JEE-MAIN

SOLUTIONS

TOPIC

ELASTICITY

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Exercise-I

1. (C)

Given L = 1 mm, $\Delta L = 6 \times 10^{-5}$ mm

$\alpha = 12 \times 10^{-6}\ k^{-1}$

then

$\Delta L = L\alpha\Delta T$

6×10^{-5} mm = (1mm) $(12 \times 10^{-6})\ \Delta T$

$\Delta T = 5°C$

2. (A)

Given

L = 25 cm, A = 0.8×10^{-4} cm^2

$\Delta T = 10°C$, $\alpha = 10^{-5}\ °C^{-1}$, $Y = 2\times10^{10} N^2$

then

$$\frac{\Delta L}{L} = \alpha\Delta T = \frac{F}{A\gamma}$$

$F = \alpha A\ \gamma\Delta t$

$= (10^{-5})(0{\cdot}8 \times 10^{-4}) \times (2 \times 10^{10}) \times 10$

= 160 N

3. (C)

$L_1 = L + L\alpha_1\ \Delta t$

$L_2 = L + L\alpha_2\ \Delta t$

$$\frac{\text{Stress}_1}{\text{Stress}_2} = \frac{Y_1 L\alpha_1\Delta t}{L}\cdot\frac{L}{Y_2 L\alpha_2\Delta t}$$

$$1 = \frac{2}{3}\frac{Y_1}{Y_2} \quad\Rightarrow\quad \frac{Y_1}{Y_2} = \frac{3}{2}$$

4. (C)

$I = CMR^2$

$dI = 2CMRdR = 2CMR\ [R\alpha\Delta T]$

$= 2\alpha I\Delta T$

5. (B)

$$F = AY\frac{\Delta L}{L} = AY\alpha\Delta T$$

$$f = k\sqrt{\frac{F}{\mu}} = K\sqrt{\frac{AY\alpha\Delta T}{\rho A}}$$

$$\Rightarrow f\ \alpha\ \sqrt{\frac{Y\alpha}{\rho}}$$

6. (B)

We know that

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times Y\ (\text{strain})^2\ \text{volume}$$

$$U = \frac{1}{2}\ Y\ (\alpha\Delta T)^2\ AL$$

$U \propto \Delta T^2$

$U \propto (t - 20)^2$

7. (B)

$$\frac{\Delta L}{L} = \alpha\Delta t = -\alpha 20$$

means read more so actual is less

8. (B)

Given L = 20 cm, ΔL_1 = 0.075 cm, ΔL_2 = 0.045 cm

$\Delta L = L\ \alpha\ \Delta T$

$0{\cdot}075 = 20\ \alpha_1\ (100)$

$0{\cdot}045 = 20\ \alpha_2\ (100)$

Let for third rod L_1 and $L_2 = 20 - L_1$

So $\Delta L_3 = \Delta L_1 + \Delta L_2$

$\Rightarrow 0{\cdot}06 = L_1\alpha_1\ 100 + (20 - L_1)\ \alpha_2\ 100$

L_1 = 10 cm.

9. (A)

Given

f = coafficient of cubical expansion

$\rho_{Spere} = \rho_l'$

$$\Rightarrow \frac{266.5}{\frac{4}{3}\pi\left(\frac{7}{2}\right)^3} = \frac{1.527}{1+35f}$$

$\Rightarrow f = 8.3 \times 10^{-4}/.c$

10. (B)

Given volume at 0°C = V_0,

cofficient of Linear expansion = a_g

cofficient of cubical expansion = γ_m

$$h = \frac{V_m' - V_b'}{A_0'} = \frac{V_0(1+\gamma\Delta T) - V_0(1+3a_g\Delta T)}{A_0(1+2a_g\Delta T)}$$

$$= \frac{V_0 T(\gamma - 3a_g)}{A_0(1+2a_g T)}$$

11. (B)

$$F = A\gamma\frac{\Delta L}{L(1+\alpha\Delta t)}$$

$$F = \frac{A\varepsilon\alpha t}{(1+\alpha t)}$$

12. (A)

At 0°C

then $\rho_l\, v_s\, g = W_0$ ...(1)

At t°C $\rho_l'\, v_s' g = W$ ...(2)

$\rho_s v_s = \rho_s'\, v_s' = m$ ...(3)

(2) – (1) $\Rightarrow (\rho_l'\, V_s' - \rho_l\, V_s)\, g = W - W_0$

$W = W_0 + (\rho_l\{1 - \gamma_l t\}\, v_s\, (1 + \gamma_s t) - \rho_l v_s) g$

$= W_0[1 - (\gamma_l - \gamma_s)t]$

13. (C)

Initially $P = \frac{V_b\rho_b}{A_c}$, $P' = \frac{V_b'\rho_b'}{A_c'}$

$$P' = \frac{V_b(1+10^{-3}\times 10)}{A_c(1+2\times 10^{-3}\times 10)} \times \frac{\rho_b}{(1+10^{-3}\times 10)}$$

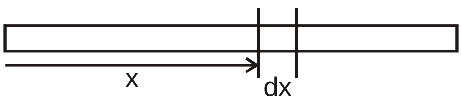
$$P' = \frac{P}{1+2\times 10^{-2}}$$

$$\left(\frac{P'}{P} - 1\right)\times 100 = \frac{1-(1+2\times 10^{-2})}{1}\times 100$$

$= -2\%$

14 (C)

$dx = \Delta dx$



$$\int_0^{\Delta L}\Delta dx = \int_0^L dx(3x+2)\times 10^{-6}(20-0)$$

$$\Delta L = (20\times 10^{-6})\left(\frac{3x^2}{2} + 2x\right)_0^L$$

$$\Delta L = (20\times 10^{-6})\left(\frac{3L^2}{2} + 2L\right) = 1{\cdot}2 \text{ cm}$$

$L_{new} = L + \Delta L$

15. (C)

Let eq^n. temp = t then

$m_R\, s_R\, t = m_s\, s_s\,(100 - t)$ ...(1)

$d_R' = d_R\,(1 + \alpha_R\, t)$ ...(2)

$d_s' = d_s[1 - \alpha_s\,(100 - t)]$ ...(3)

Now $d_R' = d_s'$ ...(4)

So. $d_R(1 + \alpha_R\, t) = d_s[1 - \alpha_s(100 - t)]$

$$t = \frac{d_s(1-\alpha_s 100) - d_R}{[d_R\,\alpha_R - d_s\,\alpha_s]}$$

Put the above value of t in eq. 1.

$$\left(\frac{m_R\, s_R}{m_S\, s_s} + 1\right)t = 100$$

$$\frac{m_s}{m_r} = \frac{23}{54}$$

16. (C)

$\alpha_x + \alpha_y$ for x – y plane

$\beta_{CDEH} = 3 \times 10^{-5}$ per °C

17. (D)

$\gamma_{oil} = \gamma_{vessel} \Rightarrow$ D.

Volume increases but mass remains same.

18. (C)

$\because\ \gamma_m < \gamma_{Al}$ $\qquad \rho_m >> \rho_{ac}$

$\Delta V_m < \Delta V_{al}$ $\qquad$ So completely Immersed

$\Delta\rho_m < \Delta\rho_{Al}$

So $W_2 > W_1$ [$\because$ Displaced mass of alchol is less]

19. (D)

Initially $\rho_s\, \rho_l$ and V density of sphere, density of liquid and volume.

$$\frac{B_T' - B_T}{B_T}\times 100 = \frac{V_s'\rho_l' g - V_s\rho_l g}{V_s\rho_l g} \times 100$$

$\Rightarrow [(1 + \gamma_s\,\Delta t)(1 - \gamma_l\,\Delta t) - 1] \times 100$

$= -0.05$ (decreases)

20. (B)

$$\frac{\Delta L}{L} \times 100 = 1 = 100\,\alpha\Delta t = 100\,\alpha(T_2 - T_1)$$

$$\frac{\Delta A}{A} \times 100 = 200\,\alpha\Delta t = 2\%$$

21. (C)

$\Delta L = \Delta L_1 + \Delta L_2$

$(3L)\alpha_{net}\,\Delta t = L\alpha\Delta t = (2L)\,(2\alpha)\,\Delta t$

$$\alpha_{net} = \frac{\alpha + 4\alpha}{3} = \frac{5\alpha}{3}$$

22. (B)

Given $\beta = 1.4 \times 10^{11}$ Pa, $\alpha = 1.7 \times 10^{-5}$°C^{-1}

$\Delta T = 30°C - 20°C = 10°C$

$$\beta = -\frac{\Delta P}{\Delta V / V} \Rightarrow \Delta P = -\beta\frac{\Delta V}{V}$$

$\Delta P = \beta(3\alpha\,\Delta T) = 1.4\times 10^{11}\times 3\times 1.7\times 10^{-5}\times 10$

$= 7.14 \times 10^7$ Pa.

23. (A)
At 40°C
1 Unit will be = $1(1 + \alpha_s \Delta t)$ units
= $1(1 + 12 \times 10^{-6} \times 40)$ Units
So 100 Unit will be = $100\ (1 + 12 \times 10^{-6} \times 40)$ = Actual
$100\ (1 + 40 \times 12 \times 10^{-6}) = l_0\ (1 + (2 \times 10^{-6})\ 40)$
$l_0 = 100\ [1 + 400 \times 10^{-6}] > 100$mm.

24. (C)
V_m denote volume of murcury
$V_{air} = V_{flask} - V_m = V'_{flask} - V_m'$
$V_{flask} - 300 = V_{flask}\ [1 + 3 \times (9 \times 10^{-6})\ \Delta t] - 300\ [1 + 8 \times 10^{-4}\ \Delta t]$

$$V_{flask} = \frac{(300 \times 1.8 \times 10^{-4})}{27 \times 10^{-6}\ \Delta t} \Delta t = 2000\ cm^3$$

25. (B)
Because floating

$$\rho_s Vg = \rho_\ell \left(\frac{V}{2}\right) g$$

$$2\rho_s = \rho_\ell$$

26. (A)
if $\gamma_L > \gamma_S$ then submerged more else come out of liquid respectively
and $\gamma_L > \gamma_s$ (always)

27. (A)
$V' = V[1 + \gamma_s \Delta t]$
$\rho_l' = \rho_l\ [1 - \gamma_l \Delta t]$

$$\rho_l \left(\frac{V}{2}\right) g = \rho_l' \left(\frac{V'}{2}\right) g$$

$$\rho_l \left(\frac{V}{2}\right) g = \rho_l (1 - \gamma_l \Delta t) \left(\frac{V}{2}\right) (1 + \gamma_s \Delta t) g$$

$(1 - \gamma_l \Delta t)\ (1 + \gamma_s \Delta t) = 1$
$(1 - \gamma_l \Delta t)\ (1 + 3\alpha_s \Delta t) = 1$
$3\alpha_s - \gamma_l = 0$

28. (A)
initially $\rho_l(A_s h)g = (\rho_s A_s h_o)g$ ...(1)
Now $\rho_l'(A_s' h)\ g = (\rho_s' A_s' h_o')g$ ...(2)
$\rho_l\ (1 - \gamma_l \Delta t)h = \rho_s(1 - 3\alpha_s \Delta t)h_o\ (1 + \alpha_s \Delta t)$
$\gamma_L = 2\alpha_s$

29. (A)

$$\rho_l' < r_s \text{ or } \frac{\rho_l}{2}$$

$$\frac{\rho_l}{1 + \gamma_l \Delta t} < \frac{\rho_l}{2}$$

$1 + \gamma_l \Delta t > 2$

$$\Delta t > \frac{1}{\gamma_l}$$

$$T_F - T > \frac{1}{\gamma_l} \Rightarrow T_F > T + \frac{1}{\gamma_l}$$

30. (C)

$$\left.\begin{array}{ll}\text{Given} & \gamma_l - \gamma_c = c \\ \text{and} & \gamma_l - \gamma_c = s\end{array}\right] \Rightarrow \gamma_s = c + \gamma_c - s = 3\alpha_s$$

$$\alpha_s = \frac{c + \gamma_c - s}{3}$$

31. (B)
ΔQ_0 = 100 × 4 × 60 = 24000 cal.
for 0°C = water
ΔQ_1 = (100 × 0.2 × 20) + (200 × 0.5 × 20) + (200 × 80)
= 18400 cal.
So let temp is t then.
24000 – 18400 = (200 × 1 + 100 × 0.2)t
t = 25.5°C

32. (C)
$\rho_{0°C}\ h_1\ g = \rho_{30°C}\ h_2\ g$
$\rho_0\ (120) = \rho_0\ (1 - \gamma 30)\ (124)$

$$\gamma = \left(1 - \frac{120}{124}\right)\frac{1}{30} = 11 \times 10^{-4}\ /°C$$

33. (A)

$$\frac{(212 - 37)°F}{(100 - 0)°C} \times 25°C = 45°F$$

34. (D)
at 0°C
$V_{0x} = 20A$
$V_{0y} = 30A$
Now at time T y read 120°C
So. $V'_{0y} = A(120) = 30A\ (1 + \gamma_m T)$
and $V'_{0x} = Ah = 20A\ (1 + \gamma_m T)$

Dividing $\frac{120}{h} = \frac{30}{20} \Rightarrow h = 80.$

MULTIPLE CHOICE QUESTION

35. (C, D)
for Adiabatic
PV^γ = const.

$$P \propto \frac{1}{V^\gamma}$$

PV = nRT

36. (B, C)
Strain → Same

Stress = $\frac{F}{A}$ = constant

$F \propto A$

$\Rightarrow \quad F \propto r^2$

Energy = $\frac{1}{2}$ stress × strain × volume

$\propto$ Area

$\propto r^2$

37. (A, C, D)

Gravitational Potential Energy U_G = Mgl

Elastic Potential Energy U_e =

$= \frac{1}{2}$ stress × strain × volume

$$= \frac{1}{2}\frac{F}{A}\frac{\ell}{\ell_0} \times V \quad \begin{vmatrix} F = mg \\ V = A\,\ell_0 \end{vmatrix}$$

$= \frac{1}{2}$ mgl

Heat Produced = U_e = $\frac{1}{2}$ Mgl

38. (A, C, D)

(A) % rise in area = $\beta\Delta T$

$= 2(\alpha\Delta T)$

$= 2 \times 0.2 = 0.4\%$

(C) % rise is volume = $3\,\alpha\Delta T$

$= 3 \times 0.2 = 0.6\%$

(D) $\alpha = \frac{0.2}{80 \times 100} = 0.25 \times 10^{-4}/°C$

Exercise-II

1. (i) $y_{air} = y_{material}$
So Hollow feel more pressure from inside and increase more due to air pressure

2 (a). $\frac{15\times60}{2}$ = 450 oscillation

(b). $T = k\sqrt{\ell}$

$T' = k\sqrt{\ell+\Delta\ell} = k\sqrt{\ell}\left(1-\frac{\Delta\ell}{\ell}\right)^{1/2}$

$= k\sqrt{\ell}\left(1+\frac{1}{2}\frac{\Delta\ell}{\ell}\right)$

$T' = T\left(1+\frac{1}{2}\frac{\Delta\ell}{\ell}\right)$

So T' time = 1 oscillation

$15 \times 60 \text{ sec} \quad = \frac{1}{T'} \times 15 \times 60$

$= \frac{15\times60}{2\left[1+\frac{1}{2}(2\times10^{5})\times(40-20)\right]}$

$= \boxed{449.9}$

(c). $\boxed{449.9}$ × 2 sec ahead

(d). (i) 2 sec extra = 1 oscillation extra
(ii) (450 + 1) Oscillation – 15 minutes

1 oscillation – $\frac{15\times60}{451}$ sec.

(d). (iii)

$dT = \frac{T}{2}\propto\Delta\theta$

$\left[2-\frac{15\times60}{451}\right] = (1)\ 2 \times 10^{-5} \times \Delta\theta$

$\Delta\theta = \frac{1}{451\times10^{-5}}$ sec

3. (a). $V_c' = Ah(1 + 3\propto_c\Delta t)$
(b). $Ah_1' = Ah[1 + y\Delta t]$
$h_1' = h[1 + y\Delta t]$
(c). (i) $y_L < 3\propto_c$
(ii) $y_L > 3\propto_c$
(iii) $y_L = 3\alpha_c$
(d). $\Delta V = V_l' - V_c' = Ah[1 + y_l\Delta\theta] - Ah[1-3\alpha_c\Delta\theta]$
$= Ah[y_l - 3\alpha_c]\Delta\theta$

3 (e). $A(h - h_1) = V_c' - V_l'$
$Ah - Ah_1 = Ah(1 + 3\alpha_c\Delta\theta) - Ah_1(1 + \gamma_L\Delta\theta)$
$0 = h\,3\alpha_c - h_1\gamma_L$
(f) (i). (i) $Ah_1 = A'h$

$h = \frac{Ah_1}{A'} = \frac{h_1}{(1+2\alpha_c\Delta\theta)}$

Now $h = h_c'[1 + \alpha_c\Delta\theta]$

So $\boxed{h_c' = \frac{h_1}{1+3\alpha_c\Delta\theta}}$

(f)(ii). $Ah_1 = V_0$
$V_0' = V_0[1 + \gamma_L\Delta\theta]$
$Ah_1' = Ah_1[1 + \gamma_L\Delta\theta]$
$h_1' = h_1[1 + \gamma_L\Delta\theta]$
(f)(iii)
(1) $\gamma_L > 3\alpha_c$
(2) $\gamma_L < 3\alpha_c$
(3) $\gamma_L = 3\alpha_c$

4. $\rho_l = \frac{156.25-56.25}{Vg}$ at 15°C

Now $\rho_l' = \frac{156.25-66.25}{V_g'}$

$\rho_l' = \rho[1 - \gamma_l\Delta t]$ at 52°C
$V_g' = V_g[1 + \gamma_g\Delta t]$

$\frac{156.25-66.25}{V_g(1+\gamma_g\Delta t)} = \frac{156.25-56.25}{V_g}[1 - \gamma_l\Delta t]$

$\Rightarrow \frac{90}{1+3\times9\times10^{-6}\times37} = 100[1 - \gamma_l \times 37]$

$\gamma_l =$

$\frac{1}{3700}\left[100-\frac{90}{1+37\times9\times3\times10^{-6}}\right]$

$\Rightarrow \gamma_l = 2.72 \times 10^{-3}/°C$

5 (a)
initially $\rho_l = \rho_b = d_o$
$F_{thrust} = \rho_l'(V_0')g$
$= d_o(1 - \gamma_l\Delta\theta)V_0(1 + 3\alpha_s\Delta\theta)g$
(b) (i) $3\alpha_s > y_L$
(ii) $3\alpha_s < y_L$
(iii) $3\alpha_s = y_L$

6. $T = k\sqrt{\ell}$

$dT = \frac{k}{2}\frac{d\ell}{\sqrt{\ell}} = \frac{k}{2}\sqrt{\ell}\alpha\Delta\theta$

$= \frac{T}{2}\alpha\Delta\theta$

In T + dT lag by = dT

In 10^6 slow by $= \frac{dT}{T+dT} \times 10^6$ sec

$= \frac{dT/T}{1+\frac{dT}{T}} \times 10^6$ Sec

$= \frac{\frac{1}{2}\alpha\Delta\theta}{\left(1+\frac{1}{2}\alpha\Delta\theta\right)} \times 10^6$ sec

$= \left(\frac{1}{2}\alpha\Delta\theta\right)\left(1-\frac{1}{2}\alpha\Delta\theta\right)\times 10^6$

$= \frac{1}{2}\alpha\,\Delta\theta \times 10^6$

$= \frac{1}{2} \times 10^{-6} \times (30 - 20) \times 10^6$

= 5 sec slow.

7. $F = Ay\frac{\Delta\ell}{\ell}$

$= Ay\,\alpha\,\Delta\theta$

$= 10^{-3} \times 10^{11} \times 10^{-6}\,(100 - 0°)$

= 10000 N

8. C.M $= y = \frac{h}{3}$ from 0

$dy = \frac{1}{3}dh$

$dy = \frac{1}{3}(h\,\alpha\,\Delta\theta)$

$\frac{dy}{d\theta} = \frac{1}{3}h\alpha = \frac{2}{3}(l\cos 30°)\alpha$

$= \left[\frac{1}{3}\times\frac{2\times\sqrt{3}}{2}\times 4\sqrt{3}\times 10^{-6}\right] m/°C$

$= 4 \times 10^{-6}$ m/°C

9. $\Delta L = \Delta L_1 + \Delta L_2$

$(3L)\alpha_0\,\Delta\theta = L\alpha_1\,\Delta\theta + 2L\alpha_2\,\Delta\theta$

$3\alpha_0 = (\alpha_1 + 2\alpha_2)$

$\alpha_0 = \frac{1}{3}(\alpha + 42) = \frac{5\alpha}{3}$

10. $g_{eff} = g_R\left(1-\frac{2h}{R}\right)$

$T = k\sqrt{\frac{\ell}{g_{eff\,h}}}$ at 20°C and height h

$T' = k\sqrt{\frac{\ell[1+\alpha(30-20)]}{g_{eff\,h}}}$ at 30° at height h

$T'' = k\sqrt{\frac{\ell(1+\alpha(30-20))}{g_R}}$

T = T"

$\sqrt{\frac{1}{1-\frac{2h}{R}}} = \sqrt{1+\alpha(30-20)}$

$\alpha = \left[\frac{2h}{R-2h}\right]\frac{1}{10}$ R >>>> h

So $\alpha = \frac{h}{5R}$

11. $y_M = 20\,\alpha_g$

$V - V_0 = V' - V_0'$

$V - V_0 = V(1 + 3\,\alpha_g\,\Delta\theta) - V_0(1 + \gamma_m\,\Delta\theta)$

$0 = 3\alpha_g\,V - \gamma_m\,V_0$

$0 = 3\alpha_g\,V - 20\,\alpha_g\,V_0 \Rightarrow \boxed{V_0 = \frac{3V}{20}}$

12. $\frac{\Delta\ell}{\ell} = \alpha_A\,\Delta\theta \Rightarrow \frac{0.05}{25} = \alpha_A\,(100)$

and $\frac{00.4}{40} = \alpha_B\,(100)$

$\Delta\ell_c = \Delta\ell_A = \Delta\ell_B$

$0.03 = \ell_A\,\alpha_A\,(50) + (50 - \ell_A)\,\alpha_B\,50$

$\ell_A = \frac{0.03-2500\alpha_B}{(\alpha_A-\alpha_B)50}$

= 10cm

13 (a) $\left.\begin{array}{l} R_x = 70-(-20) = 90° \\ R_y = (90-0) = 90° \\ R_w = (120-30) = 90° \end{array}\right]$ same

(b) 50°w < 50°y < 50°x

14. $\frac{212-32}{100-0} \times T°C = T°F - 32$

T = – 40

Exercise-III

1. (D)
Elastic energy stored in the wire is

$U = \frac{1}{2} \times$ stress × strain × volume

$= \frac{1}{2}\frac{F}{A}\times\frac{\Delta l}{L}\times AL$

$= \frac{1}{2}F\Delta l = \frac{1}{2}\times 200\times 1\times 10^{-3} = 0.1\,J$

2. (D)
Work done in stretching the wire
= potential energy stored

$= \frac{1}{2} \times$ stress × strain × volume

$= \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L}\times AL = \frac{1}{2}Fl$

3. (B)
Given S = Stress
Y = youngs modulus

We know that U = $\frac{1}{2}$ stress × strain × volume

Energy per unit volume

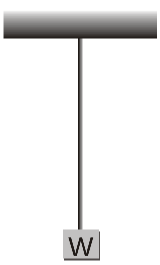
$= \frac{U}{Volume} = \frac{1}{2}$ stress × strain

(strain = $\frac{stress}{Y}$)

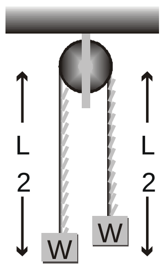
$\frac{U}{V} = \frac{1}{2}S\times\frac{S}{Y} = \frac{S^2}{2Y}$

4. (A)
Let us consider the length of wire as L and cross-sectional area A, the material of wire has Young's modulus as Y.

Case 1



Case 2



Then for 1st case $Y = \frac{w/A}{l/L}$

For 2nd case, $Y = \frac{w/A}{2l'/L}$

$\Rightarrow \quad l' = \frac{l}{2}$

So, total elongation of both sides = 2l' = l

5. (D)
$A_1 l_1 = A_2 l_2$

$\Rightarrow \quad l_2 = \frac{A_2 l_1}{A_1} = \frac{A\times l_1}{3A} = \frac{l_1}{3}$

$\Rightarrow \quad \frac{l_1}{l_2} = 3$

$\Delta x_1 = \frac{F_1}{A\gamma}\times l_1$ ...(i)

$\Delta x_2 = \frac{F_2}{3A\gamma} l_2$ ...(ii)

Here $\Delta x_1 = \Delta x_2$

$\frac{F_2}{3A\gamma} l_2 = \frac{F_1}{A\gamma} l_1$

$F_2 = 3F_1\times\frac{l_1}{l_2} = 3F_1\times 3 = 9F$

6. (C)

$\Delta L = \alpha L\Delta T = \frac{FL}{AY}$

$\Rightarrow$ Stress $= \frac{F}{A} = Y\alpha\Delta T$

7. (B)
Given d = 20 cm, t = 0°C to t = 100°C
$V = V_0(1+\gamma t)$
$V = V_0(1+3\alpha t)$ $(\gamma = 3\alpha)$
change in volume = $V - V_0$

$= 3V_0\,\alpha t$

$= 3\times\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times 23\times 10^{-6}\times 100$

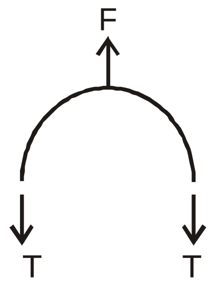
$= 3\times\frac{4}{3}\pi\left(\frac{0.2}{2}\right)^3\times 23\times 10^{-6}\times 100$

= 28.9 cc (1 cc = $10^{-6}m^3$)

8. Increase in length
$\Delta L = \alpha\Delta T$

$\frac{\Delta L}{L} = \alpha\Delta T$

Then thermal stress developed is



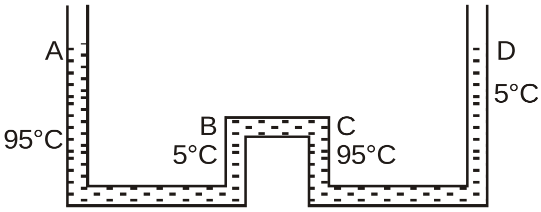
$\frac{T}{S} = \frac{\Delta L}{L} = Y\alpha\Delta T$

$T = SY\alpha\Delta T$
From FBD of one part of the wheel
$F = 2T$
Where F is force that one part of wheel is applies on the other part.
$F = 2SY\alpha\Delta T$

Exercise-IV

1. $\rho_t = \dfrac{\rho_0}{1+\gamma t}$

$\rho_0 + h_A\,\rho_{95}\,g - h\rho_5.g$
$= p_0 + h_B\,\rho_5.\,g - h\,\rho_{95}.\,g$



$$\frac{\rho_{95}}{\rho_5} = \frac{h_B + h}{h_A + h} \Rightarrow \frac{\dfrac{\rho_0}{1+95\gamma}}{\dfrac{\rho_0}{1+5\gamma}} = \frac{h_g + h}{h_A + h}$$

$\Rightarrow r = 2 \times 10^{-4}/°C, \quad \alpha = \dfrac{r}{3} = 6{\cdot}7 \times 10^{-5}/°C$

2. **(B, D)**

$\ell_0\,(1 + \alpha_B\,\Delta T) = (R + d)\theta$
$\ell_0\,(1 + \alpha_C\,\Delta T) = R\theta$

$$\frac{R+d}{d} = \frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T} \Rightarrow 1+\frac{d}{R} = 1 + (\alpha_B - \alpha_C)\,\Delta T$$

[Binomial Expansion]

$$R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}\,,\quad R \propto \frac{1}{\Delta T} \text{ and } R \propto \frac{1}{\alpha_B - \alpha_C}$$

3. **(A)**

$\ell_1\,\alpha_a\,t = \ell_2\,\alpha_s\,t$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{\alpha_s}{\alpha_a} \Rightarrow \frac{\ell_1}{\ell_1+\ell_2} = \frac{\alpha_s}{\alpha_a+\alpha_s}$$

4. $T\uparrow\ V_{cube}\uparrow\ \rho_\ell\downarrow$

Depth submerged in liquid remains same

Upthrust = Weight

$v_i\,\rho_L\,g = v_i'\,\rho_L'\,g$

$$(Ah_i)\,\rho_L\,g = A(1 + 2\alpha_s\,\Delta T)\,h_i\left(\frac{\rho_L}{1+\gamma_\ell\Delta T}\right)g$$

$\Rightarrow \quad \gamma_\ell = 2\alpha_s$

5. $\omega = \dfrac{k}{m}$ $k=0.1\,(140)^2$

$$Y = \frac{\dfrac{F}{A}}{\dfrac{\Delta L}{L}},$$

$$Y = \frac{0.1\times 140\times 140\times x\times 1\times 10}{4.9\times 10^{-7}\times x\times 10}$$

$= 4\times 10^9\,Nm^{-2}$ N=4