JEE-MAIN

TOPIC

C.M.
W.P & E.

SOLUTIONS

CIRCULAR MOTION, WORK, POWER & ENERGY

Exercise-I

(A) CIRCULAR MOTION

1. **(C)**

$\omega_i = 0$; $\omega_f = 80$ rad/sec

t = 5 sec

$\omega_f = \omega_i + \alpha t \Rightarrow \alpha = \frac{80}{5} = 16$ rad/sec^2

$\theta = \frac{1}{2}\alpha t^2 = 200$ rad

2. **(C)**

Given, $\omega_0 = 0$, t = 2 sec.

$\theta = 0$, next 2 sec., $\theta = O_2$

$\theta_1 = \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha 2^2 = 2\alpha$

$\theta_2 = \frac{1}{2}\alpha(2+2)^2 - \frac{1}{2}\alpha 2^2 = 6\alpha$

$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$

3. **(D)**

Given

a = 10m/sec^2 $\Rightarrow \alpha = 5$ rad / sec^2

$a = \alpha r$

$r = \frac{10}{5} = 2$ m

4. **(A)**

Given $\omega_0 = 0$, $\omega = 2\pi n = 2\pi \times \frac{210}{60}\frac{rad}{sec}$

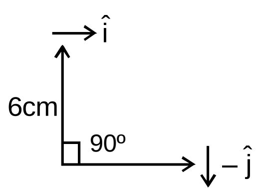
from t = 5

$\omega = \omega_0 + \alpha t$

$2\pi \times \frac{210}{60} = 0 + \alpha \times 5 \Rightarrow$

$\alpha = 1.4\pi \frac{rad}{sec^2}$

5. **(D)**



speed of the second hand

$v = \frac{2\pi r}{\text{time in one revolution}}$

$\Rightarrow v = \frac{2\pi 6}{60} = 2\pi mm/s$

Magnitude of difference of vel. = $v_i - (-v_j)$

$= v_i + v_j$; $v = \sqrt{v_i^2 + v_j^2} = \sqrt{8\pi^2} \Rightarrow 2\sqrt{2}\pi$

6. **(C)**

$\omega_1 = \frac{2\pi}{T_1}$, $\omega_2 = \frac{2\pi}{T_2}$

$\omega_1 : \omega_2 = T_2 : T_1$

$T_1 = 12 \times 60 \times 60$ sec.

$T_2 = 60$ sec.

$\omega_1 : \omega_2 = 60 : (12 \times 60 \times 60)$

$\omega_1 : \omega_2 = 1 : 720$

7. **(A)**

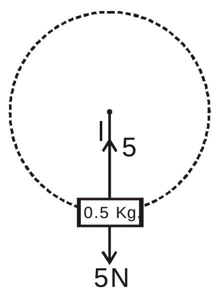
$\omega = \frac{2\pi}{t}$

where $t = 1\,\text{Day} = 24 \times 60 \times 60$ second

because earth complete one revolution is 24 hours about its own axis

$w = \left(\frac{2\pi}{60 \times 60 \times 24}\right)$ rad / s

8. **(B)**



Given l = 1m

u = 4 m/s

Tension provides necessary centripeters force so

$$T = m\omega^2 l = \frac{mv^2}{l} = \frac{0.5 \times 4^2}{1} = 8N$$

9. **(C)**

$\theta_A \propto t^2$ $\qquad \theta_B \propto t$

$\theta_A = k_1 t^2$ $\qquad \theta_B = k_2 t$

From given conditon calculate k_1 and k_2

$2\pi = k_1 \times \pi$ $\qquad \pi = k_2 \times 4\pi$

$k_1 = 2$ $\qquad k_2 = 1/4$

$\theta_A = 2t^2$ $\qquad \theta_B = t/4$

$w_A = \frac{d\theta_A}{dt} = 4t$ $\qquad w_B = \frac{d\theta_B}{dt} = \frac{1}{4}$

$\left(\frac{d\theta_A}{dt}\right)_{t=5sec} = 20$ $\qquad \left(\frac{d\theta_B}{dt}\right)_{t=5sec} = \frac{1}{4}$

$\omega_A : \omega_B = 80 : 1$

10. **(B)**

We know that

$\nu \le \sqrt{\mu r g}$

$\nu \le \sqrt{0.64 \times 20 \times 9.8}$

$\nu \le 11.2\, m/s$

11. **(B)**

$$a_c = \omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4 \times 3.14^2 \times 6400 \times 10^5}{(24 \times 60 \times 60)^2}$$

$$\omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4 \times 3.14^2 \times 6400 \times 10^5}{(24 \times 60 \times 60)^2}$$

$= 3.4 cm/sec^2$

12. **(B)**

We know the Tension provides necessary centripetal force

So $T = m\omega^2\ell$

Given $m = 0.1$, $\qquad \omega = 2\pi \times \frac{19}{\pi}$

$\ell = 1$ $\qquad \Rightarrow \qquad T = m\omega^2\ell$

$T = 0.1 \times \left(2\pi \times \frac{10}{\pi}\right)^2 \times 1$

$= 0.1 \times 4\pi^2 \times \frac{100}{\pi^2} \times 1 = 40N$

13. **(A)**

In I case $\qquad \mu mg = m\omega^2 R$ ....(1)

In I case $\qquad \mu mg = m(2\omega)^2 R'$ ....(2)

From (1) & (2) $m\omega^2 R = m4\omega^2 R'$

$R' = \frac{R}{4}$

Given R = 40 cm, $\qquad$ R' = 10 cm

14. **(C)**

Given r = 25 cm, $\qquad$ n = 2

$\omega = 2\pi \times 2\, rad/s$ $\qquad \Rightarrow \qquad a_c = \omega^2 r$

$= (4\pi)^2 \times 0.25 = 16\pi^2 \times 0.25 = 4\pi^2$

15. **(A)**

Slope should be decreasing

$\alpha = \frac{d\omega}{dt} = \tan\theta$, if $\theta\downarrow$, $\alpha\downarrow$

16. **(C)**

Given

$R = \frac{20}{\pi} m$; $\qquad v = 80\, m/sec$

$v^2 = u^2 - 2a_t s$

$u = 0$; $\qquad s = 2(2\pi R)$

$(80)^2 = 2a_t(4\pi.\frac{20}{\pi})$ $\quad \Rightarrow \quad a_t = 40\, m/s^2$

17. **(i) A (ii) A**

(i) At any moment $a_t = a_r$

$$a_t = -\frac{v^2}{R}$$

$v\frac{dv}{ds} = -\frac{v^2}{R} \Rightarrow \frac{dv}{v} = -\frac{1}{R} ds$

After integration $\log v = -\frac{S}{R} + C$ $\qquad$ ...(i)

at t = 0, s = 0, $v = v_0$

$C = \log v_0$

from eq. (1) $\log\left(\frac{v}{v_0}\right) = -\frac{S}{R}$

$v = v_0 e^{-S/R}$

(ii) At any moment $a_t = a_v$

$a = \sqrt{2} a_r = \sqrt{2}.\frac{v^2}{R}$

18. **(C)**

Given

$\omega = \theta^2 + 2\theta$

$\frac{d\omega}{d\theta} = 2\theta + 2$ $\qquad \Rightarrow \qquad \left.\frac{d\omega}{d\theta}\right|_{t=1} = 2\theta + 2 = 4$

$\alpha = \frac{\omega d\omega}{d\theta} = (\theta^2 + 2\theta).(2\theta + 2) = 12\, rad/sec^2$

19. **(C)**

The magnitude of acceleration is constant in (A) and decreasing in (B)

In (A) $\rightarrow$ r constant, $a_t = 0$;

v constant , $a_r = \frac{V^2}{R}$ constant

In (B) $\rightarrow$ r is increasing , V constant

$a_t = 0$; $a_r = \frac{V^2}{R}$ decreasing

20. **(C)**

Given $\frac{m_1}{m_2} = 1; \frac{R_1}{R_2} = \frac{1}{2}$

If centripetal force is same

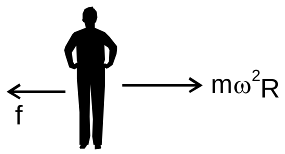
$\frac{m_1 v_1^2}{R_1} = \frac{m_2 v_2^2}{R_2}$ $\qquad \Rightarrow \qquad \frac{v_1}{v_2} = \sqrt{\frac{R_1}{R_2}} = \frac{1}{\sqrt{2}}$

21. **(D)**
constant speed and variable velocity

22. **(C)**
Car will not slip when moving with speed v

23. **(C)**
$mg < \frac{mv^2}{R}$

24. **(A)**



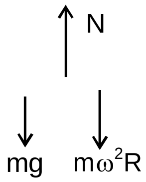
$\mu mg \geq \frac{mv^2}{R}$

$0.5\ mg \geq m \times (5)^2 \times R$

$\frac{0.5 \times 10}{25} \geq R$

$R \leq 0.2\ m$

25. **(C)**
Given
R = 10 m



m = 500 kg

$N = m\omega^2 R + mg$

$= \frac{mv^2}{R} + mg = \frac{500 \times 400}{10} + 500 \times 10$

= 25 kN

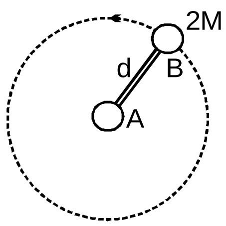
26. **(C)**
$v = \sqrt{Rg\tan\theta}$

$R = 10\sqrt{3}\ m,\ \theta = 30°$

$= \sqrt{10\sqrt{3} \times 10 \times \frac{1}{\sqrt{3}}}$ = 10 m/sec = 36 km/hr

27. **(D)**
In uniform circular motion
Force is towards centre

28. **(B)**



Given

$P = \frac{2\pi}{\omega} \Rightarrow \omega^{-1} = \frac{P}{2\pi}$

$T = 2M\,\omega^2 d = \frac{8\pi^2 Md}{P^2}$

29. **(C)**
Energy conservation

$mgR = \frac{1}{2}mv^2$

$v = \sqrt{2gR}$

30. **(A)**
Given m = 2 kg, R = 2 m
net displacement = 4 m
work done = f × 4 = 20 × 4 = 80 joule

31. **(B)**
Speed will be maximum at the lowest point of the path.

32. **(A)**
The maximum bearable Tension

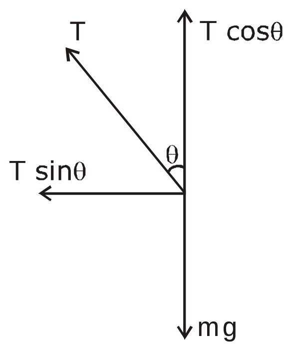
$T = \frac{mv^2}{l}$

T_{max} = 10 N, m = 1, v = ?, l = 1

$\upsilon = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{100 \times 1}{1}}$ = 10 m/s

33. **(C)**
At highest point velocity is zero.
After word it fall freely.

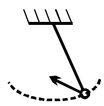
34. **(B)**



$T\sin\theta = mw^2 l$

$T\cos\theta = mg$

35. **(B)**



36. **(A)**
$\because \omega = \frac{2\pi}{T}$

T is same for both cases car's
so ratio $\omega_1 : \omega_2$ = 1 : 1

37. **(D)**
To balance the torque of the centripetal force he bend inwards.

38. **(C)**
Given that
v = 72 km/h., r = 80 m
We know that

$$\tan\theta = \frac{v^2}{rg} = \frac{20\times 20}{80\times 10} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

39. **(C)**
We know that
$\upsilon^2 = rg\tan\theta$ (θ is same)
$\Rightarrow$ $\upsilon^2 = rg$
Case 1
$r_1 = 20$ m, $v_1 = v$
$r_2 = r$, $v_2 = 1.1v$

$$\frac{v_2^2}{v_1^2} = \frac{r_2 g}{r_1 g} \quad \Rightarrow \frac{(1.1v)^2}{v^2} = \frac{r_2}{r_1}$$

$$1.21 = \frac{r}{20} \quad \Rightarrow r = 24.2\,\text{m}$$

40. **(D)**
v = 72 km = 20m/s, r = 20m, g = 10 m/s^2
To avoid skiding θ must be greater than

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan{-1}\left(\frac{20\times 20}{20\times 10}\right)$$

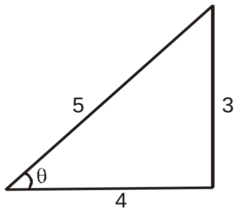
$\theta = \tan^{-1}(4)$

41. **(B)**
$F \perp dr$
$\therefore$ W.D. = 0
Force and displacement are perpendicular to each other.

42. **(C)**
Given F = 5N, d = 10 m,
we know w = Fd cos θ
25 = 5 × 10 cos θ
$\theta = 60^o$

43. **(B)**
m = 5 kg, F = 20 N, K.E. = 40 $\hat{j}$
change is K.E. = Fd cos θ
40 = 20 × 4 cos θ
$\theta = 60^o$

44. **(B)**



w = mgh, cos θ = 4/5
= 10 × 9.8 × 3 = 294 joule

45. **(C)**
Let $\vec{r} = dx\,\hat{i} + dy\,\hat{j}$, $F = 3x\,\hat{i} + 4\,\hat{j}$

$$w = \int (3x\hat{i} + 4\hat{j}).(dx\hat{i} + dy\hat{j})$$

$$= \int_{2m}^{3m} 3x\,dx + \int_{2m}^{0} 4dy = \left[\frac{3x^2}{2}\right]_{2m}^{3m} + [4y]_{3m}^{0}$$

$$= \left[\frac{3\times 9}{2} - \frac{3\times 2^2}{2}\right] + [0 - 12] = -4.5\text{ J}$$

46. **(D)**
2 K.E$_{man}$ = K.E.$_{boy}$

$$2\times\frac{1}{2}M\times v_{man}^2 = \frac{1}{2}.\frac{M}{2}v_{boy}^2$$

$$V_{man} = \frac{v_{boy}}{2} \quad \ldots(i)$$

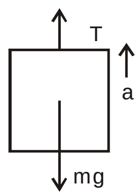
$$\Rightarrow \frac{1}{2}M(v_{man}+1)^2 = \frac{1}{2}.\frac{M}{2}v_{boy}^2$$

$$\Rightarrow (v_{man}+1)^2 = \frac{V_{boy}^2}{2} \Rightarrow$$

$$v_{man} = (\sqrt{2}+1)\,m/sec$$

47. **(A)**
From F.B.D.
T – mg = ma
T = m (g+a)

$w = \vec{F}.\vec{S}$, $S = \frac{1}{2} \otimes t^2$



$= m(g+a)\,\frac{1}{2}\,at^2$

$= \frac{m}{2}\,(g+a)\,at^2$

48. **(B)**
$f = \mu_k N$ (Tangentially)
$\Rightarrow$ W = – $2\pi r\,\mu_k N$
–ve sign indicate that f & ds is opposite

49. **(A)**

$V_0 = at_0 \qquad \Rightarrow a = \frac{v_0}{t_0}$

$\therefore v = \frac{v_0}{t_0}.t \qquad \Rightarrow w = \Delta k = k_f - k_i$

$\Rightarrow \frac{1}{2}M\frac{v_0^2}{t_0^2}.t^2$

50. **(B)**
Maximum velocity will be at Mean Position
Where $F_{net} = 0 \Rightarrow$ mg = Kx
1 × 10 = 2 × 100 × x $\Rightarrow$ x = 5 cm
$\therefore$ h = 20 – 5 = 15 cm

51. (A)

$w = \frac{1}{2}k(x_2^2 - x_1^2)$

$= \frac{1}{2}\,10\,(6^2 - 4^2) = 100$ N cm

= 1 joule

52. (B)

53. (D)

$\frac{1}{2}K(0.3)^2 = 10$

$\Rightarrow \quad K = \frac{20}{0.09} = \frac{2000}{9}$

work done = $\frac{1}{2}.\frac{2000}{9}[(0.45)^2 - (0.3)^2]$

= 12.5 J

54. (D)

55. (A)

$u = x^2 - 3x, \quad x = 0, \; x = 2$

$(u_i)_{x=0} = 0, \; (u_f)_{x=2} = 4 - 6 = -2$

$\Delta k = -\Delta u = 2$ joule

56. (D)

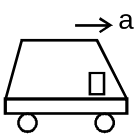
$(W.D)_{\text{by friction}} + (W.D)_{\text{by spring}} = \Delta k = k_f - k_i = 0 - k_i$

$-0.25 \times 1 \times 10 \times 4 - \frac{1}{2} \times 2.75 \times 4^2$

$= -\frac{1}{2} \times 1 \times v^2$

v = 8 m/s

57. (C)



P = F.V = (R + ma) V

58. (C)

Given m = 12000 kg, v = 4 m/sec & t = 40 sec

$P_{avg} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2} \times 12000 \times 4^2}{40}$

= 2400 w = 2.4 kw

59. (C)

Initially be in contact with the inner wall and later with the outer wall.

60. (A)

Area under force vs displacement gives work and area above x-axis taken as positive while area below x-axis taken as negative.

$W_{net} = 10 \times 1 + 20 \times 1 - 20 \times 1 + 10 \times 1 = 20$ erg.

61. (A)

P = F.v

Given

$\tan\theta = \frac{1}{100}$; v = 30 km/hr = $30 \times \frac{5}{18}$ m/s

$P = mg\sin\theta.\,v \qquad [\theta$ is very small]

$= 30,000 \times 10 \times \frac{1}{100} \times 30 \times \frac{5}{18} = 25$ kw

62. (C)

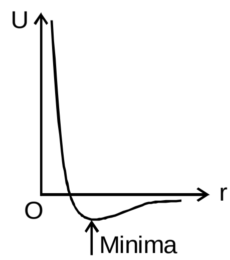
$\vec{P} = \vec{F}.\vec{V}$

$P = (10\hat{i} + 10\hat{j} + 10\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k})$

$P = (50 - 30 + 120) \Rightarrow P = 140$ J/sec

63. (A)

For stable equilibrium



$\frac{dU}{dr} = 0 \Rightarrow r_1$ and $\left(\frac{d^2U}{dr^2}\right)_{r_1} > 0$

For stable equilibrium P.E. must be Minimum at the equilibrium position

64. (A)

$2x^2 - 3x - 2 = 0$

$x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} \Rightarrow x = -\frac{1}{2}, 2$

$\frac{dF}{dx} = -\frac{d^2u}{dx^2} = 4x - 3 \Rightarrow \frac{d^2u}{dx^2} = 3 - 4x$

$\Rightarrow \left(\frac{d^2u}{dx^2}\right)_{x=-\frac{1}{2}} = 3 + 4 \times \frac{1}{2}$

= (5) > 0 (stable)

65. (B)

For light rod

$v_{top} = 0$

Using energy conservation

$\frac{1}{2}mv^2 + 0 = 0 + mg\ell$

$v = \sqrt{2g\ell}$

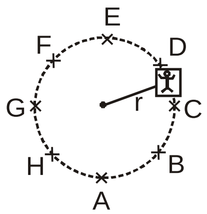
Exercise-II

1. **(B, D)**

(B) There are other forces on the particle

(D) The resultant of the other forces varies in magnitude as well as in direction.

2. **(B, C, D, E)**



$v = \sqrt{gr} \Rightarrow$ At A

$N = mg + \dfrac{mv^2}{r} = 2mg \; [v = \sqrt{gr}]$

at E

$N + \dfrac{mv^2}{r} = mg$

$\Rightarrow N = 0 \Rightarrow$ At G and C

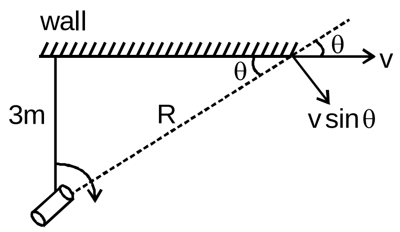
N = mg

3. **(B, C)**

$T - mg\cos\theta = \dfrac{mv^2}{L}$

Tangential Acceleration = g sin θ

4. **(A)**

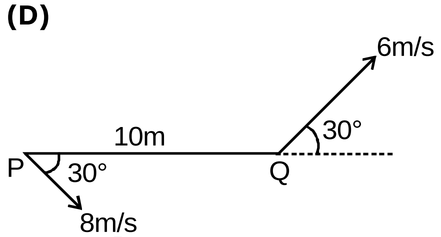


$\omega = \dfrac{V_\perp}{R}$

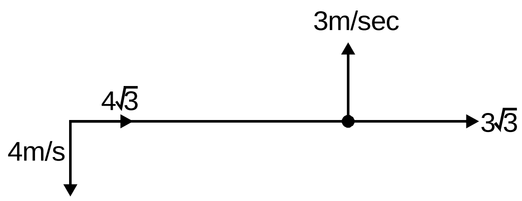
$\Rightarrow \quad \omega = \dfrac{v\sin\theta}{3/\sin\theta} \quad \Rightarrow \omega = \dfrac{v\sin^2\theta}{3}$

v = 0.6 m/s (Given $\theta = 45°$ ω = 0.1 rad/s)

5. **(D)**



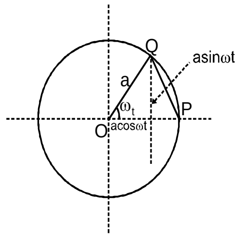
w.r.t to P



$v_{\perp rel} = 8\sin 30° + 6\sin 30° = 7$ m/s

$\omega = \dfrac{v_{\perp rel}}{R} = \dfrac{7}{10} = 0.7\,\text{rad/sec}$

6. **(D)**

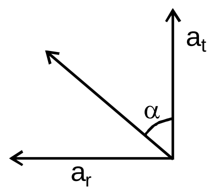


$PQ = \sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$

$= 2a\sin\left(\dfrac{\omega t}{2}\right)$

7. **(B)**

Given $v = a\sqrt{s}$

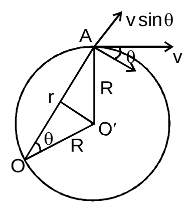


$a_t = \dfrac{vdv}{ds} = a\sqrt{s}.\dfrac{a}{2\sqrt{s}} = \dfrac{a^2}{2}$

$a_r = \dfrac{v^2}{R} = \dfrac{a^2 s}{R}$

$\tan\alpha = \dfrac{a_r}{a_t} = \dfrac{2s}{R}$

8. **(D)**



$\dfrac{r}{2} = R\cos\theta$

r = 2R cos θ

After differentiable

$\dfrac{dr}{dt} = -2R\sin\theta\dfrac{d\theta}{dt} \Rightarrow \dfrac{dr}{dt} = v_{rad} = v\sin\theta$

$\dfrac{d\theta}{dt} = \omega$ (– ve because θ decreasing)

v sin θ = 2R sin $\theta\omega$

$v = 2R\omega = 0.4$ m/s

$a = \sqrt{a_t^2 + a_r^2}$ $\quad\therefore\ \omega$ = constant

$\Rightarrow \quad a = a_r = \dfrac{v^2}{R} \quad \Rightarrow a_t = 0$

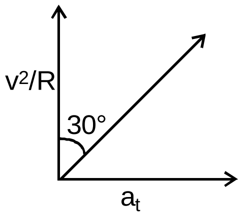
$\Rightarrow a_r = \dfrac{V^2}{R} = 32\ \text{m/s}^2$

9. **(C)**

$a_t = \sqrt{3}\,t$

$\int dV = \int \sqrt{3}t\,dt$

$v = \dfrac{\sqrt{3}t^2}{2}$



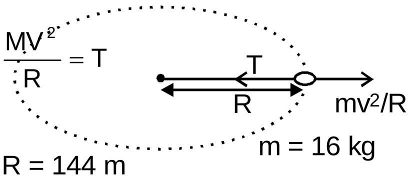
$\tan 30° = \dfrac{\sqrt{3}t.R}{\left(\dfrac{\sqrt{3}t^2}{2}\right)^2} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{4t}{\sqrt{3}t^4}$

$\Rightarrow\ t^4 = 4t \Rightarrow\ t^3 = (2)^2$

$\Rightarrow\ t = 2^{2/3}$ sec

10. **(D)**

Given

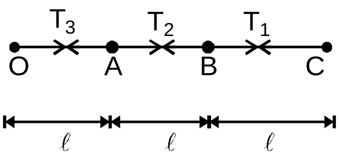


$R = 144$ m

$T_{max} = 16$ N

$v_{max} = \sqrt{\dfrac{RT}{m}} \quad \Rightarrow \quad v_{max} = \sqrt{\dfrac{16 \times 144}{16}} = 12\,\text{m/s}$

11. **(D)**



$T_1 = m\omega^2 R = m\omega^2 (3l)$ ..(1)

$T_2 = T_1 + m\omega^2 (2l)$ ...(2)

$T_3 = T_2 + m\omega^2 (l)$ ...(3)

$T_1 : T_2 : T_3 = 3 : 5 : 6$

12. **(A)**

$v = r\omega$

If $r \to r/2$

$\therefore\ v' = \dfrac{v}{2} = \dfrac{20}{2} = 10$ cm/sec

Turn table rotating uniformly $a_t = 0$

$a_r = \dfrac{v^2}{R}$; $a'_r = \dfrac{v'^2}{R/2} = \dfrac{20}{2} = 10\ \text{cm/s}^2$

13. **(B)**

Given $k = as^2$

$v^2 = \dfrac{2a}{m}s^2$

After differentiating w.r.t s $\dfrac{vdv}{ds} = \dfrac{2as}{m} = a_t$

$a_r = \dfrac{v^2}{R} = \dfrac{2as^2}{mR}$

Total force $= \sqrt{(ma_r)^2 + (ma_t)^2}$

$\therefore$ Total force $= \sqrt{m^2\left(\dfrac{2a}{mR}s^2\right)^2 + \left(m\dfrac{2as}{m}\right)^2}$

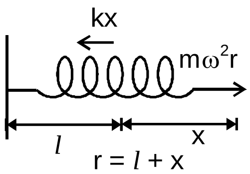
$= \sqrt{\dfrac{4a^2s^4}{R^2} + 2a^2s^2} \Rightarrow = 2as\sqrt{\dfrac{s^2}{R^2} + 1}$

$= 2as\left(1 + \dfrac{s^2}{R^2}\right)^{1/2}$

14. **(B)**

$kx = m\omega^2 r$

$kx = m\omega^2 (l + x)$



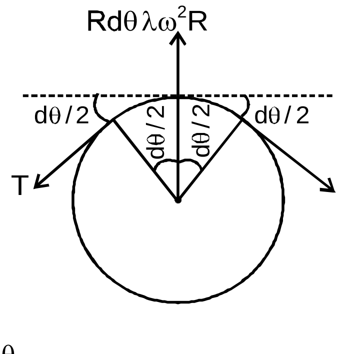
$x = \dfrac{m\omega^2\ell}{k - m\omega^2}$

15. **(C)**

$2T\sin\dfrac{d\theta}{2} = Rd\theta\lambda\omega^2R$

If $d\theta$ is small

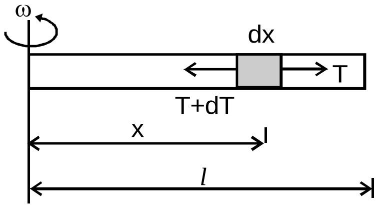
$\sin\dfrac{d\theta}{2} \simeq \dfrac{d\theta}{2}$



$2T\dfrac{d\theta}{2} = Rd\theta\lambda\omega^2R$

$T = \lambda\omega^2R^2$

16. **(D)**



$(T+dT) - T = \dfrac{m}{\ell}w^2x\,dx$

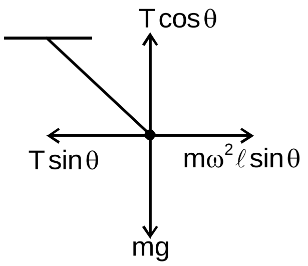
$dT = \frac{m}{\ell}.\omega^2 x dx$

Integrate with limit x to ℓ

$$T = \int_x^{\ell} \frac{m}{\ell}\omega^2 x dx$$

$$T = \frac{m\omega^2}{\ell}\left[\frac{x^2}{2}\right]_x^{\ell} = \frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2 - x^2]$$

17. (B)



T for simple pendulum = $2\pi\sqrt{\frac{\ell}{g}}$

For conical pendulum

T sin θ = m ω^2 l sin θ

$\Rightarrow$ T = mω^2 l

and T cos θ = mg

$\Rightarrow \quad T = \frac{mg}{\cos\theta}$

Now, $\frac{g}{\cos\theta} = \omega^2 l \quad \Rightarrow \omega = \sqrt{\frac{g}{l\cos\theta}}$

$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$

$\therefore \frac{T_{conical\ Pendulum}}{T_{simplePendulum}} = 2\pi\sqrt{\frac{\ell}{g}\cos\theta} \times \sqrt{\frac{g}{\ell}} \times \frac{1}{2\pi}$

Ratio = $\sqrt{\cos\theta}$

18. (A, C, D)

(A) F $\perp$ V

(C) Object is at Rest But point of application of the force moves on the object.

(D) The object moves in such a way that point of application of the force remains fixed.

19. (A, B)

(A) The spring initially compressed and finally in its N.L.

(B) Initially stretched and then in its N.L.

20. (A, B, C)

W.D. by force of friction

can be zero, positive & Negative

21. (B, D)

Total work done on a Particle positive when momentum increases & K.E increases

22. (A)

Total energy = E = K.E + P.E.

When speed of the particle is zero.

i.e., K.E = 0

$\Rightarrow$ U(x) = E

23. (A)

Angle of Inclination

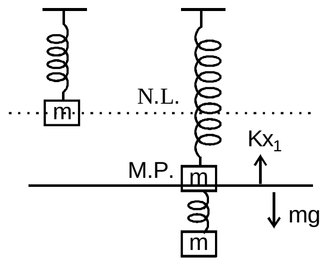
24. (D)

Only Conservative force (mg) is act.

So E.C. is done only two points

(1 and 2)

25. (B, C, D)

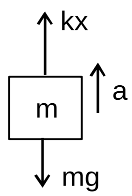


M.P. $\quad x_1 = \frac{mg}{k}$

But block further move downward due to inertia. So descending through distance

$$x = \frac{2mg}{k}$$

at M.P. at $\frac{x}{2}$ $\Rightarrow F_{net} = 0$;



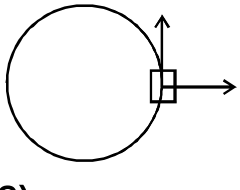
so a = 0

at lower most point

$k\left(\frac{2mg}{k}\right) - mg = ma \Rightarrow a = g$

26. (B, D)

Particle takes speed tangentially and act as a 'Projectile' (curved path)



27. (A, B, C)

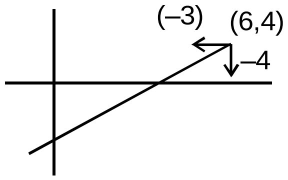
Given U = 3x + 4y

Initially particle at rest at (6,4)

So K.E = 0

E_{total} = P.E = 3 × 6 + 4 × 4 = 34 J

$F = -\frac{\partial U}{\partial y}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -3\hat{i} - 4\hat{j}$



$a = -3\hat{i} - 4\hat{j} \Rightarrow |a| = 5 m/s^2$

Let us assume particle crosses y axis after time t

$x - 6 = -\frac{1}{2} \times 3 \times t^2$

at y axis $\Rightarrow$ x = 0

$\Rightarrow$ t = 2 sec

So $y - 4 = -\frac{1}{2} \times 4 \times (2)^2 = -8$

y = – 4m

(P.E.) at y = – 4 and x = 0

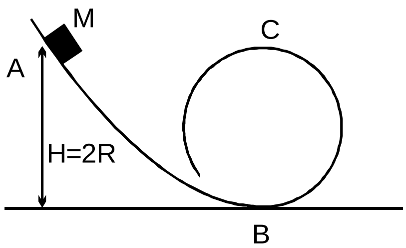
is $U_{(y=-4,\ x=0)} = -16$ J

So. K.E. = T.E. – U

$\frac{1}{2}MV^2 = 34 - (-16) = 50$

$V^2 = 100 \Rightarrow V = 10$ m/s

28. (B, D)



E.C between point A and B

$Mg\ (2R) = \frac{1}{2}MV^2$

$V = \sqrt{4gR} < \sqrt{5gR}$

$V = \sqrt{4gR} > \sqrt{2gR}$

So, doesn't complete vertical circle and break off at a height (R < H < 2R)

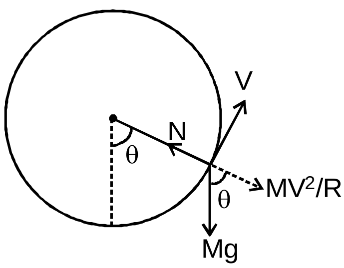
29. (A, B, D)

30. (C)

To complete vertical circle

speed at point B $\geq \sqrt{5gR}$

So. E.C.



$MgH = \frac{1}{2}M(5gR)$

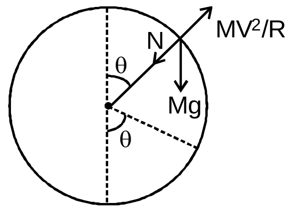
$H = \frac{5R}{2} = 2.5\ R$

$N = \frac{Mv^2}{R} + Mg\cos\theta$

N_{max} at $\theta = 0°$

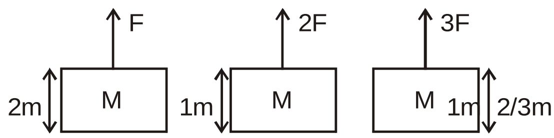
N is zero only

$\theta \geq \pi/2$ because in this



$N = \frac{MV^2}{R} - Mg\cos\theta$

31. (C)



Apply work energy theorem

$W_F + W_{mg} = \Delta K = K_f - K_i\ (K_i = 0)$

Case I : $F(2) - mg \times 2 = K.E.$

Case II : $2F(1) - mg \times 1 = K.E.$

Case III : $3F\left(\frac{2}{3}\right) - mg \times \left(\frac{2}{3}\right) = K.E.$

In case III K.E. is maximum.

32. (C)

In case of first spring $F = k_1x_1$

$x_1 = \frac{F}{K_1}$ ...(1)

In case of second spring $F = K_2x_2$

$x_2 = \frac{F}{K_2}$ ...(2)

$\therefore K_1 > K_2 \Rightarrow x_2 > x_1$

$\Rightarrow$ More work is done by this force in case of second spring.

33. (C)

$W.D. = \int \vec{F}.\vec{ds}$

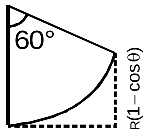
$= K\int [(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j})]$

$= K\int (ydx + xdy)$

$= K\int_{(1,5)}^{(3,5)} d(xy) = 20K$

34. (B)

$W = R\theta x\ F\cos 0°$



(by the force)

$= 10 \times \frac{\pi}{3} \times 200$

Work done by g = MgR (1 – cos 60°)

$= \frac{gRM}{2}$

K.E. = $RF\theta - \frac{gRM}{2}$

$\frac{1}{2}MV^2 = 10 \times \frac{\pi}{3} \times 200 - \frac{10 \times 10 \times 10}{2}$

$v^2 = 2 \times \frac{\pi}{3} \times 200 - 50$

V = 17.32 m/s

35. (C)

v = at

$= 10\sqrt{3}$ m/s

In ground frame

W.D. by gravity + W.D. by normal = Δk

$0 + W.D._N = \frac{1}{2} \times 1 \times (10\sqrt{3})^2 = 150J$

36. (C)

Friction is present

∴ Mechanical energy is not conserved

But work energy principle conserved

Due to extrenal friction force is working on the block.

37. (C)

The block will come to rest when work done by friction becomes equal to the change in energy stored in spring.

38. (C)

Work done by force

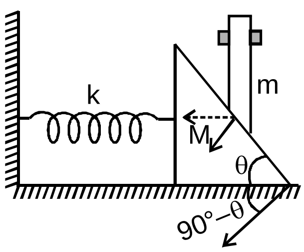
$F = 100 \times 11 \times \frac{1}{2} = 550$ J

Work done by the gravity = mgh

mgh = 550

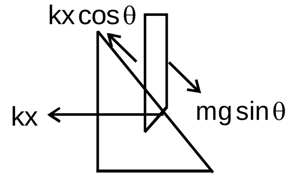
$\Rightarrow h = \frac{550}{5 \times 10} = 11m$

39. (C)



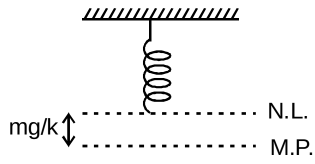
$kx \cos\theta = mg \sin\theta$

$x = m\, g \tan\theta / k$



P.E. = $\frac{1}{2}kx^2 = \frac{m^2g^2\tan^2\theta}{2k}$

40. (B)



K = $\frac{mg}{a}$(Given)

$\frac{1}{2} \times m \times v^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = mg\left(\frac{mg}{k}\right)$

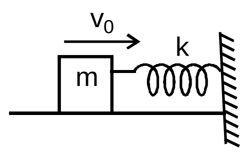
$\frac{1}{2} \times m \times v^2 + \frac{1}{2} \times \frac{mg}{a} \times \frac{m^2g^2}{m^2g^2} \times a^2 = \frac{m^2g^2}{mg} \times a$

$\frac{1}{2}mv^2 + \frac{1}{2}mga = mga$

$v^2 = ga$

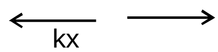
K.E. = $\frac{1}{2}mv^2 = \frac{mga}{2}$

41. (B)



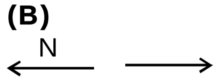
$-\frac{1}{2}mv_0^2$

42. (B)



$-\frac{1}{2}mv_0^2$

43. (B)



Net work done = $-\frac{1}{2}mv_0^2$

44. (B)

45. (C)

Velocity of block with respect to observer B is zero so K.E of block = 0

46. (B)

P.E ↑

Due to +ve work done by N

47. (B)

On comparing

F ∝ V

$F = kV$
$P = F.V = kV^2 \Rightarrow$ Now $2P = KV'^2$
$2 \times kv^2 = kV'^2 \Rightarrow V'^2 = 2V^2$
$V' = \sqrt{2}V$

48. **(B)**

$\frac{dW}{dt} = \frac{d K.E.}{dt} (K.E = 2t^2)$

$\Rightarrow P = \left(\frac{dK.E.}{dt}\right)_{at\, t=2s} = 4t = 8 watt$

$\frac{1}{2}mv^2 = 2t^2$

$\Rightarrow v = 4t$ $[\therefore m = 1 \text{ kg}]$

$\Rightarrow \frac{dv}{dt} = 2m/s^2 = a_t$

49. **(A)**

$\vec{F} = -\frac{du}{dx}\hat{i} - \frac{du}{dy}\hat{j} - \frac{du}{dz}\hat{k}$

$\vec{F} = \Delta U$ $[U = \sin(x + y)]$

$= \cos(x+y)\hat{i} + \cos(x+y)\hat{j}$

$\vec{F}_{(0,\pi/4)} = \cos\frac{\pi}{4}\hat{i} + \cos\frac{\pi}{4}\hat{j}$

$|\vec{F}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

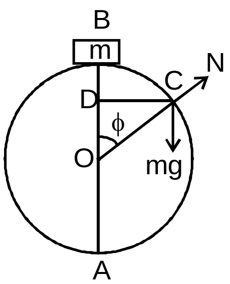
50. **(B)**

$mg\cos\phi - N = \frac{mv^2}{R}$

$N = m(g\cos\phi - \frac{v^2}{R})$ ...(i)

$\because N = 0$

$\Rightarrow \cos\phi = \frac{v^2}{Rg}$ ...(ii)



By energy conservation

$\frac{1}{2}mv^2 = mg(R - R\cos\phi) \Rightarrow v^2 = 2Rg(1-\cos\varphi)$

Using (i) & (ii) $\cos\phi = \frac{2}{3}$

height from highest Point = BD = R (1 – cos ϕ)

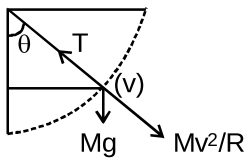
$h = R\left(1 - \frac{2}{3}\right) = \frac{R}{3}$ Ans.

51. **(C)**

$\sqrt{5Rg} = \sqrt{5 \times 2.5 \times 10} = 5\sqrt{5} > 10 m/s$

$\therefore N_2$ will be zero in part A, D, C at some point

52. **(A)**



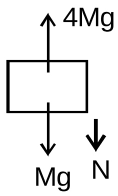
$T = \frac{Mv^2}{R} + Mg\cos\theta \Rightarrow MgR\cos\theta = \frac{1}{2}Mv^2$

$\Rightarrow Mgh = \frac{1}{2}Mv^2 \Rightarrow$

$T = \frac{2Mgh + Mgh}{R}$

Straight line

53. **(C)**

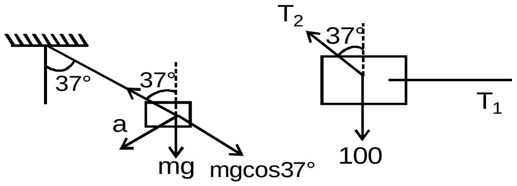


$2MgR = \frac{1}{2}Mv^2 \Rightarrow 2\sqrt{gR} = V$

$\frac{mv^2}{R} = mg + N \Rightarrow N = 3mg$

Exercise-III Level - I

1.



$T_2 \times \frac{4}{5} = 100$ $T_1 = 80$ N

$T_2 = 125$ N

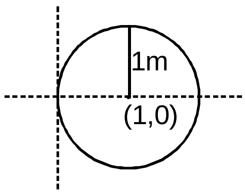
2. $\vec{v} = a\hat{i} + bt\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = b\hat{j}$

component of acceleration along v is

$a_t = \frac{\vec{a}.\vec{v}}{|\vec{v}|} - \frac{b^2 t}{\sqrt{a^2 + b^2 t^2}}$

3.



$a_t = \frac{\pi}{2} m/s^2 \Rightarrow$ from $S = ut + \frac{1}{2}at^2$

$\Rightarrow \pi = 0 + \frac{1}{2} \times \frac{\pi}{2} t^2 \quad \Rightarrow \quad t = 2$ sec

from $v = u + at \Rightarrow v = \frac{\pi}{2}.2 = 3.14$ m/s

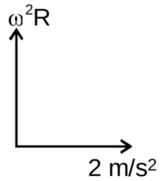
4. **Do Yourself**

5. $\alpha = \frac{a_t}{R}$ $a_r = \frac{v^2}{R}$

$a_t = 6$ m/s² $a_r = 8$ m/s²

6. $\omega_i = 12$ rad/s

$\omega_f = \frac{2\pi \times 480}{\pi \times 60} = 16$ rad/s



$\omega_f = \omega_i + \alpha t$

$\Rightarrow \alpha = 2$ rad/sec²

$12 + 2t$ for $t \le 2$ s

16 for $t \ge 2$ sec

at 0.5 sec

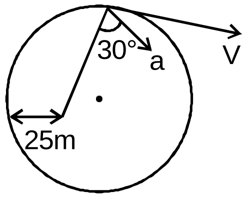
$a_t = 2$m/s²

$\omega_f = 12 + 0.5 \times 2 = 13$ rad/sec

$\sqrt{28565} \approx 169$

at t = 3 sec only $a_r = \omega^2 r$

7.

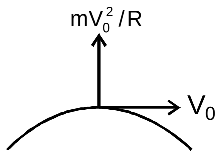


$a_r = a \cos 30° = 25 \times \frac{\sqrt{3}}{2} m/s^2$

$\because \; a_r = \frac{v^2}{r} \Rightarrow \quad v^2 = \frac{25 \times \sqrt{3}}{2} \times 2.5m$

$v = \left(125 \times \frac{\sqrt{3}}{4}\right)^{1/2} m/s \Rightarrow a_t = a\sin 30° = \frac{25}{2}\, m/s^2$

8. $\frac{MV^2}{R} = Mg$

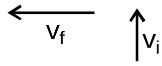


$(V_0 \cos 45°)^2 = g.\, R$

$\Rightarrow R = \frac{V_0^2}{2g}$

$\therefore a_c = \frac{V_0^2}{R} = 2g$

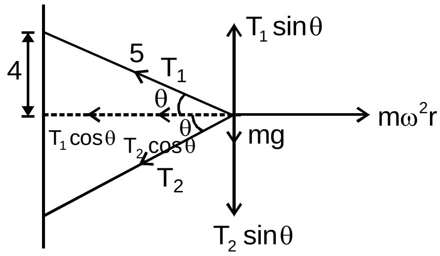
9. $\theta = \frac{\frac{V\pi R}{2V}}{R} = \frac{\pi}{2}$



$\therefore$ Average acceleration

$= \frac{\vec{v}_f - \vec{v}_i}{\frac{\pi R}{2V}} = 2\sqrt{2}\, V^2 / \pi R$

10.



$\Rightarrow \quad T_1 \cos\theta + T_2 \cos\theta = m\,\omega^2 r$ ...(1)

$T_1 \sin\theta = mg + T_2 \sin\theta$ ...(2)

11. $a_t = \alpha R = 5 \times 0.5 = 2.5$ m/s²

Normal exert on Block, N = ma

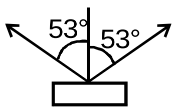
N = 1 × 2.5 = 2.5 N

$\omega = \alpha t = 5\,t$
Block slip when $f = m\omega^2 R$
$\mu N = m\omega^2 R$
$(0.05)\,(2.5) = (1)\,(5t)^2\,(0.5)$
$t = 0.1$ sec.

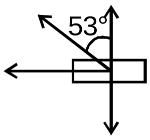
12. $M\omega^2 \times 1 = 2\,mg$

13. **Do your self**

14.



$2T\cos 53° = 20$

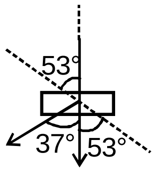


$T = \frac{50}{3}\,N$

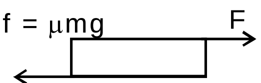
$$\text{Acceleration} = \frac{\sqrt{\left(\frac{40}{3}\right)^2 + (10)^2}}{2} = \frac{25}{3}$$

$a = g\sin 53° = 10 \times \frac{4}{5} = 8$

Ratio $= \frac{25}{24}$



15. (a) Net force on Block is zero
V = constant
$F = f = \mu mg$



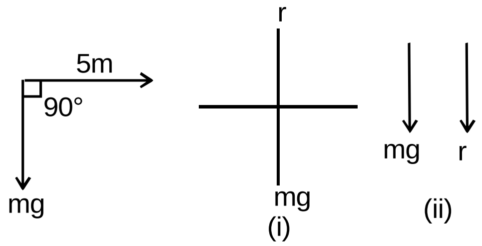
so, work done by force.
$\int \vec{F}.\vec{dr} = 0$

(b) $W = \int_0^r N.dr$

$dr \perp N$
$N.dr = N\,dr\cos 90° = 0$
$W = 0$
(c) $W = f.r$
$= f\,r\cos 180° = -f_r$
$= -\mu\,mg(vt) = -\mu\,mg\,vt$

(d) work done by $F = \vec{F}.\vec{r} = F_r$
$= (\mu\,mg)\,(vt) = \mu\,mg\,vt$

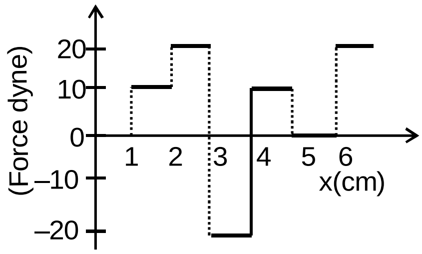
16.



m = 10 kg
w = zero.
(i) $w = (-5) \times 10\,g = -500$ J
(ii) $w = 500$ J

17. $W = \vec{F}.\vec{r} = (-2\hat{i} + 15\hat{j} + 6\hat{k}).(10\hat{j})$
$= 150$ J

18.

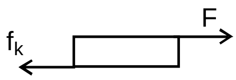


$W = 10 \times (2-1) + 20(3-2) + (-20)(4-3) + 10 \times (5-4)$
$= 20$ dyne cm $= 20$ ergs
$= 20 \times \frac{kgm^2}{sec^2} \times \frac{1}{10^3 \times 10^4} = 0.2 \times 10^{-5}$ J

19. Work done by the resistive force
$= -mgh + \frac{1}{2}mv^2$
$= -\left(\frac{1}{1000}\right)[10 \times 10^3 - \frac{1}{2} \times 50 \times 50]$
$= -8.75$ J

20. $F = 7N\,\hat{i}$

and $f = 2N(-\hat{i})$

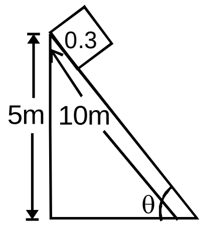


$a = \frac{7-2}{2} = \frac{5}{2}$
$S = \frac{1}{2}at^2 = \frac{1}{2}.\frac{5}{2}.(10)^2 = 125\,\hat{i}$
(a) $\vec{F}.\vec{r} = (7\hat{i})(125\,\hat{i}) = 875$ J
(b) $f_k\,r = -2 \times 125 = -250$ J
(c) $(F - f_k)r = 5 \times 125 = 625$ J
(d) $\Delta K = k_f - k_i$
= work done by net force
= 625 J

21. $\mu = 0.15$
$g = 9.8\ m/s^2$

$\sin\theta = \frac{1}{2} \qquad \theta = 30°$

$\mu\, g \cos\theta < g \sin\theta$



Block will move on the horizontal plane.

(a) Work done by Gravitational force in round trip.

$= -mgh + mgh = 0$

(b) $w_1 = mgh + \mu\, mg \cos\theta.\, r$

$= 0.3 \times 10 \times 5 + 0.15 \times 0.3 \times \frac{\sqrt{3}}{2} \times 10 \times 10$

$= 15 + 3.82 = 18.82$

(c) $w_{f_{net}} = w_{f_1} + w_{f_2}$

$= -2 \times \mu mg \cos\theta.r$

$= -2 \times 0.15 \times 0.3 \times 10 \times \frac{\sqrt{3}}{2} \times 10$

$= -7.64$ J

(d) K.E. of a body

$= mgh - \mu\, mg \cos\theta$

$= 15 - 3.8$

$= 11.2$ J

22. (a) $v_f = \frac{F}{m} \times t$

(b) $v'_f = v_c = \frac{F}{m} t$

(c) $\Delta k = \frac{1}{2} m (F/m \times t)^2 - \frac{1}{2} m (0)^2$

(d) $\Delta k' = \frac{1}{2} m \left(v_c + \frac{F}{m} t\right)^2 - \frac{1}{2} m v_c^2$

(e) $S = \frac{1}{2}\left(\frac{F}{m}\right) t^2$

(f) $S' = v_c t + \frac{1}{2}\left(\frac{F}{m}\right) t^2$

(g) $w = F \times S$

$w' = F \times S'$

(h) K.E. is more for the ground frame.

(i) K.E. of a body is different in different different frame.

and

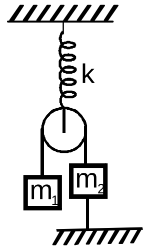
work-energy theorem hold for the moving observer.

23. $T_1 = m_1 g$

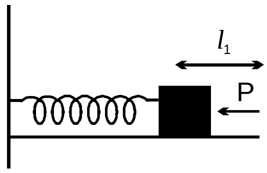
$kx = 2m_1 g$

$= \frac{1}{2} k . \frac{4m_1^2 g^2}{k^2}$

$= \frac{2\, m_1^2\, g^2}{k}$



24. $Kx = P$



$\ell_1 = \frac{P}{K} \Rightarrow K = \frac{P}{\ell_1}$

$P(\ell_1 + \ell_2) = \frac{1}{2} k \ell_2^2 - \frac{1}{2} k \ell_1^2$

$P(\ell_1 + \ell_2) = \frac{1}{2} k (\ell_2 + \ell_1)(\ell_2 - \ell_1)$

$P = \frac{1}{2} k (\ell_2 - \ell_1) \Rightarrow P = \frac{1}{2} P(\ell_2 / \ell_1 - 1)$

$\frac{\ell_2}{\ell_1} = 3$

25. $P = 3t^2 - 2t + 1$

$dW = \int_2^4 (3t^2 - 2t + 1)\, dt$

W.D. $= [t^3]_2^4 - [t^2]_2^4 + [t]_2^4$

$= (64 - 8) - (16 - 4) + 2$

$= 46$ J = change in K.E.

26. $P_{av} = \frac{\text{Total work done}}{\text{total time}}$

$= \frac{100 \times 1 \times 6 \times 9.8}{2 \times 60} = 49$ w

27. $P \times t = w$

$10 \times 10^3 \times t = 200 \times 10 \times 40$

$t = 8$ sec

28. $a = \frac{F}{m}$

$P = FV \Rightarrow V = \frac{P}{F} = \frac{P}{ma}$

$a = \frac{P}{mV} \qquad \Rightarrow V = \frac{F}{m} t$

$S = \frac{1}{2} \frac{F}{m} t^2 \qquad \Rightarrow \frac{F}{m} t = \sqrt{\frac{2P}{m}} . \sqrt{t}$

$\frac{dv}{dt} = \frac{P}{mv} \qquad \Rightarrow vdv = \frac{P}{m} dt$

$\frac{v^2}{2} = \frac{P}{m} t \qquad \Rightarrow v = \sqrt{\frac{2P}{m}} . \sqrt{t}$

$dx = \sqrt{\frac{2P}{m}}.\sqrt{t}\,dt \Rightarrow x = \sqrt{\frac{2P}{m}}.\frac{t^{3/2}}{3/2}$

$\text{Ratio} = \frac{\frac{1}{2}\frac{F}{m}.t^2.\frac{3}{2}}{\sqrt{\frac{2P}{m}}.t^{3/2}} = \frac{3}{4}$

29. $A = 10^{-2}\ m^2$

Mass coming in one second

$Av\rho = m$

$Av = 0.2\ m^3$

$10^{-2}\ V = 0.2\ m^3$

$v = 20$ m/s.

Energy required in one second

$= mgh + \frac{1}{2}mv^2$

$= 0.2\times1000\times10\times20 + \frac{1}{2} \times 0.2 \times 1000\ (20)^2$

= 80 kw

30. P = F.v

P = mav

$a = \frac{P}{mv} \Rightarrow a = \frac{P}{mv}$

$\frac{vdv}{dx} = \frac{P}{mv}$

$\Rightarrow \int_{u_i}^{u_f} \frac{v^3}{3} = \frac{P}{m}x = \frac{6^3 - 3^3}{3} = \frac{P}{m}\times 252$

$\frac{m}{p} = 4$

$v\frac{dv}{dt} = \frac{p}{m}$

$_{u_i}^{u_f}[v^2/2] = \frac{P}{m}t \Rightarrow \frac{8^2 - 3^2}{2} = \frac{t}{4}$

t = 54 sec

31. $\vec{F} = x^2y^2\hat{i} + x^2y^2\hat{j}\,(N)$

act on a particle which moves in the xy plane.

If F is conservative.

$\oint F.dr = 0$

otherwise $\oint F.dr \neq 0$

$w = \int x^2y^2dx + \int x^2y^2dy$

$\& \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y^2 & x^2y^2 & 0 \end{vmatrix}$

$\hat{i}\left(0 - \frac{\partial x^2y^2}{\partial z}\right) + \hat{j}\left(\frac{\partial x^2y^2}{\partial z} - 0\right) + \hat{k}\left(\frac{\partial x^2y^2}{\partial z} - \frac{\partial x^2y^2}{\partial y}\right)$

$2(xy^2 - 2yx^2)\hat{k}$

Force is conservative if and only if x = y.

(b) $w_{ADC} = \int_0^a 0.dy + \int_0^a a^2x^2dx = \frac{a^5}{3}$

$w_{ABC} = \int_0^a 0.dx + \int_0^a a^2y^2dy = \frac{a^5}{5}$

$w_{AC} = \int_0^a x^4dx + \int_0^a y^4dy = \frac{2a^5}{5}$ (x = y)

32. $F_y = -\frac{\partial x}{\partial y}$

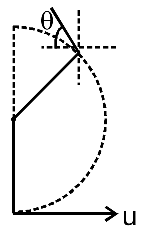
(a) $F(y) = +\omega$

(b) $F(y) = -3ay^2 + 2by$

(c) $F(y) = -\beta\ U_0 \cos\beta\ y$

33. $\sqrt{3g\ell} < \sqrt{5g\ell}$

$\cos\theta = \frac{u^2 - 2gR}{3gR} = \frac{3gR - 2gR}{3gR} = \frac{1}{3}$



$mgR(1+\cos\theta) = \frac{1}{2}m(3g\ell) - \frac{1}{2}mv^2$

$gR.\frac{4}{3} = \frac{1}{2}(3g\ell - v^2)$

$\frac{8}{3}gR = 3g\ell - v' \Rightarrow v' = \sqrt{\frac{g\ell}{3}}$

$u_{min} = v\cos\theta = \frac{1}{3}\sqrt{\frac{g\ell}{3}}$

34. $x = v\sqrt{\frac{2.2R}{g}}$

$v = x\sqrt{\frac{g}{4R}} \Rightarrow \frac{1}{2}mu^2 = mg.2R + \frac{1}{2}mv^2$

$\frac{1}{2}mu^2 = mg.2R + \frac{1}{2}m\frac{(3R)^2g}{4R}$

$\frac{1}{2}u^2 = 2gR + \frac{9Rg}{8R} \Rightarrow u = \frac{5}{2}\sqrt{gR}$

For x_{min} v should be min.

$\therefore u_{min} = \sqrt{5gR} \Rightarrow v = \sqrt{gR}$

$x = \sqrt{gR}\ .\ \sqrt{\frac{2.2R}{2}} = 2R$

35. $a_{av} = \frac{v_f - v_i}{\text{Total time}} = \frac{20 - 0}{3} = \frac{20}{3}$

$F_{avg} = m\ a_{avg} = 1 \times \frac{20}{3} = \frac{20}{3}N$

0.02 kg fuel has energy = $\frac{1}{2}mv^2$

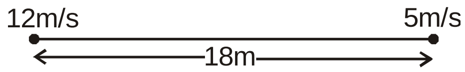
$= \frac{1}{2} \times 1 \times (20)^2 \Rightarrow$ 200 J

1 kg fuel has energy = $\frac{200}{0.02} = 10^4$ J

Energy content per unit mass of fuel
= 10,000 J/kg

Exercise-III Level - II

1



Given $a = -Kx$ $\Rightarrow$

$\frac{vdu}{dx} = -Kx$

$\Rightarrow \int_{12}^{5} v\,dv = -K\int_{0}^{18} x\,dx \Rightarrow 119 = K(324)$

$K = \frac{119}{324}$

Acceleration of particel at point

$A = \frac{119}{324} \times 18 = \frac{119}{18}$

$a_t = \frac{119}{18}$

$a_{net} = 10\,m/s^2 = \sqrt{a_t{}^2 + a_N{}^2}$

$\Rightarrow 10 = \sqrt{\left(\frac{119}{10}\right)^2 + a_N{}^2}$

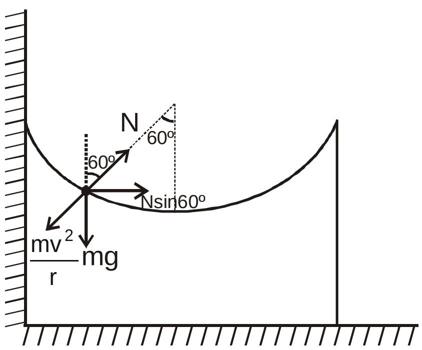
$\Rightarrow$ $a_N = 7.5\,m/s^2 \Rightarrow$ $\frac{V^2}{R} = 7.5$

$\Rightarrow$ $R = \frac{(5)^2}{7.5} = \frac{25}{7.5}$ $\Rightarrow$ R = 3.3 m

2 $N = \frac{mv^2}{r} + mg\cos 60^o$ ...(1)

from E.C. $mg\cos 60^o = \frac{1}{2}mv^2$

$v^2 = 5$ ...(2)



$\Rightarrow$ from (1) & (2)

N = 15 N

Now force on the wedge due to wall

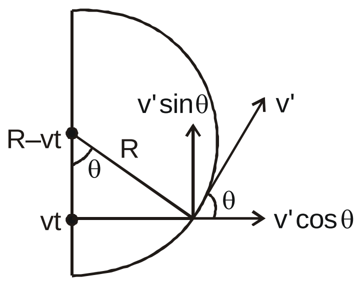
$= N \sin 60^o = 15 \times \frac{\sqrt{3}}{2} N$

3 $\therefore$ $v' \sin\theta = v$

$v' = \frac{v}{\sin\theta}$

$v' = \frac{vR}{(2Rvt - v^2t^2)^{1/2}}$

$a_N = \frac{v'^2}{R}$ **=** $\frac{v^2R^2}{(2Rvt - v^2t^2)}/R$



$a_N = \frac{vR}{(2Rt - vt^2)}$

$a_t = \frac{dv'}{dt} = -\frac{(2Rv - 2v^2t)}{(2Rvt - v^2t^2)} \times \frac{1}{2(2Rvt - v^2t^2)^{1/2}}$

$a_t = \frac{-(Rv - v^2t)}{(2Rvt - v^2t^2)^{3/2}}$

4 Given $U(x) = ax^3 - bx$

so $F(x) = -\frac{dU(x)}{dx} = b - 3ax^2$

at maximum kinetic energy U = minimum

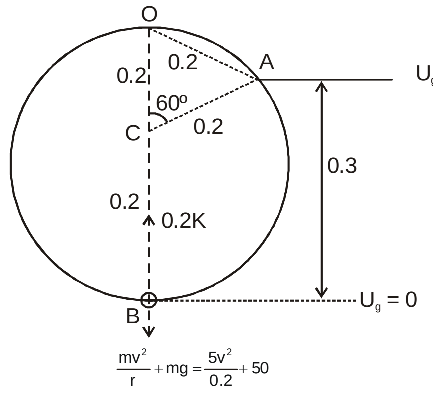
F = 0

$b - 3ax^2 \Rightarrow x = \sqrt{\frac{b}{3a}}$

5 at point B

$\Rightarrow$ $0.2k = 50 + 25v^2$ ...(1)

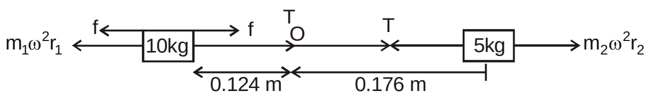
from E.C. = $15 = \frac{1}{2}K(0.2)^2 + \frac{1}{2} \times 5 \times v^2$ ....(2)



from (1) & (2) we get

k = 500 N/n

6. (i)



Now $T = m_2 \omega^2 r_2$ ...(1)

$T + f = m_1 \omega^2 r_1$ ...(2)

from (1) & (2) $f = m_1 \omega^2 r_1 - m_2 \omega^2 r_2$

$f = 10.(10)^2(0.124) - 5(10)^2(0.176)$

$124 - 88$

$f = 36N$

(ii) for slipping condition friction should be maximum

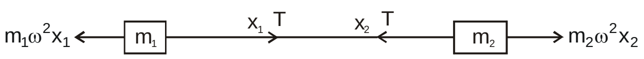
$f_{max} = 50N$

$50 = m_1 \omega^2 r_1 - m_2 \omega^2 r_2$

$50 = 0.36\,\omega^2$

$\omega^2 = \frac{50}{0.36} \Rightarrow \omega = 11.78\,\text{rad/sec}$

(iii)



$\Rightarrow m_1 \omega^2 x_1 = m_2 \omega^2 x_2$

$\frac{x_1}{x_2} = \frac{m_2}{m_1} = \frac{5}{10} \Rightarrow x_2 = 2x_1$

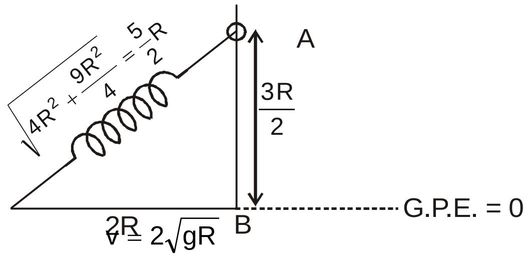
$x_1 + x_2 = 0.3$

$\Rightarrow x_1 = 0.1\,m \quad x_2 = 0.2\,m$

7 Extension is string $x = \frac{5}{2}R - 2R = \frac{R}{2}$

Now from energy conservation between point A & B.

$$mg\left(\frac{3R}{2}\right) + \frac{1}{2}\frac{4mg}{R}.\frac{R^2}{4} = \frac{1}{2}mv^2$$



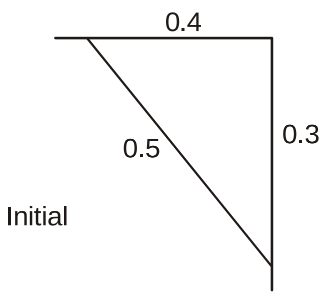
$v = 2\sqrt{gR}$

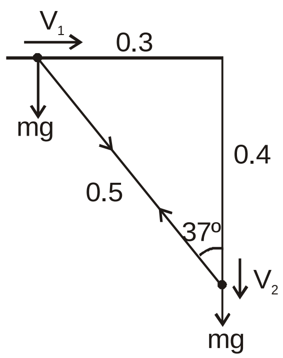
$W_f = Wmg = \Delta K$

$W_f = \frac{1}{2}m(4gR) - \frac{3}{2}mgR \Rightarrow$

$W_f = \frac{1}{2}mgR$

8.





from energy conservation

$$mgh = \frac{1}{2}m\left(v_1^2 + v_2^2\right) \qquad ...(1)$$

$$1 \times 10 \times 0.1 = \frac{1}{2}\left[v_1^2 + v_2^2\right]$$

Now $x^2 + y^2 = \ell$

$$2xv_1 + 2yv_2 = 0$$

$$0.3v_1 = 0.4v_2 \Rightarrow 3v_1 = 4v_2 \qquad ...(2)$$

from (1) & (2)

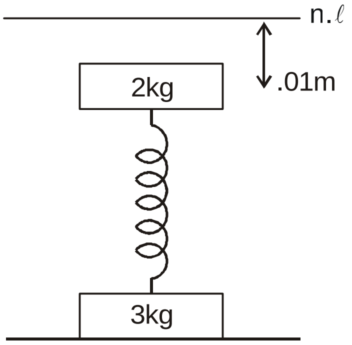
$$v_1 = \frac{4\sqrt{2}}{5}m/sec, \quad v_2 = \frac{3\sqrt{2}}{5}m/sec$$

Now $3a_1 = 4a_2$ ...(3)

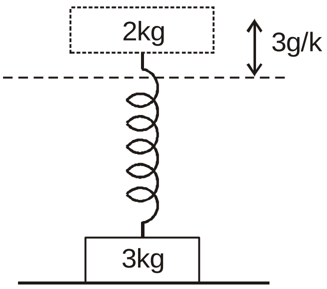
9. at equilibrium kx = mg × 2 × 10

$$k = \frac{2 \times 10}{.01} = 2000 N/m$$

To just lift the 3kg block force on the 3 kg block is upward direction kx = 3g $\Rightarrow$ x = 3kg/k



i.e.



from energy conservation

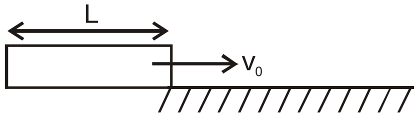
$$\frac{1}{2}k(.01 + x)^2 = \frac{1}{2}k\left(\frac{3g}{k}\right)^2 + 2g(.01 + x + 3g/k)$$

$$\Rightarrow 1000(.01 + x)^2 = \frac{1}{2}\left(\frac{90}{2000}\right) + 0.2 + 20x + \frac{6 \times 100}{2000}$$

after solving $x^2 = \frac{25}{40 \times 1000}$

x = 2.5 cm

10



(a) When x length lies on the rough surface than mass on the rough surface

$$m' = \frac{m}{L} \times x$$

$$\Rightarrow f = -\mu mg = -\mu\frac{m}{L}xg$$

(b) As the block's part enter the rough surface friction force increases so f as a function of x is.

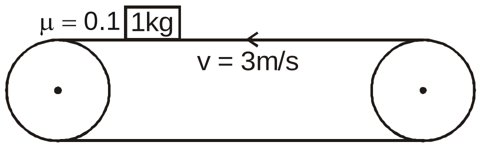
$$f = \frac{-\mu m}{L}xg, \quad a = \frac{-\mu}{L}xg$$

$v_i = v \qquad v_f = 0 \qquad a = \frac{-\mu}{L}xg$

$$\Rightarrow v\frac{du}{dx} = -\frac{\mu}{L}xg \Rightarrow \int_{v_0}^{0} v\,du = -\frac{\mu}{L}g\int_0^L xdx$$

$$v^2 = \frac{\mu}{L}gL^2 \Rightarrow v = \sqrt{\mu gL}$$

11.



Maximum heat liberated then all kinetic energy is loss in heat due to friction

$$\frac{1}{2}mv^2 = \text{work done by friction}$$

$$\Rightarrow \frac{1}{2}v^2 = 8(0.1)(g)$$

$$v^2 = 16 \Rightarrow v = 4m/s$$

Velocity with respect to belt = 7 m/s

v = 0 u = 7 m/s a = μg

$$\Rightarrow v^2 - u^2 = 2as$$

$$\Rightarrow s = 49/2$$

Heat liberate = μgs = 24.5 J

When but velocity is 5 m/s then

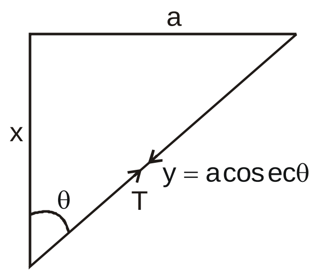
u = 9 m/s v = 0 a = μg

$$\Rightarrow s = 81/2$$

$\Rightarrow$ Heat = 40.5 J

12. **(a)** T = ky $\Rightarrow$ $T = \frac{2mg}{a} \times a\,\text{cosec}\,\theta$

T = 2mg cosec θ
At equilibrium
T cos θ = mg



2 mg cot θ = mg
cot θ = 1/2

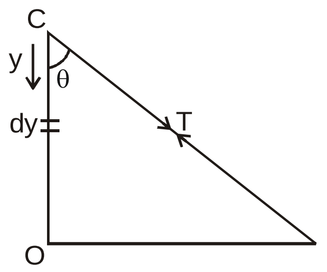
By fig cot θ = $\frac{x}{a}$

$\therefore \frac{x}{a} = \frac{1}{2} \Rightarrow x = \frac{1}{2}$

(b) $dF_{Tension} = Kydy$

$F_{Tension} = \int_0^a kydy = k\left[\frac{y^2}{2}\right]_0^a = k\frac{a^2}{2} = \frac{2mg}{a} \times \frac{a^2}{2}$

$F_{Tension}$ = mga



$W_{total} = \Delta KE$

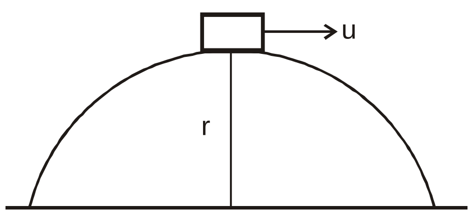
$W_{Tension} + W_{gravity}$ = KF
mga + mga = 1/2 mv^2

2mga = 1/2 mv^2 ; $v = 2\sqrt{ag}$ Ans.

For maximum path $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

x = 2a

13.

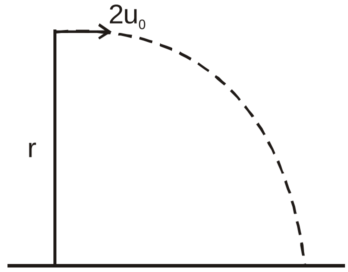


Particle leave the surface at top when

$$U = \sqrt{rg}$$

Now

$$T = \sqrt{\frac{2r}{g}}$$



$R = 2u_0\sqrt{\frac{2r}{g}} \Rightarrow R = 2.\sqrt{rg}\sqrt{\frac{2r}{g}} = 2\sqrt{2}\,r$

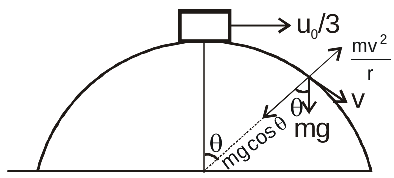
Now when $U = U_0/3$
from energy conservation

$$\frac{1}{2}m\frac{U_0^2}{9} + mgR(1-\cos\theta) = \frac{1}{2}mv^2 \quad ...(1)$$

force balance $\frac{mv^2}{R} = mg\cos\theta$ ...(2)

from equation (1) & (2)

$$\frac{3}{2}rg\cos\theta = rg + \frac{U_0^2}{18}$$



put $U_0 = \sqrt{rg} \Rightarrow \cos\theta = \frac{19}{27}$

Height from the ground at which it leaves the hamisphere = $r\cos\theta = \frac{19}{27}r$

Exercise-IV Level - I

1. Using the relation

$$\frac{mv^2}{r} = \mu R, R = mg$$

$$\Rightarrow \frac{mv^2}{r} = \mu mg \text{ or } v^2 = \mu rg$$

$$\Rightarrow v^2 = 0.6 \times 150 \times 10$$

or $v = 30 ms^{-1}$

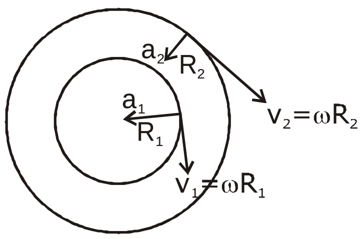
2. **(B)**
For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the centre of circle or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector, therefore, velocity will be perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle.

3. **(D)**

$$a_1 = \frac{v_1^2}{R_1} = \frac{\omega^2 R_1^2}{R_1} = \omega^2 R_1$$

$$a_2 = \frac{v_2^2}{R_2} = \omega^2 R_2$$

Taking particle of mass equal



$$\frac{F_1}{F_2} = \frac{ma_1}{ma_2} = \frac{\omega^2 R_1}{\omega^2 R_2} = \frac{R_1}{R_2}$$

4. **(D)**
$s = t^3 + 5$

$\therefore$ Speed, $v = \frac{ds}{dt} = 3t^2$

and rate of change of speed, $a_t = \frac{dv}{dt} = 6t$

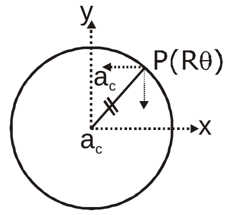
$\therefore$ Tangential acceleration at t = 2s,
$a_t = 6 \times 2 = 12 ms^{-2}$
and at t = 2s, $v = 3(2)^2 = 12 ms^{-1}$
$\therefore$ Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{144}{20} ms^{-2}$$

$\therefore$ Net acceleration $= \sqrt{a_t^2 + a_i^2} \approx 14 \, ms^{-2}$

5. **(C)**
For a particle in uniform circular motion,

$$\vec{a} = \frac{v^2}{R} \text{ towards centre of circle}$$



$$\therefore \vec{a} = \frac{v^2}{R}(-\cos\theta\hat{i} - \sin\theta\hat{j})$$

or $\vec{a} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

6. **(C)**
As their period of revolation is same, so is their angular speed. Centripetal acceleration is circular path, $a = \omega^2 r$.
Thus,

$$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

7. **(B)**
The work is stored as the PE of the body and is given by,

$$U = \int_{x_1}^{x_2} F_{external} dx$$

$$U = \int_{x_1}^{x_2} kx dx$$

$$= \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{800}{2}[(0.15)^2 - (0.05)^2]$$

= 400 [0.2×0.1]
= 8 J

8. **(B)**
P = constant

$\Rightarrow \quad Fv = P \quad [\because P = \text{force} \times \text{Velocity}]$

$\Rightarrow \quad Ma \times v = P$

$\Rightarrow \quad v \times \left[\frac{vdv}{ds}\right] = \frac{P}{M} \quad \left[\because a = \frac{vdv}{ds}\right]$

$$\Rightarrow \int_0^v v^2 dv = \int_0^s \frac{P}{M} ds$$

[Assuming at t = 0 it starts from rest, ie, from s = 0]

or $\frac{v^3}{3} = \frac{P}{M}s$

or $v = \left(\frac{3P}{M}\right)^{1/3} s^{1/3}$

$\Rightarrow \frac{ds}{dt} = ks^{1/3}$ $\left[k = \left(\frac{3P}{M}\right)^{1/3}\right]$

$\Rightarrow \int_0^s \frac{ds}{s^{1/3}} = \int_0^t Kdt$

or $\frac{s^{2/3}}{2/3} = Kt$ or $s^{2/3} = \frac{2}{3}Kt$

or $s = \left(\frac{2}{3}K\right)^{3/2} \times t^{3/2}$ or $s \propto t^{3/2}$

9. **(B)**

$W_1 = \frac{1}{2}kx_1^2 = \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2$

= 6.25 J

$W_2 = \frac{1}{2}k(x_1 + x_2)^2$

$\frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2} + 5 \times 10^{-2})^2 = 25$ J

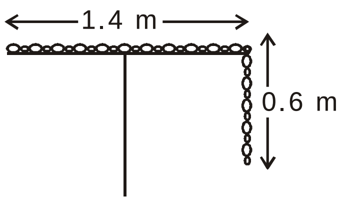
Net work done = W_2 - W_1
= 25 - 6.25
= 18.75 J
= 18.75 N-m

10. **(B)**

Work done in displacing the particle

$W = \vec{F}.\vec{r} = (5\hat{i} + 3\hat{j} + 3\hat{k}).(2\hat{i} - \hat{j}) = 7$ J

11. **(B)**



Mass per unit length = $\frac{M}{L}$

$= \frac{4}{2} = 2 kg/m$

The mass of 0.6 m of chain
= 0.6 × 2 = 1.2 kg

The height of centre of mass of hanging part

$h = \frac{0.6 + 0}{2} = 0.3 m$

Hence, work done in pulling the chain on the table = work done against gravity force.
ie, W = mgh
= 1.2 × 10 × 0.3
= 3.6 J

12. **(A)**

From given information a = - kx, where a is acceleration, x is displacement and k is a
proportionality constant,

$\frac{vdv}{dx} = -kx \quad \Rightarrow vdv = -kxdx$

Let for any displacement from 0 to x, the velocity changes from v_0 to v.

$\Rightarrow \int_{v_0}^{v} dv = -\int_0^x kxdx \quad \Rightarrow \frac{v^2 - v_0^2}{2} = -\frac{kx^2}{2}$

$\Rightarrow m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mkx^2}{2}$

$\Rightarrow \Delta K \propto x^2$ [ΔK is loss in K.E.]

13. **(A)**

Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE. According to conservation of energy,

$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2 \Rightarrow kL^2 = \frac{(Mv^2)}{M}$

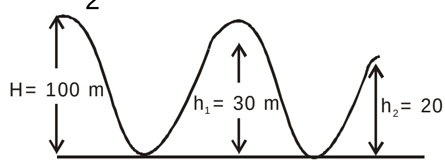
or $MkL^2 = p^2 \quad (\because p = Mv)$

$\Rightarrow P = L\sqrt{Mk}$

14. **(A)**

According to conservation of energy,

$mgH = \frac{1}{2}mv^2 + mgh_2$



$\Rightarrow mg(H - h_2) = \frac{1}{2}mv^2$

or $v = \sqrt{2g(100 - 20)}$

or $v = \sqrt{2 \times 10 \times 80} = 40 m/s$

15. **(A)**

$F = ma = \frac{mv}{T} \quad \left(\therefore a = \frac{v - 0}{T}\right)$

Instantaneous poer = F_v
= mav

$= \frac{mv}{T}.at \quad = \frac{mv}{T}.\frac{v}{T}.t \quad = \frac{mv^2}{T^2}t$

16. **(D)**

According to work - energy theorem,
W = ΔK

Case I $-F \times 3 = \frac{1}{2}m\left(\frac{v_o}{2}\right)^2 - \frac{1}{2}mv_0^2$

Where F, is resistive force and v_0 is initial speed.

Case II Let, the further distance travelled by the bullet before coming to rest is s.

$$\therefore -F(3+s) = K_f - K_i = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \quad -\frac{1}{8}mv_0^2(3+s) = -\frac{1}{2}mv_0^2$$

or $\frac{1}{4}(3+s) = 1$

or $\frac{3}{4} + \frac{s}{4} = 1$

or s = 1 cm

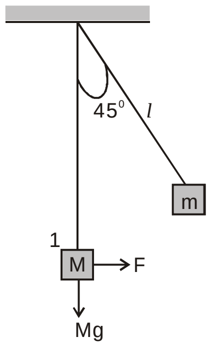
17. (D)

Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem. Let us assume that body is taken slowly so that its speed does not change, then

$\Delta K = 0$

$= W_F + W_{Mg} + W_{tension}$

[symbols have their usual meanings]



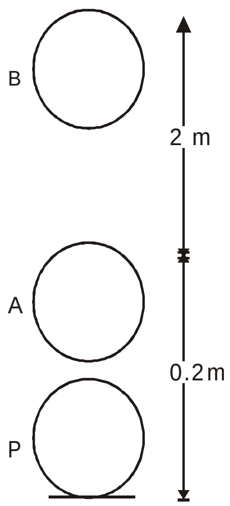
$W_F = F \times l \sin 45^0 = \frac{Fl}{\sqrt{2}}$

$W_{Mg} = Mg\,(l - l\cos 45^0)$, $W_{tension} = 0$

$\therefore F = Mg(\sqrt{2} - 1)$

18. (D)

The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA is a ms^{-2} [assumed to be constant] in upward direction and velocity of ball at A is v m/s.



Then for PA,

$v^2 = 0^2 + 2a \times 0.2$

For AB, $0 = v^2 - 2 \times g \times 2$

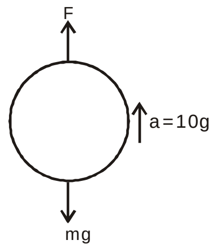
$\Rightarrow v^2 = 2g \times 2$

From above equations,

$a = 10g = 100ms^{-2}$

Then for PA, FBD of ball is F - mg = ma

[F is the force



exted by hand on ball]

$\Rightarrow F = m(g+a)$

$= 0.2(11g)$

$= 22$ N

Alternate using work energy theorem

$W_{mg} + W_F = 0$

$\Rightarrow -mg \times 2.2 + F \times 0.2 = 0$

or F = 22N

19. (A)

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$$

For minimum value of V, $\frac{dv}{dx} = 0$

$$\Rightarrow \quad \frac{4x^3}{4} - \frac{2x}{4} = 0$$

$\Rightarrow x = 0, \ x = \pm 1$

So, $\quad V_{min}(x = \pm 1) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$ J

Now, $K_{max} + V_{min}$ = Total mechanical energy

$$\Rightarrow \quad K_{max} = \left(\frac{1}{4}\right) + 2$$

or $\quad K_{max} = \frac{9}{4}$

or $\quad \frac{mv^2}{2} = \frac{9}{4}$

or $\quad v = \frac{3}{\sqrt{2}} ms^{-1}$

20. (A)

$$a = \frac{F_k}{m} = \frac{15}{2} = 7.5 \, m/s^2$$

Now, $ma = \frac{1}{2}kx^2$

$\Rightarrow 2 \times 7.5 = \frac{1}{2} \times 10000 \times x^2$

or $\quad x^2 = 3 \times 10^{-3}$

or $\quad x = 0.055$ m

or $\quad x = 5.5$ cm

21. (B)

Question is somewhat based on approximations. Let mass of athlete is 65 Kg.

Approx velocity from the given data is 10m/s.

So, $KE = \frac{65 \times 100}{2} = 3250\,J$

So, option (d) is the most probable answer.

22. (D)

Given, $\frac{dk}{dt} = \text{constant}$

$\Rightarrow\ k \propto t$

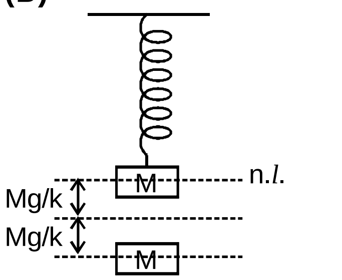
$\Rightarrow\ v \propto \sqrt{t}$

Also, $P = Fv = \frac{dk}{dt} = \text{constant}$

$\Rightarrow\ F \propto \frac{1}{v}$ $\Rightarrow\ F \propto \frac{1}{\sqrt{t}}$

Exercise-IV Level - II

1. (B)



Maximum Extension = $\dfrac{2Mg}{K}$

2. Ball will lose contact with inner sphere A

$\cos\theta = \dfrac{2}{3} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{2}{3}\right)$

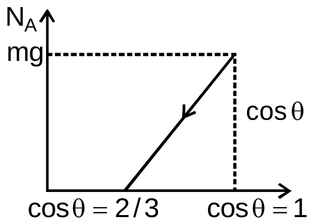
Now, N starts acting towards the centre and make contact with outer sphere.

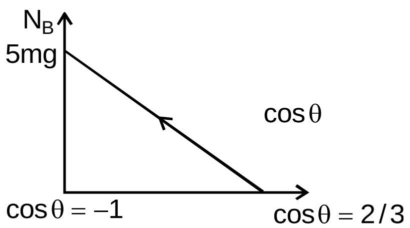
$\therefore\ \theta \le \cos^{-1}\left(\dfrac{2}{3}\right)$

$N_A = mg\,(3\cos\theta - 2)$; $N_B = 0$

and for $\theta \ge \cos^{-1}\left(\dfrac{2}{3}\right)$

$N_A = 0$ and $N_B = mg(2 - 3\cos\theta)$



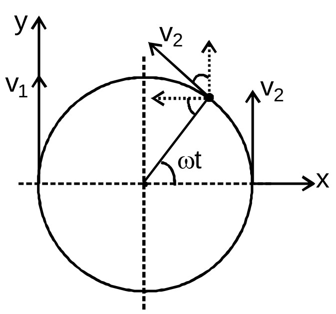


3. (A)

In a conservative field work done does not depends upon the path. The gravitational field is conservative field.

$\therefore\ W_1 = W_2 = W_3$

4.



$P = m\,|\vec{v}_2 - \vec{v}_1|$

$= m[-v_2 \sin\omega t\,\hat{i} + v_2\cos\omega t - v_2)\hat{j}]$

5. (A)

$F = kx \quad \Rightarrow \quad \dfrac{du}{dx} = -kx$

$u = -\dfrac{kx^2}{2}$

6. (D)

$\dfrac{1}{2}5mg\ell = \dfrac{1}{2}m\dfrac{5g\ell}{4} + mg(1-\cos\theta)$

$\cos\theta = -7/8$

Hence, $3\pi/4 < \theta < \pi$

7. 8

$a = g/3$, $T = 4.8$ N, $S = 1/2\,at^2 = 5/3$ m

$\Rightarrow$ W = TS = 8 (in joule)

8. (D)

$T = m\omega^2\ell$

$324 = 0.5\,\omega^2\,(0.5)$

$\omega = 36$ Radian/S

9. Applying energy conservation

$\dfrac{1}{2}kx^2 + \mu Nx = \dfrac{1}{2}mv^2$

$\Rightarrow \dfrac{1}{2} \times 2 \times (0.06)^2 + 0.1 \times 1.8 \times 0.06$

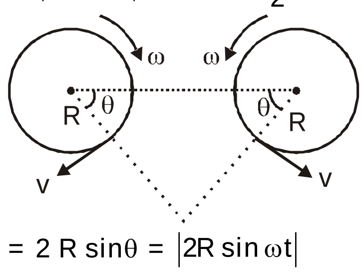
$= \dfrac{1}{2} \times 0.18 \times \left(\dfrac{N}{10}\right)^2$

or N = 4

10. (A)

$V_2 - V_1 = V_{Rel}$

$\therefore\ \left|\vec{V}_2 - \vec{V}_1\right| = 2R\sin\dfrac{(2\theta)}{2}$



$= 2R\sin\theta = \left|2R\sin\omega t\right|$