
13.



$$U = 0 - \frac{Gm^2}{1} - \frac{Gm^2}{a} - \frac{Gm^2}{a} - \frac{3GMm}{R}$$

$$2R \cos 30^\circ = a$$

$$= \frac{-3GM}{R} \left[\frac{m}{\sqrt{3}} + M \right]$$

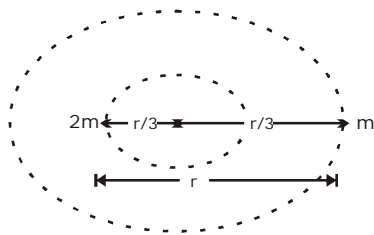


$$\frac{GMm}{R^2} + \frac{\sqrt{3}Gm^2}{3R^2} = \frac{mv^2}{R}$$

$$\frac{GMm}{R^2} + \frac{\sqrt{3}Gm^2}{3R^2} = \frac{mv^2}{R}$$

$$v = \sqrt{\frac{G}{R} \left(\frac{m}{\sqrt{3}} + M \right)}$$

14.



$$\frac{G2m^2}{r^2} = m\omega^2 \frac{2}{3}$$

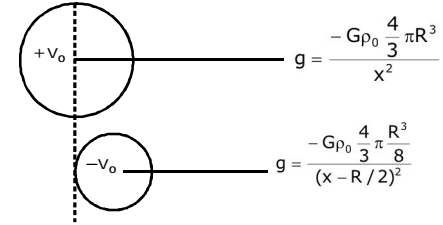
$$\frac{3Gm^2}{r^3} = \omega^2$$

Given $m = \frac{M_s}{3}$

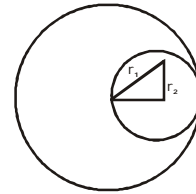
then $\frac{3GM_s}{r^3 \cdot 3} = \frac{GM_s}{R^3}$

$r = R$

15.



$$g_{net} = \frac{\pi G \rho_0 R^3}{6} \left[\frac{1}{(x - \frac{R}{2})^2} - \frac{8}{x^2} \right]$$



$$g_{net} = \frac{-G \frac{4}{3} \pi R^3 \rho_0 \vec{r}_1}{R^3} + \frac{G \frac{4}{3} \pi \frac{R^3}{8} \rho_0 \vec{r}_2}{R^3}$$

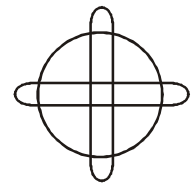
$$= \frac{4G\rho_0\pi}{3} \left[\vec{r}_2 - \vec{r}_1 \right] = \frac{-2}{3} \pi G \rho_0 R \hat{i}$$

16.

$v = \sqrt{\frac{GM_e}{r}}$

max^m distance

$= r\sqrt{2}$



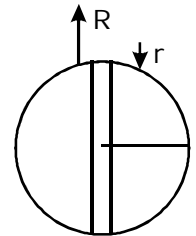
$v_{relative} = \sqrt{2} \sqrt{\frac{GM_e}{r}}$

17.

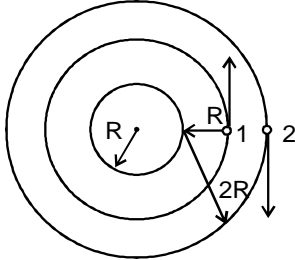
$$\frac{-GMm}{2R} = \frac{-GMm}{R} + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM}{R}} \therefore \text{Time} = \frac{2R}{v}$$

$$= 2 \times \sqrt{\frac{R^3}{GM}}$$



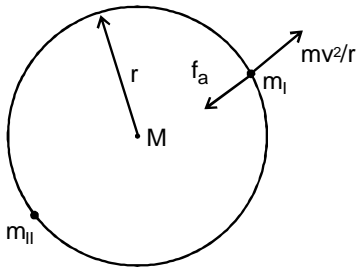
1 $\omega_1 = \frac{\sqrt{GM}}{(2R)^{3/2}}$; $\omega_2 = \frac{\sqrt{GM}}{(3R)^{3/2}}$
with respect to one time taken



$$t = \frac{2\pi}{\omega_1 + \omega_2} \Rightarrow t = \frac{2\pi R^{3/2}(6\sqrt{6})}{\sqrt{GM}(2\sqrt{2} + 3\sqrt{3})}$$

2 Attraction force $F_a = \frac{mv^2}{r}$

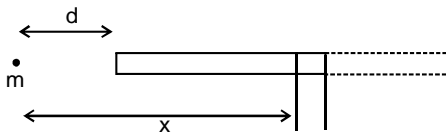
$$\frac{GMm}{r^2} + \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$$



$$\frac{G}{4r}[4M + m] = v^2 \text{ Now}$$

$$T = \frac{2\pi r}{v} = \frac{4\pi r^{3/2}}{\sqrt{G(4M + m)}}$$

3 $dF = G\left(\frac{\lambda}{x}\right) \frac{dx \cdot m}{x^2}$



$$F = Gm\lambda \int_{x=d}^{x=\infty} \frac{dx}{x^3} \Rightarrow F = -Gm\lambda \left[\frac{1}{x^2} \right]_d^{\infty} = -\frac{Gm\lambda}{2d^2}$$

4 In a cavity gravitational field

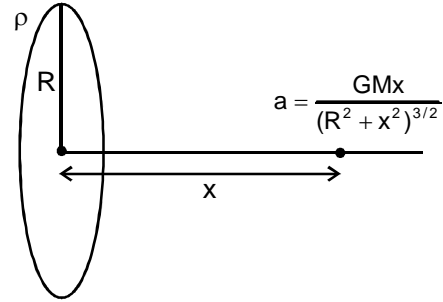
$$a = \frac{\rho r(4\pi r)}{3}; r = R/2 \Rightarrow a = \frac{4\pi\rho RG}{6}$$

$$\text{from } v^2 - u^2 = 2as \Rightarrow v^2 = 2\left(\frac{4\pi\rho RG}{6}\right) \frac{R}{2}$$

$$\Rightarrow v = \sqrt{\frac{2}{3}\pi G\rho R^2}$$

5 $\frac{da}{dx} = 0$ for $a_{\max} \Rightarrow x = \frac{R}{\sqrt{2}}$

$$a = \frac{aMR}{\sqrt{2}\left(\frac{3R^2}{2}\right)^{3/2}} = \frac{2GM}{(3)^{3/2}R^2}$$



$$M = (\pi r^2)(2\pi r)\rho \Rightarrow a = \frac{4\pi^2 G r^2 \rho}{(3)^{3/2} R}$$

Now $F = ma$

6 $R_{\max} = 4 = \frac{u^2}{g} \Rightarrow u = \sqrt{4g}$

According to problem $\sqrt{4g} = \sqrt{2g_p R_p}$

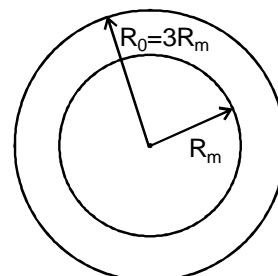
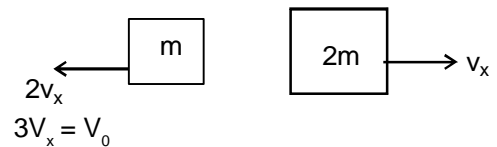
$$4g = 2g_p R_p ; \frac{GM_e}{R_e^2}(2) = \frac{GM_p}{R_p^2} \cdot R_p$$

$$\frac{\left(\frac{4}{3}\pi R_e^3\right)\rho(2)}{R_e^2} = \frac{\left(\frac{4}{3}\pi R_p^3\right)(2\rho)R_p}{R_p^2}$$

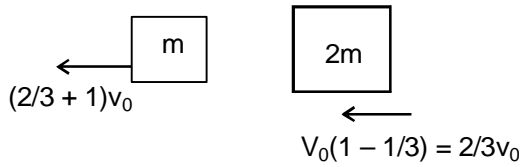
$$R_p = \sqrt{R_e} \Rightarrow R_p = \sqrt{6.4} \text{ Km}$$

7 w.r to COM of ship & pad

$$V_r = V_0$$



$$V_x = V_0/3$$



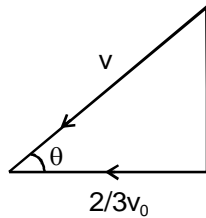
$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(2m)\left(\frac{2}{3}v_0\right)^2 + \left[\frac{-GM(2m)}{3R_m}\right] = \frac{1}{2}(2m)v^2 + \left[\frac{-GM(2m)}{R_m}\right] \quad \dots(1)$$

$$\frac{GM(m_2)}{R_0^2} = \frac{(2m)v_0^2}{R_0} \Rightarrow v_0 = \sqrt{\frac{GM}{R_0}} = \sqrt{\frac{GM}{3R_m}} \quad \dots(2)$$

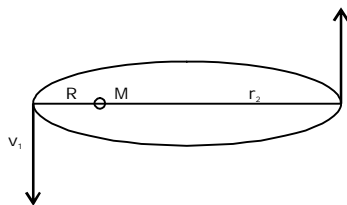
from (1) & (2)

$$\therefore v = \sqrt{\frac{40}{27}} \sqrt{\frac{GM}{R_m}}$$



$$\cos \theta = \frac{1}{\sqrt{10}} \quad \& \quad \sin \theta = \frac{3}{\sqrt{10}} \quad \text{Ans.}$$

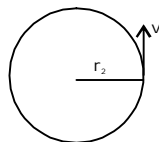
8



$$v_1 r_1 = v_2 r_2; \quad \sqrt{\frac{6GM}{5R}} \times R = v_2 r_2$$

from energy conservation

$$\frac{1}{2}m \frac{6GM}{5R} - \frac{GMm}{R} = \frac{-GMm}{r_2} + \frac{1}{2}mv_2^2$$



$$\frac{mv^2}{r_2} = \frac{GMm}{r_2^2} \quad v = \sqrt{\frac{GM}{r_2}}$$

9.

$$-\frac{GMm}{R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R+h} + \frac{1}{2}mv^2 \quad \dots(1)$$

$$v_0 = \sqrt{\frac{3GM}{2R}} \quad \dots(2)$$

(given)

conserved about centre of earth

$$\therefore m R v_0 \sin 60^\circ = m(R+h)v \quad \dots(3)$$

Solving these (3) equations

$$\text{we get } v = \frac{R}{R+h} \frac{\sqrt{3}}{2} v_0 \quad \& \quad h = R \left(1 + \frac{\sqrt{7}}{2}\right)$$

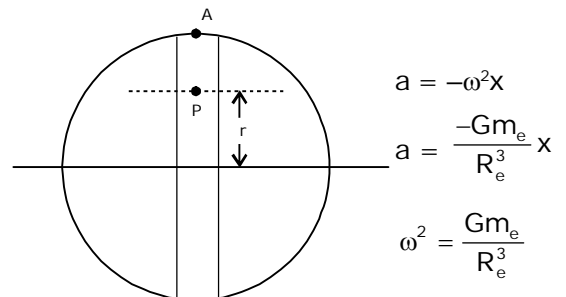
Ans. (i)

$$R_c = \frac{v^2}{a_\perp} = \frac{R^2}{(R+h)^2} \times \frac{3}{4} \times \frac{3GM}{2R} \times \frac{(R+h)^2}{GM}$$

$$R_c = \frac{9}{8}R$$

$$R_c = 1.13R \quad \text{Ans.}$$

10.



$$a = -\omega^2 x$$

$$a = \frac{-Gm_e}{R_e^3} x$$

$$\omega^2 = \frac{Gm_e}{R_e^3}$$

From W.E.T

$$Wg = kt - ki$$

$$-\int_r^R \frac{Gm_e m r}{R_e^3} dr = \frac{1}{2}mv_p^2 - \frac{1}{2}mv_A^2$$

$$\frac{1}{2}mv_p^2 = \frac{1}{2}mv_A^2 + \frac{Gm_e m}{R_e^3} (R^2 - r^2)$$

$$v_p^2 = v_A^2 + \omega^2 (R^2 - r^2)$$

$$\frac{-dr}{dt} = \sqrt{2gR_e + \omega^2 (R^2 - r^2)}$$

$$\left[v_A = \sqrt{2gR_e} \right]$$

$$t = \left(\sin^{-1} \frac{1}{\sqrt{3}} \right) \sqrt{\frac{R_e}{g}}$$

Exercise-IV

Level - I

1. (B)

When gravitational force becomes zero, then centripetal force required cannot be provided. So, the satellite will move with the velocity as it has at the instant when gravitational force becomes zero, i.e. it moves tangentially to the original orbit.

2. (A)

$$\text{Escape velocity} = \sqrt{2gR_e}$$

So, escape velocity is independent of m . So, it depends upon mass m as m^0 .

3. (C)

The minimum kinetic energy required to project a body of mass m from earth's surface to infinity is known as escape energy. Therefore

$$KE = \frac{GM_e m}{R} = mgR \quad \left(\text{as } gR = \frac{GM_e}{R} \right)$$

4. (D)

Gravitational potential energy of body will be

$$E = -\frac{GMm}{r}$$

Where M = mass of earth,
 m = mass of the body,
 R = radius of earth.

At $r = 2R$,

$$E_1 = -\frac{GMm}{(2R)}$$

At $r = 3R$,

$$E_2 = -\frac{GMm}{(3R)}$$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

$$\Delta E = \frac{GMm}{R} \left[\frac{1}{2} - \frac{1}{3} \right] = \frac{GMm}{6R}$$

5. (C)

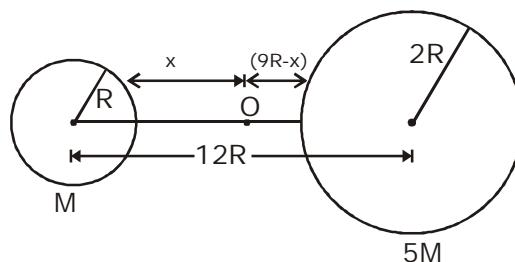
The escape velocity is independent of angle of projection, hence, it will remain same i.e. 11 km/s.

6. (C)

Let at 'O' there will be a collision. If smaller sphere moves x distance to reach at O, then bigger sphere will move a distance of $(9R-x)$.

$$F = \frac{GM \times 5M}{(12R - x)^2}$$

$$a_{\text{small}} = \frac{F}{M} = \frac{G \times 5M}{(12R - x)^2}$$



$$a_{\text{big}} = \frac{F}{5M} = \frac{GM}{(12R - x)^2}$$

$$x = \frac{1}{2} a_{\text{small}} t^2 = \frac{1}{2} \frac{G \times 5M}{(12R - x)^2} t^2 \quad \dots (i)$$

$$(9R - x) = \frac{1}{2} a_{\text{big}} t^2 = \frac{1}{2} \frac{GM}{(12R - x)^2} t^2 \quad \dots (ii)$$

Thus, dividing Eq. (i) by Eq. (ii) we get

$$\frac{x}{9R - x} = 5$$

$$\text{or } x = 45R - 5x$$

$$\text{or } 6x = 45R$$

$$\text{or } x = 7.5R$$

7. (C)

According to Kepler's law

$$T^2 \propto r^3$$

$$\Rightarrow 5^2 \propto r^3 \quad \dots (i)$$

$$\text{and } (T')^2 \propto (4r)^3 \quad \dots (ii)$$

From Eqs. (i) and (ii), we have

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

$$\text{or } T' = \sqrt{1600}$$

$$\text{or } T' = 40 \text{ h}$$

8. (A)

The necessary centripetal force required for a planet to move round the sun = gravitational force exerted on it

$$\text{ie, } \frac{mv^2}{R} = \frac{GM_e m}{R^n}$$

$$\text{or } v = \left(\frac{GM_e}{R^{n-1}} \right)^{1/2}$$

$$\text{Now, } T = \frac{2\pi R}{V}$$

$$= 2\pi R \times \left(\frac{R^{n-1}}{GM_e} \right)^{1/2}$$

$$= 2\pi \left(\frac{R^2 \times R^{n-1}}{GM_e} \right)^{1/2}$$

$$= 2\pi \left(\frac{R^{(n+1)/2}}{(GM_e)^{1/2}} \right)$$

$$\text{or } T \propto R^{(n+1)/2}$$

9. (B)

Gravitational potential energy of body on earth's surface

$$U = -\frac{GM_e m}{R}$$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_e m}{(R+h)} = -\frac{GM_e m}{2R} \quad (\text{as } h = R)$$

Where M_e = mass of earth,
 m = mass of body,
 R = Radius of earth

∴ Gain in potential energy = $U_h - U$

$$= -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R} \right)$$

$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R}$$

$$= \frac{GM_e m}{2R} = \frac{gR^2 m}{2R} \quad \left(\text{as } g = \frac{GM_e}{R^2} \right)$$

$$= \frac{1}{2} mgR$$

10. (A)

Time period of satellite

$$T = 2\pi \sqrt{\frac{(R+h)^3}{GM_e}}$$

where $R + h$ = orbital radius of satellite,
 M_e = mass of earth.

Thus, time period does not depend on mass of satellite.

11. (D)

The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

Where M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so,

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_0^2}{(R+x)}$$

(where v_0 is orbital speed of satellite)

$$\Rightarrow \frac{GM_e m}{(R+x)} = mv_0^2$$

$$\text{or } \frac{gR^2 m}{(R+x)} = mv_0^2 \quad \left(\because g = \frac{GM_e}{R^2} \right)$$

$$\text{or } v_0 = \sqrt{\left[\frac{gR^2}{(R+x)} \right]} = \left[\frac{gR^2}{R+x} \right]^{1/2}$$

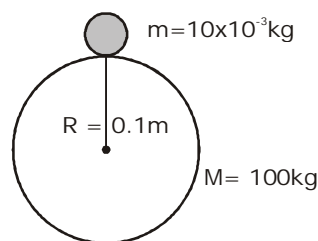
12. (D)

$$U_i = -\frac{GMm}{r}$$

$$U_i = -\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1}$$

$$U_i = -\frac{6.67 \times 10^{-11}}{0.1}$$

$$= -6.67 \times 10^{-10} \text{ J}$$



We know that

$$W = \Delta U$$

$$= U_f - U_i \quad (\because U_f = 0)$$

$$\Rightarrow W = -U_i = 6.67 \times 10^{-10} \text{ J}$$

13. (C)

$$g_h = g \left(1 - \frac{2h}{R} \right) \quad \dots (i)$$

$$g_d = g \left(1 - \frac{d}{h}\right) \quad \dots(ii)$$

As per statement of the problem,

ie, $g_h = g_d$

$$g \left(1 - \frac{2h}{R}\right) = g \left(1 - \frac{d}{R}\right)$$

$$\Rightarrow 2h = d$$

14. (C)

$$g = \frac{GM}{R^2}; \quad M = \left(\frac{4}{3} \pi R^3\right) \rho$$

$$\therefore g = \frac{4G}{3} \frac{\pi R^3}{R^2} \rho$$

$$\text{or } g = \left(\frac{4G\pi R}{3}\right) \rho \quad (\rho = \text{average density})$$

$$\Rightarrow g \propto \rho \quad \text{or} \quad \rho \propto g$$

15. (C)

According to Millikan's oil drop experiment, electronic charge is given by,

$$q = \frac{6\pi\eta r(v_1 + v_2)}{E}$$

Which is independent of g .

$$\text{So, } \frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$$

16. (A)

Correct option is (a) you can make an analogy with Gauss's law in electrostatics.

17. (A)

Mass of planet, $M_p = 10M_e$, where M_e is mass of earth.

Radius of planet, $R_p = \frac{R_e}{10}$, Where R_e is radius of earth.

$$\text{Escape speed is given by, } v_p = \sqrt{\frac{2GM}{R}}$$

$$\text{For planet, } v_p = \sqrt{\frac{2G \times M_p}{R_p}}$$

$$= \sqrt{\frac{100 \times 2GM_e}{R_e}}$$

$$= 10 \times v_e$$

$$= 10 \times 11 = 110 \text{ Km s}^{-1}$$

18. (A)

$g' = \frac{Gm}{(R+h)^2}$, acceleration due to gravity at height h

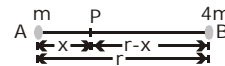
$$\Rightarrow \frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+h)^2} = g \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$$\Rightarrow 3R = R+h \Rightarrow 2R = h$$

19. (C)

Let gravitation field is zero at P as shown in figure.



$$\therefore \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\Rightarrow 4x^2 = (r-x)^2$$

$$\Rightarrow 2x = r-x$$

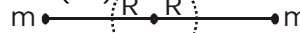
$$\Rightarrow x = \frac{r}{3}$$

$$\therefore V_p = -\frac{Gm}{x} - \frac{G(4m)}{r-x} = -\frac{3Gm}{r} - \frac{6Gm}{r} = -\frac{9Gm}{r}$$

20. (B)

Gravitational force provides necessary centripetal force,

$$\text{ie, } \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$$



$$\Rightarrow v = \sqrt{\frac{Gm}{4R}}$$

21. (D)

Potential energy on earth surface is $-mgR$ while in free space it is zero. So, to free the spaceship, minimum required energy is

$$K = mgR = 10^3 \times 10 \times 6400 \times 10^3 \text{ J} = 6.4 \times 10^{10} \text{ J}$$

22. (C)

Energy = $\Delta u + KE$

$$= \left[\frac{GMm}{R} - \frac{GMm}{3R} \right] + \frac{1}{2} M \left(\frac{Gm}{3R} \right)$$

$$= \frac{5}{6} \frac{GMm}{R}$$

Exercise-IV

Level - II

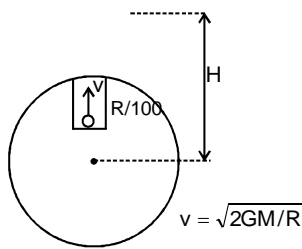
1. Gravitational force is conservative so work done only depends on position not a path taken

2. from E.C.

$$-\frac{GMm}{2R^3} \left[3R^2 - \left(\frac{99}{100} \right) R^2 \right] + \left[\left(\frac{1}{2} \right) m \left(\frac{2GM}{R} \right) \right]$$

$$= -\frac{GMm}{H}$$

after solving $H = 100.5R$

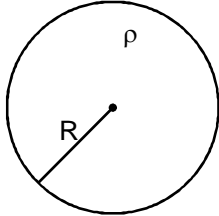


Height from the surface $H = H - R$
 $H' = 99.5R$

3. In theory $T_A = T_B$

4. for $r < R$

$$\frac{GMr}{R^3} = \frac{mv^2}{r}$$



$v \propto r$

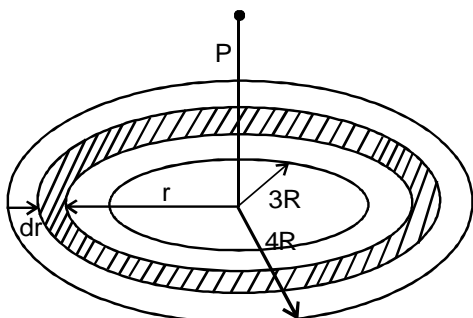
for $r \geq R \Rightarrow \frac{GM}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 \propto \frac{1}{r}$

5. Astronaut feel weight lessness when only gravitational field is act

6. $\sigma = \frac{M}{\pi(4R)^2 - \pi(3R)^2} = \frac{M}{7\pi R^2}$

$dm = \sigma 2\pi r dr$

$$U_p = -\int_{3R}^{4R} \frac{Gdm(1)}{(16R^2 + r^2)^{1/2}} = -\int_{3R}^{4R} \frac{2\pi G\sigma r dr}{(16R^2 + r^2)^{1/2}}$$



after solving $U_p = -\frac{2GM(4\sqrt{2} - 5)}{7R}$

work done by external agent $= U_\infty - U_p$
 $= \frac{2GM(4\sqrt{2} - 5)}{7R}$

7. $\frac{L}{L_B} = \frac{m_A v_A r_A + m_B v_B r_B}{m_B v_B r_B}$
 $= \frac{m_A v_A r_A}{m_B v_B r_B} + 1 = \frac{m_B}{m_A} + 1 = 6$

8. Given $\frac{GM_p}{R_p^2} = \frac{\sqrt{6} GM_e}{11 R_e^2} \Rightarrow \frac{M_p R_e^2}{M_e R_p^2} = \frac{\sqrt{6}}{11}$
(1)

$\rho_p = \frac{2}{3} \rho_e$

$\frac{M_p}{4/3 \pi R_p^3} = \frac{2}{3} \frac{M_e}{4/3 \pi R_e^3} \Rightarrow \frac{M_p R_e^3}{M_e R_p^3} = \frac{2}{3}$
(2)

$\sqrt{\frac{2GM_e}{R_e}} = 11 ; \sqrt{\frac{2GM_p}{R_p}} = v_{esc}$

$\Rightarrow \frac{11}{v_{esc}} = \sqrt{\frac{R_p M_e}{R_e M_p}}$
(3)

from (1), (2), (3) $v_{sc} = 3 \text{ km/sec}$

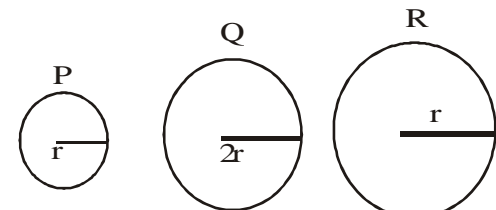
9. $\frac{GM_e M_s}{r^2} = \frac{m_s v^2}{r} \Rightarrow \frac{G M_e}{r} = v^2$

Object escape when its P.E equal to its

K.E. \Rightarrow K.E. $= \frac{G M_e m_0}{r}$

K.E. $= m_0 v^2 = mv^2$

10. B,D



$M_p = M$

$M_Q = 8M$

$M_R = 9M$

Radius of R will be slightly larger than 2r,

$V = \sqrt{2gR} = \sqrt{\frac{2GM}{R}} \propto R$

Hence $V_R > V_Q > V_P$

Also $\frac{V_p}{V_Q} = \frac{1}{2}$.

11. (B)

From energy conservation

$$\frac{GMm}{L} + \frac{GMm}{L} + \frac{GM^2}{2L} = \frac{1}{2}mv^2 + \frac{GM^2}{2L}$$

$$\frac{2GMm}{L} = \frac{1}{2}mv^2$$

$$v = 2\sqrt{\frac{GM}{L}}$$

$$E_i = \frac{1}{2}mv^2 \quad E_f = 0.$$

In one particle potential energy should not be considered.

Energy of m is not constant.

