

JEE-MAIN

TOPIC

ERROR IN MEASUREMENT

# SOLUTIONS

## ERROR IN MEASUREMENT

### Exercise-I

1. (C)

M.S.D. = 1 mm

L.C. = 0.02 cm  
= 0.2 mm

L.C. = M.S.D. - V.S.D.

= 1 mm -  $\frac{m}{n}$  = 0.02

$\Rightarrow \frac{m}{n} = 0.08$

2. (A)

3. (A, B, C)

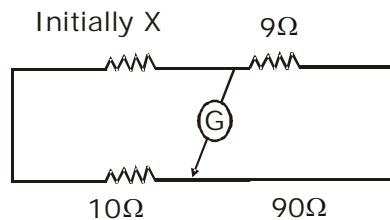
4. (A)

High Resistivity for low current flow & low Heating And low temp coef . so that Resistance doesn't vary with temperature

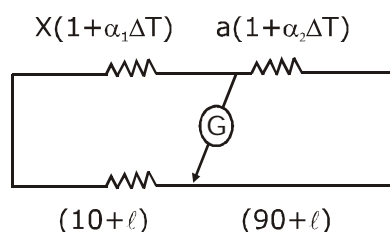
5. (A)

$x(90) = 9(10)$

$x = 1\Omega$



In second condition



Now  $x(1 + \alpha_1 \Delta T)(90 - \ell) = 9(1 + \alpha_2 \Delta T)(10 + \ell)$

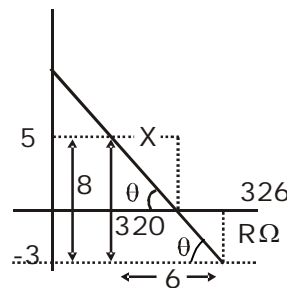
$\frac{(1 + \alpha_1 \Delta T)}{(1 + \alpha_2 \Delta T)} = \frac{9(10 + \ell)}{(90 - \ell)}$

$1 + (\alpha_1 - \alpha_2)\Delta T = \left[1 + \frac{\ell}{10}\right] \left[1 + \frac{\ell}{90}\right]$

$9(\alpha_1 - \alpha_2)\Delta T = \ell$

6. (B)

deflection



$\tan \theta = 8/6 = \frac{4}{3}$

$\theta = 53^\circ$

$\approx \tan \theta = 5/x \Rightarrow x = 3.75$

So  $R = 320 + x = 323.75 = 3.24\Omega$

7. (D)

ensures zero Magnetic field.

Exercise-II

1. (A) 3, (B) 3, (C) 4, (D) 4, (E) 3, (F) 5, (G) 2, (H) 4

Notes

2. (A)

$$\begin{array}{r} 703.00 \\ + 7.00 \\ + 0.66 \\ \hline 710.66 \end{array}$$

$\approx 711$

(B)

$$2.21 \times 0.3 = 0.663 = 0.7$$

(C)  $12.4 \times 84 = 1041.6 = 1.0 \times 10^3$

(D)  $14.28 / 0.714 = 20 = 20.0$

6.50

3. (i)  $\frac{-6.32}{0.18}$

$\approx 0.2$

$\sqrt{0.2} = 0.4$

(ii)  $\frac{2.91 \times 0.3842}{0.080}$

$= 13.975 = 14$

4. L.C. = M.S.D. - V.S.D.

$$= 1\text{mm} - \frac{9}{10}\text{mm} = 0.1\text{mm}$$

ZERO ERROR

Z.E. = 0.7mm

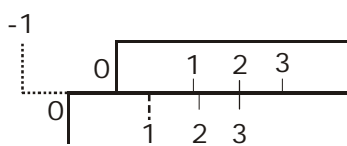
Reading  $\rightarrow$

$(\text{M.S.R.} + n \times \text{L.C.}) - \text{Z.E.}$

$= (31\text{mm} + 4 \times 0.1) - 0.7$

$= 31.4 - 0.7 = 30.7$

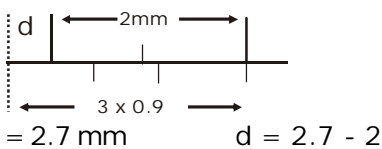
5. (i)



M.S.R. + V.S.R.  $\times$  L.C.

$-1 + 3 \times 0.1 = -0.7$

OR

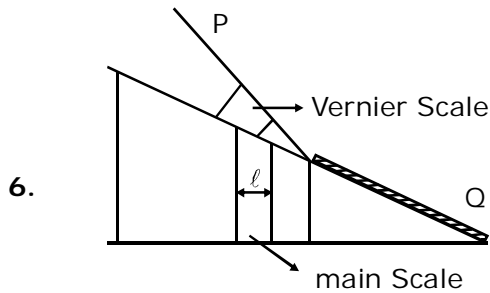
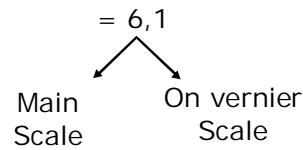


(ii)

ACTUAL READING

$= \text{READING} - \text{Z.E.}$

$= 5.4 - (-0.7)$



6.

I.M.S.D. =  $\frac{l}{\cos \theta}$

I.V.S.D. =  $l$

L.C. =  $-l + \frac{l}{\cos \theta}$

$= l \frac{(-\cos \theta + 1)}{\cos \theta}$

7.

L.C. =  $\frac{0.5\text{mm}}{50\text{division}}$

$= 0.01\text{mm}$

Reading = L.S.R.

+ L.C.  $\times$  V.S.R.

$= 5 \times 0.5 + 34 \times 0.01$

$= 2.5 + 0.34$

$= 2.84\text{mm}$

8.

L.C. =  $\frac{1\text{mm}}{50\text{div}} = 0.02\text{mm}$

Z.E. = L.S.R. + L.C.  $\times$  VSR

$= -1 + 0.02 \times 44$

$= -1 + 0.88$

$= -0.12$

Now Reading

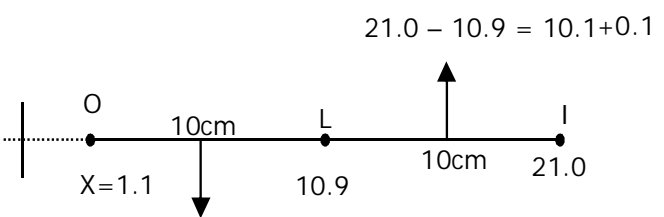
$= \text{L.S.R.} + \text{L.C.} \times \text{V.S.R.} - \text{Z.E.}$

$= 3 + 0.02 \times 26 + 0.12$

$= 3.64\text{mm}$

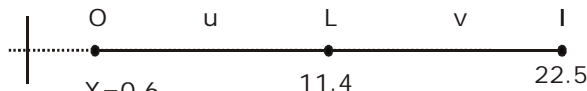
9.

Ist data



$10.9 - 1.1 = 9.8$

error =  $-0.2$



$$U = (11.4 - 0.6) + 0.2 = 11 \text{ cm}$$

$$V = 22.5 - 11.4 - 0.1 = 11 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow f = 5.5 \text{ cm}$$

$$\Delta f = f^2 \left( \frac{\Delta V}{V^2} + \frac{\Delta U}{U^2} \right)$$

$$= 5.5 \times 5.5 \left( \frac{0.1}{11^2} + \frac{0.1}{11^2} \right) = 0.05 \text{ cm}$$

10. (CD, AB, EF)

11. av.  $V = \frac{13.8}{4.0} = 3.45 = 3.4 \text{ m/s}$

$$\frac{\Delta V}{V} = \frac{\Delta S}{S} + \frac{\Delta t}{t}$$

$$\Rightarrow \frac{\Delta V}{3.4} = \frac{0.2}{13.8} + \frac{0.3}{4.0} \Rightarrow \Delta V = 0.3$$

So  $V = 3.4 \pm 0.3$

12.  $S = (1.20 \pm 0.18) \text{ cm}$

13. (i) series

av.  $R = 5 + 10 = 15 \Omega$

$$\Delta R = \Delta R_1 + \Delta R_2 = 0.3 \Omega$$

So

$$R = 15 \pm 0.3$$

OR  $\frac{0.3}{15} \times 100 = 2\% \Rightarrow R = 15 \pm 2\%$

(ii) av.  $R = \frac{R_1 R_2}{R_1 + R_2}$

So  $R = \frac{10}{3} = 3.3$

Now  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

So  $\Delta R = 0.1$

In % age  $\frac{\Delta R}{R} \times 100 = \frac{0.1}{3.3} = 3\%$

14.  $\rho = \frac{m}{v} = \frac{5.74}{1.2} = 4.78 = 4.8 \text{ (Rounding)}$

15.  $T = 2\pi\sqrt{l/g}$

$$g = \frac{4\pi^2 l}{T^2}; \quad 100 \frac{\Delta g}{g} = 100 \frac{\Delta l}{l} + 200 \frac{\Delta T}{T}$$

$$100 \times \frac{0.1}{10} + 200 \times \frac{0.02}{0.5}$$

$1 + 8 = 9\%$

16.

$$P = \frac{4\pi^2 A^3 B^2}{\sqrt{C} \times D}$$

$$100 \frac{\Delta P}{P} = \left( 3 \frac{\Delta A}{A} \right) \times 100 + \left( \frac{2\Delta B}{B} \right) \times 100$$

$$+ \frac{1}{2} \frac{\Delta C}{C} \times 100 + 100 \frac{\Delta D}{D}$$

$$= 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 2\% + 4\%$$

$$= 3 + 6 + 1 + 4 = 14\%$$

$$\Delta P = \frac{14P}{100} = \frac{14 \times 3.73}{100}$$

$$= 0.52$$

17.

$$\mu = \frac{\sin\left(\frac{A + S_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}, \quad \mu = \frac{\sin 45}{\sin 30} = \sqrt{2}$$

$$\ln \mu = \ln \sin\left(\frac{A + \delta m}{2}\right) - \ln \sin\left(\frac{A}{2}\right)$$

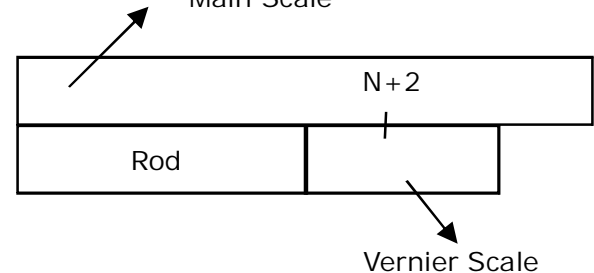
$$\frac{\Delta \mu}{\mu} = \frac{\cos\left(\frac{A + \delta m}{2}\right)}{\sin\left(\frac{A + \delta m}{2}\right)} \times \Delta\left(\frac{A + \delta m}{2}\right)$$

$$= \tan\left(\frac{A + \delta m}{2}\right) \times \frac{1}{2} \Delta(\delta m)$$

$$= \tan 45^\circ \times \frac{1}{2} \times \frac{\pi}{180} \text{ rad} \times 100\% = \frac{5\pi}{18} \%$$

18.

Let M.S.D. = 1 mm  $\hat{A}$   
Main Scale



Now  $(N+2) \times 1 \text{ mm} = l \text{ (Rod)} + 2 \times 0.9$

$$\Rightarrow N + 2 = l + 1.8$$

Now change for  $(N+2)$  mm on M.S. is equal to change of 1.8 mm on V.S.

$$\Rightarrow \alpha_1 (N+2) \Delta T \Rightarrow \alpha_2 (1.8) \Delta T$$

$$\Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{1.8}{N+2}$$

Exercise-III

1. (A)

$$a = 1.2 \times 10^{-2}$$

$$V = a^3 = (1.2)^3 \times 10^{-6}$$

$$= 1.728 \times 10^{-6}$$

$$= 1.7 \times 10^{-6}$$

2.  $a/(n+1)$

L.C. = M.S.D. - V.S.D.

$$= a - \frac{an}{n+1} = \frac{a(n+1-n)}{n+1}$$

$$= \frac{a}{n+1}$$

3. (D)

$$\rho = \frac{m}{v} = \frac{m}{\pi r^2 l}$$

$$= 100 \frac{\Delta \rho}{\rho} = 100 \frac{\Delta m}{m} + 200 \frac{\Delta r}{r} + 100 \frac{\Delta l}{l}$$

$$= 100 \times \frac{0.003}{0.3} + 200 \times \frac{0.005}{0.5} + \frac{100 \times 0.06}{6}$$

$$= 1 + 2 + 1 = 4\%$$

4.  $d = 0.050 \text{ cm} \pm 0.001 \text{ cm}$

$$F = 50 \text{ N}$$

$$l = 0.125 \text{ cm} \pm 0.001 \text{ cm}$$

$$L = 110.0 \text{ cm} \pm 0.1 \text{ cm}$$

$$\text{Now } \frac{F}{A} = \frac{Yl}{L}$$

$$\Rightarrow Y = \frac{Fl}{Al} = \frac{50 \times 1.10}{\frac{3.14}{4} (5 \times 10^{-4})^2}$$

$$Y = 2.24 \times 10^{11} \text{ N/m}^2$$

Now

$$Y = \frac{FL}{\pi d^2 l}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2 \frac{\Delta d}{d} + \frac{\Delta l}{l}$$

$$= \frac{1}{1100} + \frac{2}{50} + \frac{1}{125}$$

$$\Delta Y = Y \left( \frac{1}{1100} + \frac{1}{25} + \frac{1}{125} \right)$$

$$= 6.47 \times 10^9 \text{ N/m}^2$$

5.  $L.C. = \frac{1 \text{ mm}}{100 \text{ div}} = 0.01 \text{ mm}$

Reading = L.S.R. + C.S.R. x L.C.

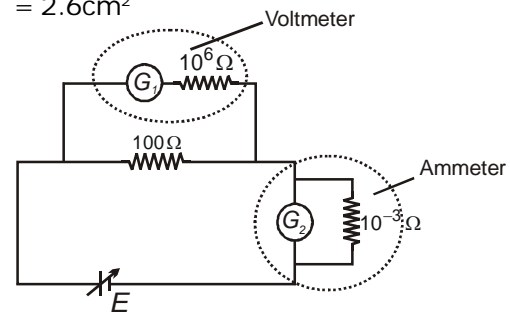
$$= 1 \text{ mm} + 47 \times 0.01$$

$$= 1.47 \text{ mm}$$

$$C.S.R. = 2\pi r l = \pi d l$$

$$= 3.14 \times \frac{1.47}{10} \times 5.6$$

$$= 2.6 \text{ cm}^2$$



6.

7. (D)

$$V = f\lambda = 2f(l_2 - l_1)$$

$$\Delta V = 2f(\Delta l_2 + \Delta l_1)$$

$$= 2 \times 512 (0.1 + 0.1)$$

$$= 512 \times 0.4$$

$$= 204.8 \text{ cm/s}$$

8. L.C. = 0.1 mm

$$\text{Reading} = 10 + 1 \times 0.1$$

$$= 10.1 \text{ mm}$$

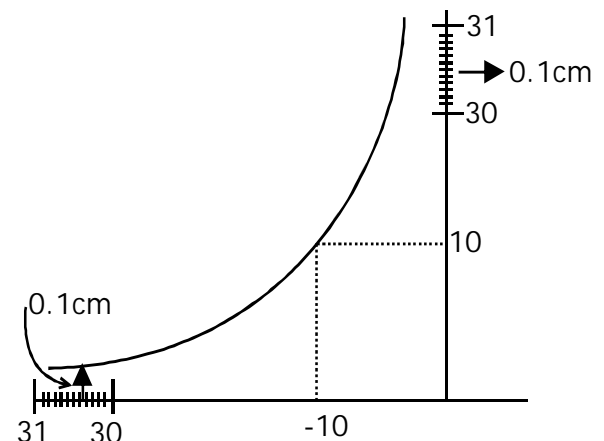
$$\rho = \frac{m}{v} = \frac{2.736}{(10.1)^3}$$

$$= \frac{2.736 \text{ gm}}{1.03 \text{ cm}^3}$$

$$= 2.6563 \text{ g/cm}^3$$

$$= 2.66 \text{ g/cm}^3$$

9. (C)



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$$

$$f = 5 \text{ cm}$$

$$\text{L.C. } 0.1 \text{ cm}$$

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Rightarrow \Delta f = 25 \left( \frac{0.1}{100} + \frac{0.1}{100} \right) = 0.05$$

10. (C)

$$\text{L.C.} = \frac{0.5 \text{ mm}}{50 \text{ division}} = 0.01 \text{ mm}$$

$$\text{Z.E.} = 0 + 5 \times 0.01$$

$$= 0.05$$

$$\text{Reading} = 2 \times 0.5 + 25 \times 0.01 - \text{Z.E.}$$

$$= 1 + 0.25 - 0.05$$

$$= 1.20$$

11. (D)

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

$\Delta l$  is least in (D) (i.e 1mm)  
and

$$\frac{\Delta T}{T} = \frac{0.1}{50} \text{ is also least in D}$$

So least error is in D.

We can also see that amplitude is also less so it is more closes to S.H.M.

12. (B)

13. (A) P, Q; (B) R, S; (C) R, S; (D) R, S

14. (B)

$$\frac{F}{A} = Y \frac{l}{L}$$

$$\Rightarrow Y = \frac{FL}{Al} = \frac{mgL}{Al}$$

$$Y = \frac{1 \times 9.8 \times 2}{\frac{\pi \times d^2}{4} + 0.4 \times 10^{-3}}$$

$$= 2 \times 10^{11} \text{ N/m}^2$$

$$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + 2 \frac{\Delta d}{d}$$

$$= \frac{0.05}{0.8} + 2 \times \frac{0.01}{0.4}$$

$$= \frac{0.09}{0.8} = \frac{9}{80}$$

$$\Delta Y = \frac{9Y}{80} = \frac{9 \times 2}{80} \times 10^{11} = 0.225 \times 10^{11}$$

$$\Rightarrow Y = (2 \pm 0.2) \times 10^{11}$$

15. (B)

$$g = \frac{4\pi^2 l}{T^2}$$

$$E = \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

$$E_I = \frac{0.1}{64.0} + \frac{2 \times 0.1}{128}$$

$$E_{II} = \frac{0.1}{64.0} + 2 \times \frac{0.1}{64}$$

$$E_{III} = \frac{0.1}{20} + 2 \times \frac{0.1}{36}$$

Clearly  $E_I < E_2 < E_3$ .

16. (C,D)

17. (A,C)

18. (D)

19. (C)

$$20. Y = \frac{4MgL}{\pi \ell d^2}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{\Delta \ell}{\ell} + \frac{2\Delta d}{d}$$

$$\Delta \ell = \Delta d \text{ (same instrment)}$$

$$\text{so } \frac{\Delta \ell}{\ell} = \frac{2\Delta d}{d} \text{ hence both contribute same}$$

in error

21. (B)

$$\text{L.C.} = \left( 0.05 - \frac{2.45}{50} \right) \text{ cm} = 10^{-3} \text{ cm.}$$

$$\text{Reading} = 5.10 \text{ cm} + 24 \times 10^{-3} \text{ cm.}$$

$$= 5.124 \text{ cm.}$$