JEE-MAIN

TOPIC

CENTRE OF MASS

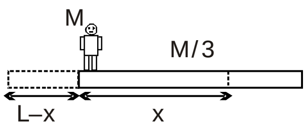
SOLUTIONS

CENTRE OF MASS

Exercise-I

1. (B)
Let x be the displacement of man. Then displacement of plank is L – x.
For centre of mass to remain stationary

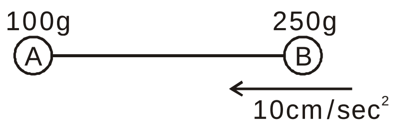
$\frac{M}{3}(L - x) = M \cdot x$



$\Rightarrow x = \frac{L}{4}$

2. (C)
Centre of mass of two particle system lies on the line joining the two particles

3. (A)
$\vec{F}_{net} = 0$



so $\vec{a}_{com} = 0$

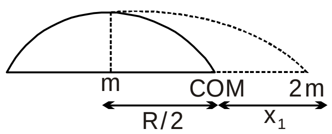
$m_1\vec{a}_1 + m_2\vec{a}_2 = 0$

$100 \times a_1 + 250(-10) = 0$

$a_1 = 25 \text{ cm/sec}^2$ east

4. (C)
For square plate ABCD centre of mass is at O but when two point masses of 3 kg placed at C & D then centre of mass shifts on the line OY because centre of mass is in that part which has higher mass.

5. (C)
Centre of mass hits the ground at the position where original projectile would have landed.



$\frac{m.R}{2} = 2mx_1 \Rightarrow x_1 = \frac{R}{4}$

$\therefore$ Distance $= R + \frac{R}{4} = \frac{5R}{4}$

6. (C)
Centre of mass will not move in horizontal direction. Let x be the displacement of boat.

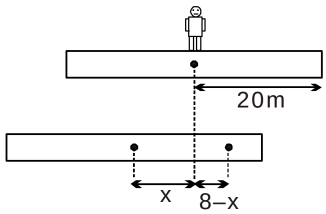
$80(8 - x) = 200x$

$640 - 80x = 200x$

$x = 2.3$ m

Now, Required distance from the shore.

$= 20 - (8 - x)$



$= 20 - (8 - 2.3)$

$= 20 - 5.7$

$= 14.3$ m

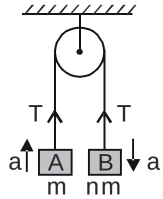
7. (C)
If mass of block A is m and the mass of block B is nm.

$nmg - T = nma$

$T - mg = ma$

After solving

$a = \frac{(n-1)g}{n+1}$



acceleration of the centre of mass of system.

$a_{COM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$= \frac{ma - nma}{(n+1)m} = \frac{a - na}{n+1} = \left(\frac{n-1}{n+1}\right)^2 g$

8. (B)
When internal force acts.
Net force is zero.

$\therefore F = \frac{dP}{dt}$ So momentum is conserved.

Therefore internal force will not change the linear momentum.
But due to force, K.E. increases.

9. (D)
Speed is constant so K.E. $\rightarrow$ Constant
Gravitational potential energy change.
$\because$ Momentum = $m\vec{v}$
$\because$ Direction of $\vec{v}$ changes
$\therefore$ Momentum changes

10. (D)

$\frac{P^2}{2m}$ = K.E.

$\ln \frac{P^2}{2m} = \ln$ K.E.

$2\ln P - \ln(2m) = \ln$ K.E.
So the graph between lnp & lnk is straight line with intercept.

11. (D)
The acceleration of both balls = – g

$a_{com} = \frac{m_1(-g) + m_2(g)}{m_1 + m_2} = -g$

12. (B)
Here net force = 0
means momentum is conserved.
$p_i = p_f$

$0 = \vec{p}_1 + \vec{p}_2 \quad \Rightarrow \quad \vec{p}_1 = -\vec{p}_2$

K.E. $= \frac{p^2}{2m} \quad \Rightarrow \quad \therefore \frac{K_1}{K_2} = \frac{m_2}{m_1}$

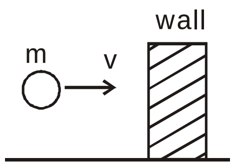
13. (A)
According to Newton's second law of motion.

$\vec{F} = \frac{d\vec{p}}{dt}$

If $\vec{F}_{net} = 0$

then $\vec{p}$ = conserved

14. (A)



Initial momentum of body = mv
& final momentum of body = – mv
Change in momentum = 2mv

15. (C)

$\vec{F}_{net} = 0$

then $\vec{p}$ = conserved

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

$\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$m\vec{v}_3 = -m(\vec{v}_1 + \vec{v}_2)$

$\therefore \vec{v}_3 = -\left[(3\hat{i} + 2\hat{j}) + (-\hat{i} - 4\hat{j})\right]$

$\vec{v}_3 = -2\hat{i} + 2\hat{j}$

16. (A)

$\vec{F}_{net} = 0$

then $\vec{p}$ = conserved
$p_i = p_f$
$m_1v = m_2(0) + (m_1 - m_2)\, v_1$

$v_1 = \frac{m_1 v}{(m_1 - m_2)}$

17. (A)
As $f_{net} = 0$ from momentum conservation

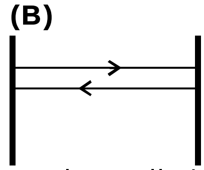
$(A-4)v_1 = 4v \quad \Rightarrow \quad v_1 = \frac{4v}{(A-4)}$

18. (C)
C_1 will move but C_2 will be stationary with respect to the ground.

19. (a) [B] (b) [C]
(a) It could be non-zero, but it must be constant.
(b) It could be non-zero and it might not be constant.

20. (a) [C]
$(F_{net})_x = 0$
$\therefore$ Momentum component parallel to the track is conserved.
but in y direction F_{net} is not equal to zero.
So momentum is not conserved in y direction.

21. (B)



Total travelled distance = 2d
then

Time between two collisions = $\frac{2d}{v_0}$

So no. of collision/sec = $\frac{v_0}{2d}$

Impulse in one collision

$= mv_0 - (-mv_0) = 2mv_0$

$$F = 2mv_0 \times \frac{v_0}{2d} = \frac{mv_0^2}{d}$$

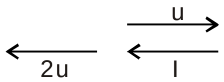
22. (B)

Impulse = change in momentum

$-I = -m2u - mu$

$I = 3mu$

W.D. = change in K.E.



$$\text{W.D.} = \frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2$$

$$= \frac{3}{2}mu^2 \quad \Rightarrow \quad \text{W.D.} = \frac{Iu}{2}$$

23. (C)

Impulse = change in momentum

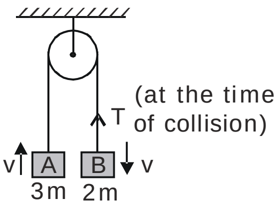
$$\int F.dt = \Delta P$$

Given $\int F.dt = J$

Now, Contact time is twice than the earlier.

$$\int \vec{F}.2dt = J' \Rightarrow J' = 2J$$

24. (D)



Impulse = change in momentum

So, $-T\Delta t = 2mv - mu$ (for bullet)

$I = T\Delta t = 3mv$ (for mass 3m)

$3mv = 2mv - mu$

$$v = u/5 \quad \Rightarrow \quad I = \frac{3mu}{5}$$

25. (B)

If e = 1 then angle = 45°

If 0 < e < 1 then angle is less than 45° with the horizontal. So 30° is not possible.

26. (A)

In inelestic collision, due to collision some fraction of mechanical energy is retained in form of deformation potential energy.

$\therefore$ thus K.E. of particle is not conserved.

In absence of external forces momentum is conserved.

27. (C)

$\because$ e = 1

As collision is elastic therefore $v_i = v_f$

$$\text{So } \Delta K = 0 \quad \Rightarrow \quad k_f = k_i = \frac{1}{2}m\left(u_1^2 + u_2^2\right)$$

28. (C)

In elastic collision e = 1, Energy is conserved because colliding particles regain their shape and size completely after collision. Due to F_{net} on the system is zero, momentum is conserved.

29. (C)

In absence of external force. Momentum of the system is conserved.

30. (C)

If e = 1 and $m_1 = m_2$ then after collision velocity interchange

31. (B)

from energy conservation

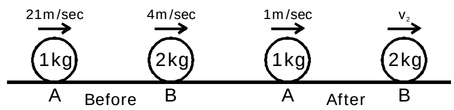
$$mgl = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gl}$$

from momentum conservation

$$m\sqrt{2gl} = mv' \quad \Rightarrow \quad v' = \sqrt{2gl}$$

$$KE = \frac{1}{2}m \times 2gl = mgl$$

32. (B)



$$21 \times 1 - 4 \times 2 = 1 + 2v_2$$

$$21 - 8 = 1 + 2v_2$$

$$2v_2 = 12 \quad \Rightarrow \quad v_2 = 6\text{m/sec}$$

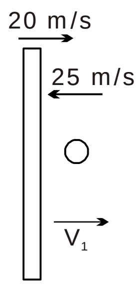
$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{6-1}{21+4} = \frac{5}{25} = \frac{1}{5}$$

e = 0.2

33. (A)

Let v_1 is the velocity of wall after collision.

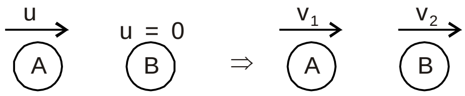
$$e = \frac{V_1 - 20}{20 - (-25)} \quad (e = 1)$$



$v_1 = 65$ m/s

34. (A)

Before Collision After Collision



$e = \dfrac{v_2 - v_1}{u} \Rightarrow eu = v_2 - v_1$ .... (1)

Now from momentum conservation

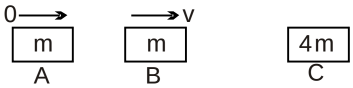
$mu = mv_1 + mv_2$

$u = v_1 + v_2$ .... (2)

from (1) and (2) $\dfrac{v_1}{v_2} = \dfrac{1-e}{1+e}$

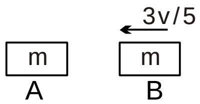
35. (A)

1^{st} Collision



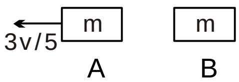
2^{nd} Collision

Velocity of B $v = \dfrac{mv + 4m(0-v)}{5m} = \dfrac{3m}{5}$



After collision of A and B.

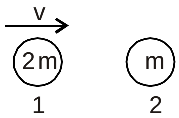
$\therefore$



36. (B)

Let mass of ball 2 is m and mass of ball 1 is 2 m.

$v_1 = \dfrac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2}$



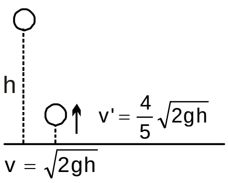
$\dfrac{v}{3} = \dfrac{2mv + em(0-v)}{3m} \Rightarrow e = 1$

So elastic collision.

37. (C)

Just before collision, speed of ball $v = \sqrt{2gh}$

and just after collision $v' = \dfrac{80}{100}\sqrt{2gh} = \dfrac{4}{5}\sqrt{2gh}$



$v^2 - u^2 = 2aS$

Let h' is the maximum height after collision.

$0 - \left(\dfrac{4}{5}\sqrt{2gh}\right)^2 = 2x\,(-g) \times h'$

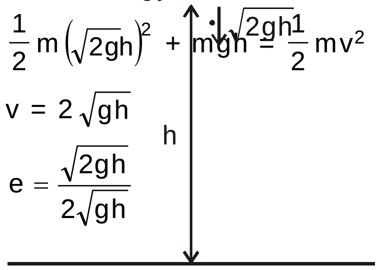
$\dfrac{16}{25} \times 2gh = 2gh$

$h' = \dfrac{16}{25}h$

38. (A)

From energy conservation

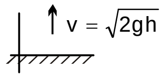
$\dfrac{1}{2}m\left(\sqrt{2gh}\right)^2 + mgh = \dfrac{1}{2}mv^2$



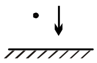
$v = 2\sqrt{gh}$

$e = \dfrac{\sqrt{2gh}}{2\sqrt{gh}}$

$\therefore\ e = \dfrac{1}{\sqrt{2}}$,



39. (C)



$\sqrt{2 \times 10 \times 5} = 10$ m/sec.

$\therefore\ \dfrac{10}{10} + \dfrac{2 \times e \times 10}{10} + \dfrac{2 \times e^2 \times 10}{10} + \ldots.$

$1 + 2\,[e + e^2 + \ldots.]$

$1 + \dfrac{2e}{1-e} = 3$ sec.

40. (A)

$v = Av_1 + v_2$ (1)

$1 = \dfrac{v_1 - v_2}{v} \quad \Rightarrow \quad v = v_1 = v_2$

$v_1 = \dfrac{2v}{A+1} \qquad v_2 = v\left(\dfrac{1-A}{1+A}\right)$

41. (C)

$5 \times 10 = \dfrac{5}{2}(0) + \dfrac{5}{2}(v_1)$

$\Rightarrow v_1 = 20$ m/sec

$KE = \dfrac{1}{2} \times \dfrac{5}{2}(20)^2 - \dfrac{1}{2} \times 5(10)^2$

$= 500 - 250 = 250$ J.

42. (B)

$E_i = \dfrac{1}{2}mu_1^2 + \dfrac{1}{2}mu_2^2$

$m(u_1 - u_2) = 2mu \quad \Rightarrow \quad u = \dfrac{u_1 - u_2}{2}$

Energy loss $= \dfrac{1}{2} \times \dfrac{2m}{4}(u_1 - u_2)^2 - \dfrac{1}{2}m\left(u_1^2 + u_2^2\right)$

43. (D)

$mu = nmu_1 + 1mu_2$

$$1 = \frac{u_1 - u_2}{u}$$

$u = n(u+u_2) + u_2$

$u = nu + nu_2 + u_2$ ....(1)

$u = nu_1 + u_1 - u$

$2u = (n+1)\, u_1$ ....(2)

$$\frac{\frac{1}{2}nmu_1^2}{\frac{1}{2}mu^2} = \frac{n\frac{4u^2}{(n+1)^2}}{u^2} = \frac{4n}{(n+1)^2}$$

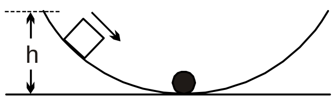
47. (C)

$\Delta p = 0.1\ (6+4)$

$= 0.1 \times 10 = 10$ NS

48. (A)

$$mgh = \frac{1}{2}mv^2$$



$v = \sqrt{2gh}$

By momentum conservation

$m\sqrt{2gh} + 0 = 2mv'$

$$v' = \frac{\sqrt{2gh}}{2}$$

By energy conservation

$$\frac{1}{2}(2m)v'^2 = 2mgh', \quad m\frac{(2gh)}{4} = 2mgh'$$

$$h' = \frac{h}{4}$$

49. (B)

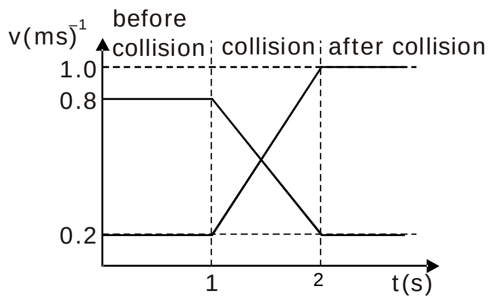
Let mass of ball 2 is m and mass of ball 1 is 2 m.

$$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2}$$

$\xrightarrow{v}$ (2m) 1 (m) 2 $\quad \frac{v}{3} = \frac{2mv + em(0 - v)}{3m} \Rightarrow e = 1$

So elastic collision.

50. (D)



(i) $\because$ v is +ve for both.

(ii) Yes (when maximum compression)

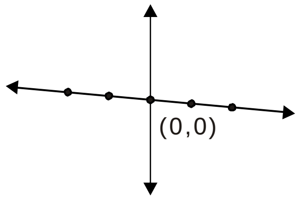
(iii) $\because$ S have greater velocity after collision then R have before collision and K.E. of S will be less then initial K.E. of R

$$\frac{1}{2}m_sV_s^2 < \frac{1}{2}m_R(V_R)^2$$

but $V_S > V_R$ So $m_s < m_R$

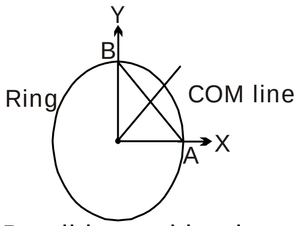
Exercise-II

1. (C, D)



2. (B, D)

Center of mass of ring is at centre and centre of mass of chord AB is at its mid point so centre of mass of this combination lie at the line which makes 45° with x axis.



Possible combination

$$\left(\frac{R}{3},\frac{R}{3};\frac{R}{4},\frac{R}{4}\right)$$

3. (A, B)

In case C & D centre of mass of rod me be at centre but in A & B centre of mass is not at centre because of non-uniform distribution. (Density continously changes.)

4. (C)

Centre of mass of uniform semi-circular disc is at $\frac{4R}{3\pi}$

Centre of mass of uniform semi-circular ring is at $\frac{2R}{\pi}$

Centre of mass of solid hemi-sphere is at $\frac{3R}{8}$

Centre of mass of hemi-sphere shell is at $\frac{R}{2}$

C	T	H	R	S	D
$\frac{h}{4}$	$\frac{h}{3}$	$\frac{R}{2}$	$\frac{2R}{\pi}$	$\frac{3R}{8}$	$\frac{4R}{3\pi}$

5. (B, C)

$F_{net} = 0$

$\Rightarrow ma_{com} = 0$

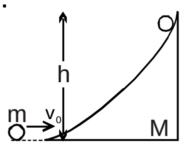
It means $a_{com} = 0$ $\Rightarrow V_{com}$ = constant.

6. (B)

As net force in x direction is zero. So from momentum conservation.

$mV_0 = (M + m)V_2$

$$V_2 = \frac{MV_0}{M+m}$$



7. (B, D)

Velocity of center of mass

$$V_{COM} = \frac{MV+mV}{M+m} = V$$

So both are at rest with respect to centre of mass. And kinetic energy is converted into potential energy.

8. (C)

By Energy conservation

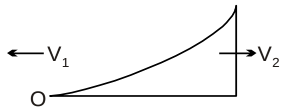
$$\frac{1}{2}mv_0^2 = \frac{1}{2}(M+m)\left(\frac{mv_0}{M+m}\right)^2 + mgh$$

After solving

$$\Rightarrow h = \left(\frac{M}{M+m}\right)\frac{V_0^2}{2g}$$

9. (C)

V_1 is the velocity of particel and V_2 is the velocity of wedge.



$(V_1 + V_2)$ = vel. of particle w.r.t. wedge

$$\Rightarrow -\left(\frac{mV_0+M(-V_0)}{M+m}\right) + \left(\frac{mV_0+mV_0}{M+m}\right)$$

$= V_0$

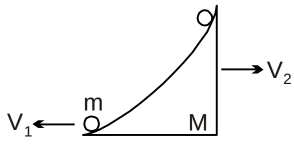
10. (B, C)

As net force in x direction is zero.

So by momentum conservation

$Mv_2 - mv_1 = mV_0$

and $V_1 + V_2 = V_0$



11. (B)

As net force in x direction is zero.

So by momentum conservation

$MV_2 - mV_1 = mV_0$ ....(1)

$V_1 + V_2 = V_0$ ....(2)

By solving

$$V_1 = V_0\left(\frac{M-m}{M+m}\right)$$

12. (A, B, C, D)

(a) From Q. 9

$\because V_1 + V_2 = V_0$

$V_2 = V_0 - V_0\left(\frac{M-m}{M+m}\right)$

$= \frac{(M+m)V_0 - V_0M + V_0m}{M+m}$

$= \frac{2mV_0}{M+m}$

$K.E. = \frac{1}{2} \times M \times \frac{4m^2V_0^2}{(M+m)^2}$

$[\because h = \frac{M}{(m+M)}\frac{V_0^2}{2g}]$

$\therefore K.E. = \frac{4m^2}{(m+M)}gh$

(b) $V_2 = \frac{2mv_0}{M+m}$

(c) $\Delta K.E. = k_f - k_i$

$= \frac{1}{2}M\left(\frac{4m^2V_0^2}{(M+m)^2}\right) - 0$

$= \frac{4mM}{(m+M)^2}\left(\frac{1}{2}mV_0^2\right)$

(d) $\because$ vel. of wedge $V_2 = \frac{2mV_0}{M+m}$

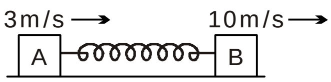
Vel. of particle $V_1 = V_0\left(\frac{M-m}{M+m}\right)$

$V_{COM} = \frac{MV_2 + (-mV_1)}{M+m}$

$= \frac{mv_0}{M+m}$

13. (B, C)

Maximum extension when they have same velocity.



Momemtum conservation

$5 \times 3 + 2 \times 10 = 7V$

$\Rightarrow V = 5$ m/s

From energy conservation

$\frac{1}{2} \times 5\ (3)^2 + \frac{1}{2} \times 2 \times 10^2$

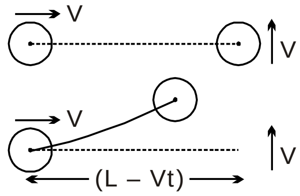
$= \frac{1}{2} \times (1120) \times x^2 + \frac{1}{2} \times 7 \times 5^2$

$45 + 200 = 1120x^2$

x = 25 cm

$T = 2\pi\sqrt{\frac{5 \times 2}{7 \times 1120}} = 0.071\ \pi$

14. (A, C)



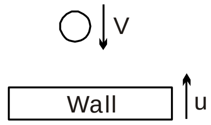
$\sqrt{(Vt)^2 + (L - vt)^2} \le L$

$2V^2t^2 + L^2 - 2LVt \le L^2$

$Vt - L \le 0$

$t \le \frac{L}{V}$

15. (B, C)



$e = 1 \rightarrow$ Given

Before collision

$u_1 = -v$

$u_2 = u$

After collision

$v_1 = ?$

$v_2 = u$

On solving $e = \frac{V_2 - V_1}{u_1 - u_2} \Rightarrow v_1 = v + 2u$

$\int F \cdot \Delta t = \Delta P$

Average elastic force

$\frac{\Delta P}{\Delta t} = \frac{m(V+2u) + mu - (-mv + mu)}{\Delta t}$

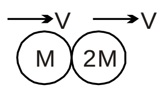
$= \frac{2m(u+v)}{\Delta t}$

(ii) Kinetic energy of the ball increases by

$= K_f - K_i$

$= \frac{1}{2}m(2u + v)^2 - \frac{1}{2}mv^2 = 2mu\ (u + v)$

16. (A, B, D)



For minimum kinetic energy

$MV_0 = 3MV$

$\Rightarrow V = V_0/3$

$\therefore \Delta K = -\left[\frac{1}{2}3m\left(\frac{V_0}{3}\right)^2 - \frac{1}{2}mv_0^2\right]$

= 2 Joule

17. (A, B, C)

Momentum conservation
$1 \times 21 - 2 \times 4 = 1 \times 1 + 2 \times V'$
$V' = 6$ m/s

$$e = \frac{6-1}{21+4} = \frac{1}{5}$$

Loss of kinetic energy = $k_f - k_i$

$$= \frac{1}{2} \times 1 \times (1)^2 + \frac{1}{2} \times 2 \times (6)^2 - \left(\frac{1}{2} \times 1 \times (21)^2 + \frac{1}{2} \times 2 \times (4)^2\right)$$

= 200 J

18. (D)

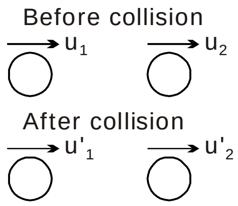
When velocity is same means maximum compression.
$\therefore$ Maximum loss
$M_R \times 8 = M_R \times 0.4 + M_s \times 1$
$0.4M_R = M_S$
$\therefore M_R > M_S$

19. (A, B, C, D)

Inelastic collision
$0 < e < 1$

20. (B, D)

Given

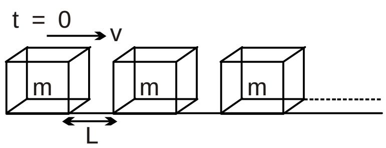


$u_2 - u_1 = v_1$ and $u_2' - u_1' = v_2$

$$e = \frac{u_2' - u_1'}{u_1 - u_2}$$

$\vec{v}_1 = -\vec{v}_2$ (elastic collision, e = 1)
In general for all cases
$\vec{v}_1 = -k\vec{v}_2 \;\; k \geq 1$

21. (A, C)



Since completely inelastic
[e = 0]
By momentum conservation

$$mv + 0 = 2mv' \Rightarrow v' = \frac{v}{2}$$

Similarly for next state
$2mv' + 0 = 3mv_1$

$$v_1 = \frac{v}{3} \qquad \left[v' = \frac{v}{2}\right]$$

$\therefore$ The centre of mass of the system will have a final speed

$$= \frac{v}{n}$$

Last block start moving at

$$t = \frac{L}{V} + \frac{2L}{V} + \frac{3L}{V} + \ldots\ldots \frac{(n-1)L}{V}$$

$$= \frac{L}{V}[1 + 2 + 3 + \ldots\ldots(n-1)]$$

It is an A.P. $S_{AP} = \left(\frac{a+\ell}{2}\right)n$

or $\frac{n}{2}[2a + (n-1)d]$

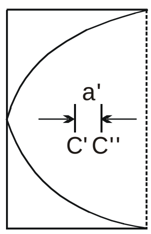
$$t = \frac{n(n-1)L}{2V}$$

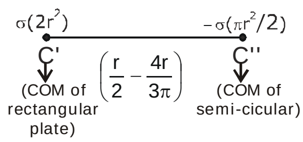
22. a. **(A, C)**

(a) Since the speed remains same for both sand and car at same instant
$\therefore$ Momentum is conserved in both A and C point

(b) B
Car maintains the same speed.

23. (D)





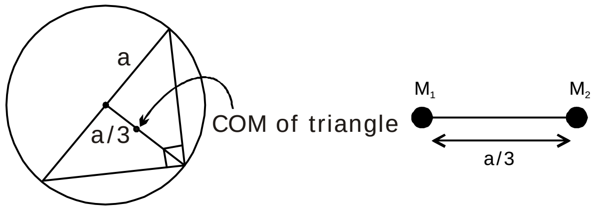
$$a'^{(\text{COM of system})} = \frac{(\sigma 2r^2)\left(\frac{r}{2} - \frac{4r}{3\pi}\right)}{\sigma 2r^2 - \frac{\sigma\pi r^2}{2}}$$

$$a' = \frac{2(3\pi r - 8r)}{3\pi(4-\pi)}$$

Required Ans (COM from O) = $a' + \frac{4r}{3\pi}$

$$= \frac{2(3\pi r - 8r)}{3\pi(4-\pi)} + \frac{4r}{3\pi} = \frac{2r}{3(4-\pi)}$$

24. (C)
COM of circle is at O. Let M_1 is mass of circle and M_2 is mass of triangle

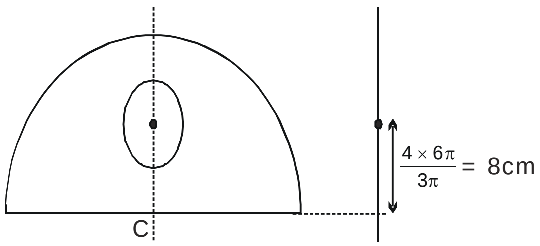


Distance of COM from centre of circle

$$r_1 = \frac{M_2\ell}{M_1 + M_2} = \frac{-\sigma a^2}{\sigma\pi a^2 - \sigma a^2} \times \frac{a}{3}$$

$$= \frac{a^2 \times a}{3a^2(\pi - 1)} = \frac{a}{3(\pi - 1)}$$

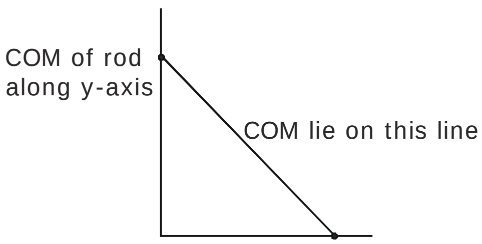
25. (B)



COM of semic circular disc = $\frac{4R}{3\pi}$

So from point C distance of COM is 8 cm.
Center of mass coincides

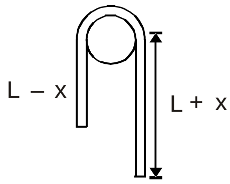
26. (D)



27. (A)

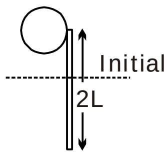
$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

Let m_1 = (L + x)λ and m_2 = (L - x)λ
where λ is mass per unit length



$$a = \frac{2x}{2L} g = \frac{x}{L} g$$

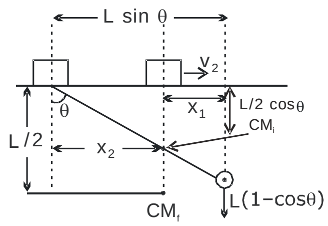
28. (C)
from energy conservation



$$mg\frac{\ell}{2} = \frac{1}{2}mv^2 \Rightarrow u = \sqrt{g\ell}$$

29. (A)
As F_{net} in x direction = 0
$mx_1 = mx_2$ [$\because F_x = 0$]
$x_1 = x_2$
Now $x_1 + x_2 = L \sin\theta$

$$\Rightarrow CM_f = \frac{L\sin\theta}{2}$$



30. (D)
V_{CMx} = 0 and F_x = 0
from momentum conservation
$mv_1 = mv_2 \Rightarrow v_1 = v_2 = v$(let)
Now energy conservation

$$mg\ell\,(1 - \cos\theta) = 2\left[\frac{1}{2}mv^2\right]$$

$v^2 = g\ell\,(1 - \cos\theta)$

Distance from centre of mass = R = $\frac{\ell}{2}$

So $T = \frac{mv^2}{R} = \frac{mg\ell(1-\cos\theta)}{\ell/2}$

T = 2mg (1 – cos θ)

31. (A)
from previous question

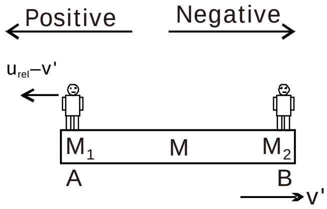
$v_{max} = V = \left[g\ell(1-\cos\theta)\right]^{1/2}$

32. (B)
Only in vertical direction
[$\because f_x$ = 0 always]

So displacement $= \frac{L}{2} - \frac{L}{2}\cos\theta$

$= \frac{L}{2}[1 - \cos\theta]$

33. (D)



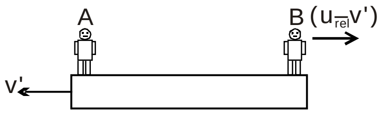
By momentum conservation

$O = m_1(u_{rel} - v') - (m_2v' + Mv')$

$m_1(u_{rel} - v') = m_2v' + Mv'$

$$v' = \frac{m_1u_{rel}}{m_1 + m_2 + M}$$

34. (D)



from momentum conservation COM remains stationary

$m_2(u_{rel} - v') = (m_1 + M)v'$

$m_2u_{rel} - m_2v' = m_1v' + Mv'$

$v'(m_1 + M + m_2) = m_2u_{rel}$

$$v' = \frac{m_2u_{rel}}{m_1 + M + m_2}$$

35. (A)

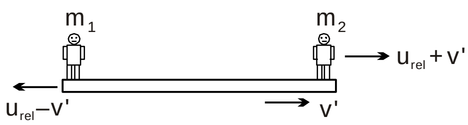
$\vec{F}_{net} = 0 \quad \vec{V}_{com} = 0$

$\therefore$ COM is at rest.

$-m_1u + m_2u + Mv = 0$

$$v' = \frac{(m_1 - m_2)}{M}u$$

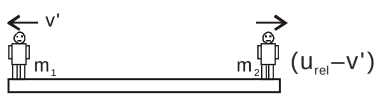
36. (A)



$m_2(U_{rel} + v') + Mv' = m_1(u_{rel} - v')$

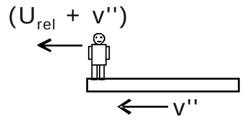
$$v' = \frac{|m_1 - m_2|U_{rel}}{m_1 + m_2 + M}$$

37. D



$m(u_{rel} - v') = (M + m)v'$

$$v' = \frac{mu_{rel}}{(M + 2m)}$$



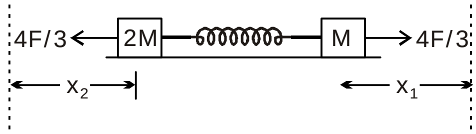
from momentum conservation

$$m(u_{rel} + v'') + Mv'' = \frac{(M + m)mu_{rel}}{(M + 2m)}$$

38. (B)

$$a_{COM} = \frac{F}{3M}$$

w.r. to COM



$$\frac{4F}{3}x_1 + \frac{4F}{3}x_2 = \frac{1}{2}k(x_1 + x_2)^2$$

$$\frac{8F}{3K} = (x_1 + x_2)$$

39. (B)

(i) From M.C. mv = 2mv'

v' = v/2

(ii) from M.C. mv = 2mv'

v' = v/2

(iii) Impulse = mv = 3mv'

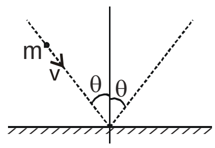
$$v' = \frac{v}{3}$$

40. (B)

$\Delta P = 2mv\cos\theta$

F_{avg} unit volume

$= (2mv\cos\theta)(nv)$



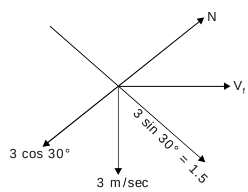
$= 2mnv^2\cos\theta$

$$\text{Pressure} = \frac{F_{\perp}}{\text{area}} = 2mnv^2\cos\theta\cos\theta$$

41. (B)

$mV_f\cos 30° = 1.5m$

$V_f\cos 30° = 1.5$



$v_f = \sqrt{3}$ m / s

42. (D)

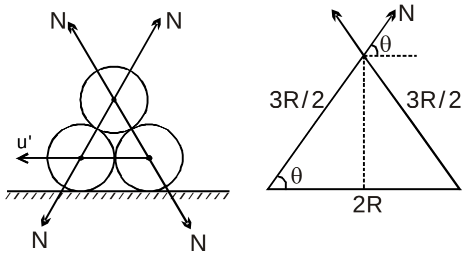
mgh = $KE_A + KE_B$

$0.25 \times 0.45 \times 10 = 1 + \frac{1}{2}(0.25)v^2$

v = 1 m/s

Ball B is heavy so ball A velocity is towards left

43. (C)



$\int 2N\sin\theta\,.dt = Mv_0$ ....(i)

$\int N\cos\theta\,.\,dt = Mu'$

$\int N.dt = \frac{mv_0}{2\sqrt{5}}\,.\,3$

$\sin\theta = \frac{\sqrt{(3/2)R^2 - R^2}}{3/2R}$

$\sin\theta = \frac{\sqrt{5}}{3}$; $\cos\theta = \frac{2}{3}$

$\frac{mv_0 3}{2\sqrt{5}}\frac{2}{3} = mv' \Rightarrow v' = \frac{v_0}{\sqrt{5}}$

44. (C)

Impulse = change in momentum

$\int 2N\sin\theta dt = \frac{mv_0}{2}$ ....(i)

$\int N.\cos\theta dt = mv'$

....(ii)

from (i) and (ii)

$\int 2N \times \frac{\sqrt{5}}{3}dt = \frac{mv_0}{2}$

$\int \frac{2N}{3}dt = mv'$

On dividing

$\frac{2N\times\sqrt{5}}{3} \times \frac{3}{2N} = \frac{v_0}{2v'}$

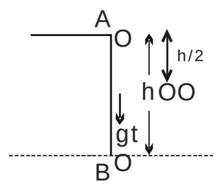
$v' = \frac{v_0}{2\sqrt{5}}$

45. (D)

time to reach $\frac{h}{2}$ from top by A

$t = \sqrt{\frac{h}{g}}$

for body B



$\frac{h}{2} = v\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\sqrt{\frac{h}{g}}\right)^2$

$h = v\sqrt{\frac{h}{g}}$, $v = \sqrt{hg}$

velocity of body B at $\frac{h}{2}$

$v_f = \sqrt{hg} - g\sqrt{\frac{h}{g}}$

$v_f = 0$

Now

momentum conservation

mg.t = 3mv'

gt/3 = v'

Energy conservation

$\Rightarrow \frac{1}{2}3m\,(gt/3)^2 + 3mg\,\frac{h}{2} = \frac{1}{2}3m.v_1^2$

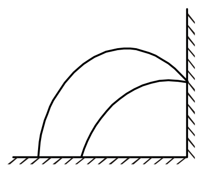
$v_1 = \frac{\sqrt{10gh}}{3}$

46. (D)

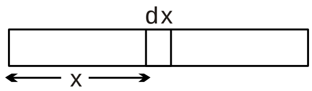
Infinite

47. (B)

$\frac{2v\cos\theta}{g}$



Exercise-III Level-I

1. 

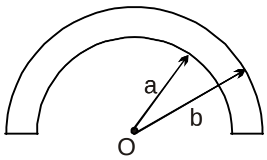
$\lambda = Ax + B$
when $x = 0$, $\lambda = \lambda_0$
$\Rightarrow B = \lambda_0$
and when $x = \ell$, $\lambda = 2\lambda_0$
$\Rightarrow 2\lambda_0 = A\ell + \lambda_0$

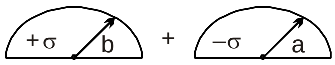
$A = \dfrac{\lambda_0}{\ell}$

$\therefore \lambda = \dfrac{\lambda_0}{\ell}x + \lambda_0$

$$x_{COM} = \frac{\int_0^{\ell} \lambda \cdot dx \cdot x}{\int_0^{\ell} \lambda \cdot dx} = \frac{5}{9}\ell$$

2. So C.M. from 0





$$y = \frac{\sigma\left(\frac{\pi b^2}{2}\right)\left(\frac{4b}{3\pi}\right) - \sigma\left(\frac{\pi a^2}{2}\right)\frac{4a}{3\pi}}{\sigma\frac{\pi b^2}{2} - \sigma\left(\frac{\pi a^2}{2}\right)}$$

$$y = \frac{4}{3\pi}\left[\frac{b^3 - a^3}{b^2 - a^2}\right]$$

3. $y = \pm kx^2$

$$x_{COM} = \frac{\int_0^a x\ dm}{\int_0^a dm}$$

$\because dm = (2y\ dx)\sigma = +2kx^2dx$

So $$x_{COM} = \frac{\sigma 2k\int_0^a x \cdot x^2 dx}{\sigma 2k\int_0^a x^2 dx}$$

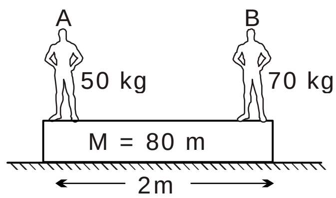
$$= \frac{a^4 \times 3}{4 \times a^3} = \frac{3a}{4}$$

Now, $Y_{COM} = 0$ [$\because$ Symmetric for x-axis]

4. $V_{COM} = \dfrac{m \times 50 + m \times 30}{2m} = 40$ m/s

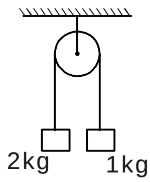
$V_{COM} = -g$

$H_{COM} = \dfrac{(40)^2}{2 \times 10} = 80$ m

5. 

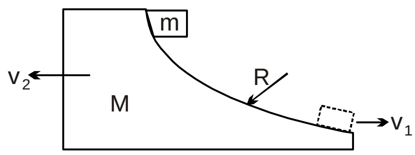
(i) zero
(ii) Right
(iii) $50(2 + x) + 80(x) = 70(2 - x)$
$100 + 50x + 80x = 140 - 70x$
$x(50 + 80 + 70) = 140 - 100$
$x = 0.2$ m or $x = 20$ cm
(iv) Distance moved by A with respect to ground is $= 2 + x = 2.2$ m
(v) Distance moved by B with respect to ground $= (2 - x)$
$= 2 - 0.2 = 1.8$ m

6. $2g - T = 2a$
$T - g = a$
$g = 3a$
$a = g/3$



$a_{COM} = \dfrac{2 \times \frac{g}{3} - \frac{g}{3}}{3} = \dfrac{g}{9}\downarrow$ (downwards)

7. By momentum conservation



$mv_1 = Mv_2$
By energy conservation

$$mgR = \frac{1}{2}mv_1^2 - \frac{1}{2}Mv_2^2$$

$$mgR = \frac{1}{2}mv_1 + \frac{1}{2}M\left(\frac{mv_1}{M}\right)^2$$

$$v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

8. Given : Trolley + child = 200 kg ;
mass of child = 20 kg
u = 36 km/hr
Let the new velocity of trolley = V
New velocity of boy = (V + 10) m/s
By momentum conservation

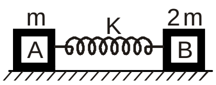
$$200 \times 36 \times \frac{1000}{3600}$$
$$= 20\,(v + 10) + (200 - 20)v$$
$$2000 = 20v + 200 + 200v - 20v$$

$$v = \frac{1800}{200} = 9 \text{ m/s}$$

$$\text{Time taken} = \frac{\text{Length of Trolley}}{\text{Relative speed of boy}}$$

$$= \frac{10}{10} = 1 \text{ sec.}$$

Required distance = 9 m

9. 

(a) By momentum conservation
mv = 2mv'

$$v' = \left(\frac{v}{2}\right)$$

By energy conservation

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}k\,\frac{x_0^2}{4}$$

$$kx_0^2 = mv^2 + \frac{mv^2}{2} + \frac{kx_0^2}{4}$$

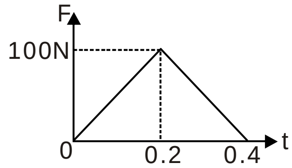
$$v = \sqrt{\frac{kx_0^2}{2m}}$$

(b) Work done = ΔK

$$\text{W.D.} = \frac{1}{2}m\left(\frac{kx_0^2}{2m}\right) - 0$$

$$= \frac{kx_0^2}{4}$$

10. (i) By graph

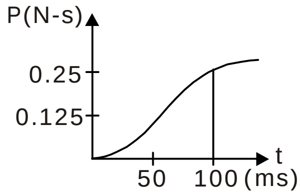


Area = ΔP

$$= \frac{1}{2} \times 0.2 \times 100 + \frac{1}{2} \times 0.2 \times 100$$
= 20 Ns.

(ii) $F_{avg.}\,\Delta t = \Delta P$
$F_{avg}\,\Delta t = 20$

$$F_{avg} = \frac{200}{4} = 50 \text{ N}$$

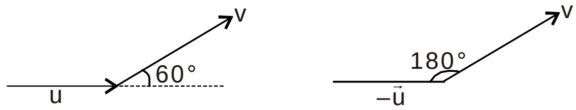
11. 

12. $F_{Th} = V_r\frac{dm}{dt}$ $\quad\left[\because m = \rho Ax\ \frac{dm}{dt} = \rho Av\right]$

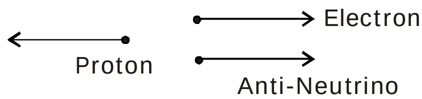
$= \rho Av^2$
$= 1000 \times 300 \times 10^{-6} \times (25)^2$
$= 187.5$ N

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{187.5}{300 \times (10^{-3})^2}$$

= 625 kPa

13. 

$$|\Delta P| = m\sqrt{m^2 + v^2 - uv}$$

14. 

(a) By momentum conservation
$m_pv_p = m_ev_e + m_Av_A$
$m_Pv_P = P_1 + P_2$

$$v_p = \frac{P_1 + P_2}{m_P} = \frac{1.4\times10^{-28} + 65\times10^{-27}}{1.67\times10^{-27}}$$

= 12.3 m/s

(b) $\dfrac{\sqrt{P_1^2 + P_2^2}}{m_P} = 9.4$ m/s

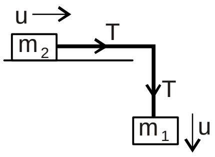
15. Speed at time of collision
$v = \sqrt{2gh} = \sqrt{80}$

(a) So $I = \Delta P = mv - m(-v)$
$= 2mv$
$= 4\sqrt{5}$ m/s

(b) $I = A_{avg}\,\Delta t = 4\sqrt{5}$

$$F_{avg} = \frac{4\sqrt{5}}{0.002} = 2000\sqrt{5}\ \text{N}$$

16.

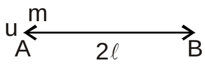


$(0.7 - 0.25)$m = 0.45 m
So speed of A after height = 0.45 m
Let then,
$v^2 = 0^2 + 2g\,(0.45)$
$v = \sqrt{9} = 3$ m/sec
Now, tension will give impulsive force and both block move with same velocity.
So $I = m_1(-u) - m_1(-v)$ .... (1)
$I = m_2 u$ .... (2)
from (1) and (2)
$m_2 u = -m_1 u + m_1 v$

$$u = \frac{m_1 v}{m_1 + m_2} = \frac{2\times 3}{2+3} = \frac{6}{5}$$

and $I = m_2 u = 3 \times \frac{6}{5} = 3.6$

17. (a)



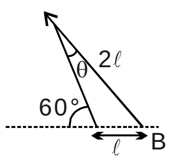
$mu = 2mv'$

$$v' = \frac{u}{2}$$

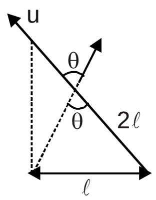
$I = \Delta P$
$I = mu/2$

(b) $mu\cos\theta = 2mv'$

$$v' = \frac{u\cos\theta}{2}$$



(c) $mu\cos\theta = 2mv'$



$$v' = u\,\frac{\sqrt{3}}{4}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

18. t = 0   P ● 2m   m ● → 5u

t = T   v' → ● 2m   ← u ●

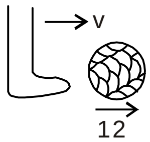
$m\,5u = 2mv' - mu$
$v' = 3u$

$$\text{W.D.} = \frac{1}{2}mu^2 + \frac{1}{2}.2m\,(3u)^2 - \frac{1}{2}m(5u)^2$$
$= -3mu^2$

19. This is a case of purely inelastic collision for x-direction.
$p_i = p_f$
$mv = (m + \rho Ax)$

$$v = \frac{mv}{m + \rho Ax}$$

20. $e = \dfrac{v - 0}{12 - v}$
$v = 6$ m/sec.



21. $m_2 v_2 + m_3 v_3 = (m_2 + m_3)v'$
$1 - 4 = 3v'$
$v' = -1$ m/s

$$\Delta K = \frac{1}{2}\times 1\times 1^2 + \frac{1}{2}\times 2\times 2^2 - \frac{1}{2}\times 3 \times 1^2$$

$\Delta K = 3$

$\boxed{A}$	$\boxed{B+C}$
2kg	3kg
1m / s →	← 1m / s

$\Delta P_A = (2\times -1/5) - (2\times 1) = 12/5$Ns
$2\times 1 + 3\times -1 = 5V_1 \Rightarrow V_1 = -1/5$ m/s

22. $\overset{u}{\rightarrow}$ (m_1)   u = 0 (m_2) $\Rightarrow$ $\overset{v}{\rightarrow}$ (m_1)(m_2)

$P_i = P_f$
Given $m_1 u = (m_1 + m_2)v$ ....(i)
$K.E_f = 2/3 K.E_i$

$$\frac{1}{2}(m_1 + m_2)v = \frac{2}{3}\times\frac{1}{2}m_1 u^2 \quad ....(ii)$$

Solving $\dfrac{m_1}{m_2} = \dfrac{2}{1}$

23. (a) Total distance

$$= \frac{10^2}{g} + \frac{e^2 10^2}{g} + \frac{e^4 10^2}{g} + ...$$

$$= \frac{10^2}{g}\{1 + e^2 + e^4 + ...\}$$

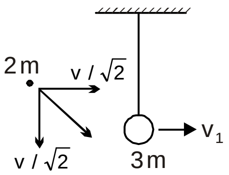
$$= 5\left\{\frac{1}{1-\frac{1}{4}}\right\} = \frac{40}{3}\text{ m}$$

(b) Time elapsed $= \frac{2u}{g} + \frac{2(eu)}{g} + \frac{1}{2} \times \frac{2(e^2u)}{g}$

$$= \frac{2\times 10}{10} + \frac{2\times 5}{10} + \frac{1}{2} \times 2 \times \frac{\left(\frac{1}{4}\times 10\right)}{10}$$

$$= \frac{13}{4} = 3.25 \text{ s}$$

24. By energy conservation



$\frac{1}{2} \cdot 3mv_1^2 = (3m)g\ell$

$v_1 = \sqrt{2g\ell}$

In horizontal direction, momentum conserved.

$2m\frac{v}{\sqrt{2}} = 3mv_1$

$v_1 = \frac{\sqrt{2}}{3}v$..... (2)

From (1) & (2)

$\sqrt{2g\ell} = \frac{\sqrt{2}}{3}v$

$v = \sqrt{9g\ell} = 3\sqrt{g\ell}$

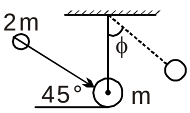
(b) By energy conservation

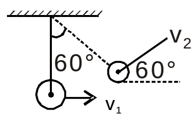
$\frac{1}{2}\cdot 3mv_1^2 = \frac{1}{2}\cdot 3mv_2^2 + 3mg\ell\,[1 - \cos 60°]$

$v_1^2 = v_2^2 + g\ell$

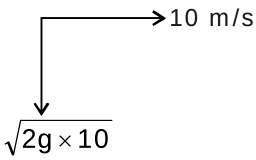
$\frac{2}{9} \times 9g\ell = v_2^2 + g\ell$

$v_2^2 = g\ell \Rightarrow v_2 = \sqrt{g\ell}$

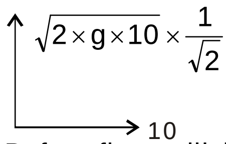




25.



after impact



$e = \frac{1}{\sqrt{2}}$

(a) Before first collision

$u_x = 10$ m/s $\quad u_y = \sqrt{2gh}$

$v = \sqrt{u_x^2 + u_y^2} = \sqrt{(10)^2 + \left(10\sqrt{2}\right)^2} = 10\sqrt{3}$

(b) $\tan\theta = \frac{u_y}{u_x} = \frac{10\sqrt{2}}{10}$

$\theta = \tan^{-1}(\sqrt{2})$

(c) After striking

$V_x = u_x$; $v_y = eu_y$

$\tan\phi = \frac{v_y}{v_x} = \frac{eu_y}{u_x} = \frac{1}{\sqrt{2}} \times \frac{10\sqrt{2}}{10}$

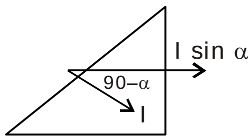
$\phi = 45°$

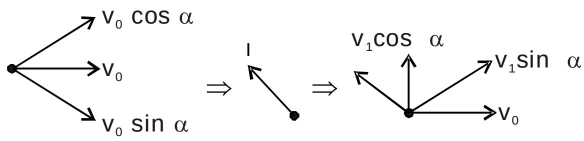
(d) $R' = \frac{2u_x(ev_y)}{g} = \frac{2\times 10 \times \frac{1}{\sqrt{2}} \times 10\sqrt{2}}{10}$

$R' = 20$ m

26. $I \sin\alpha = 2mv_2$

$I = \frac{mv_0}{\sin\alpha}$





Now, $v_1 \sin\alpha = v_0 \cos\alpha$

$v_1 = v_0 \cot\alpha$

$$e = \frac{v_1\cos\alpha + v_2\sin\alpha}{v_0\sin\alpha}$$

$$e = \frac{v_0\sin\alpha\,[\cot^2\alpha + 1/2]}{v_0\sin\alpha}$$

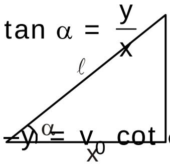
$$= \frac{2\cot^2\alpha}{2} = \frac{3}{4}$$

$v_{pw} = v_p - v_w$

$v_{pw} = v_0 \cot\alpha\,\hat{j} - \frac{v_0}{2}\,\hat{i}$

Let particle hit after length ℓ and time t on wedge, then

$\tan\alpha = \frac{y}{x}$ .... (i)



$-y = v_0 \cot\alpha \; t - \frac{1}{2}gt^2$ .....(2)

and $x = \frac{v_0}{2} \cdot t$ .....(3)

on solving $-2 = \frac{10\cot\alpha - 5t}{5}$

$t = 3$ sec.

27. (a) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.5}$

$v = \sqrt{30}$

$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2}$

$= \frac{2 \times \sqrt{30} + 4 \times \frac{3}{4}\left(-\sqrt{30}\right)}{2 + 4}$

$v_1 = -\frac{\sqrt{30}}{6} = -\sqrt{\frac{30}{36}}$

$v_1 = -\sqrt{\frac{g}{12}}$

(b) $v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$

$= \frac{2 \times \sqrt{30} + 2 \times \frac{3}{4}\left(\sqrt{30}\right)}{6}$

$= \frac{\sqrt{30}}{6}\left(2 + \frac{3}{2}\right) = \frac{7\sqrt{30}}{12}$

$S_{max} = \frac{u^2}{2a}$ $\qquad a = \frac{F_x}{m_2} = 5$

$= \frac{49 \times 30}{12 \times 12 \times 2 \times 5} = \frac{49}{48}$

28. (a) $mv_0 = 2mv'$

$v = \frac{v_0}{3} \geq \sqrt{5g\ell}$

(b) $mv_0 = 3mv'$

$v' = \frac{v_0}{3}$

For complete motion

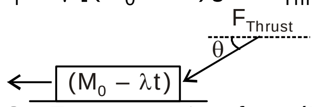
$\frac{v_0}{3} \geq \sqrt{5gR}$

or $v_0 = v_3\sqrt{5gR}$

29. $F_{thrust} = v_{rel}\frac{dm}{dt} = u\lambda$

At time t

$f_r = \mu[(M_0 - \lambda t)g + F_{Thrust}\sin\theta]$



So $F_{thrust}\cos\theta - f_r = (M_0 - \lambda t)a$

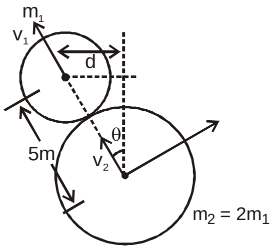
$$\int_0^t \frac{\mu\lambda\left[\cos\theta - \mu\sin\theta\right]}{(M_0 - \lambda t)} - \mu g = \int_0^v dv$$

$$v = u[\cos\theta - \mu\sin\theta]\ln\left(\frac{M_0}{M - \lambda t}\right) - \mu gt$$

Exercise-III Level - II

1. here $\sin\theta = d/5$

$$\cos\theta = \frac{\sqrt{5^2 - d^2}}{5}$$



After collision

Momentum conservation in line of impact

$m_1 u\cos\theta = m_1 v_1 + (2m_1)v_2$

$u\cos\theta = v_1 + 2v_2$ ...(1)

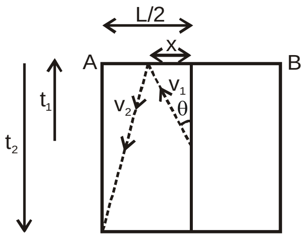
again given

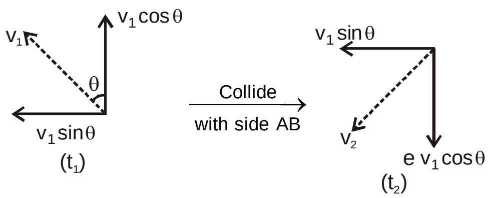
$e = \frac{2}{3} = \frac{v_1 - v_2}{u\cos\theta} \Rightarrow \frac{2}{3}u\cos\theta = v_1 - v_2$ ...(2)

eq. (1) + 2 × (2)

$\left(\frac{2}{3}\times 2 + 1\right)u\cos\theta = 3v_1$

or $+v_1 = \frac{7}{9}u\cos\theta$ ...(3)





Now $\frac{L}{2} = v_1 \sin\theta(t_1 + t_2)$ ...(4)

(in x-direction)

in y direction $\frac{L}{2} = v_1\cos\theta\, t_1$ ...(5)

and $L = \frac{2}{3}v_1\cos\theta\, t_2$ ....(6)

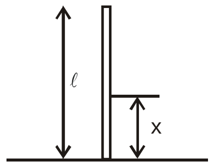
from (5) and (6) in (4) value of t_1 & t_2

$$\frac{L}{2} = v_1\frac{d}{5}\left[\frac{L}{2v_1}\frac{5}{\sqrt{25-d^2}} + \frac{3L}{2v_1}\frac{5}{\sqrt{25-d^2}}\right]$$

$\Rightarrow 1 = \frac{4d}{\sqrt{25-d^2}} \Rightarrow 25 - d^2 = 16d^2$

$\Rightarrow d^2 = \frac{25}{17} \Rightarrow d = \frac{5}{\sqrt{17}}$ **Ans.**

2.



At time t_1 x length dropped and next dt time dx part dropped, then force on table

f_T = [due to weight of the chain + $\frac{dp}{dt}$]

$f_T = x\lambda g + \frac{dp}{dt}$,

here x = v_t , $f_T = vt\lambda g + \frac{dp}{dt}$ ...(1)

$(\Delta p)_c = 0 - \lambda dx(v_2)$, $(\Delta p)_c = 0 - \lambda V^2 dt$

$\frac{(\Delta p)_c}{\Delta t} = -\lambda v^2$ on chain

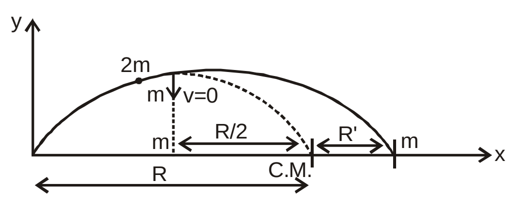
so on table = $-\frac{(dp)_c}{dt} = \lambda v^2$ ...(2)

so form eq. (1) and (2)

$f_T = Vt\lambda g + \lambda v^2$, $\lambda = \frac{m}{\ell}$

$f_T = \frac{m}{\ell}v\,(gt + v)$

3.



∵ Explosion is an internal process so it will not effect the position of C.M. and because no force in horizontal direction so

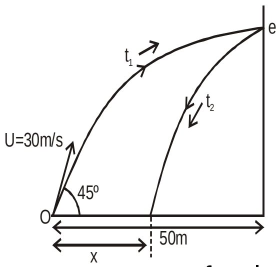
$$(x_{C.M})_{explosion} = (x_{C.M})_{without\ explosion}$$

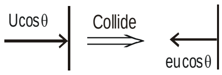
$$\frac{m(R/2)+(R+R')}{2m} = R \quad \Rightarrow \quad R+R' = \frac{3R}{2}$$

so striking point for second particle

$$= R+R' = \frac{3R}{2} = \frac{3}{2}\left(\frac{U^2 \sin 2\theta}{g}\right) = 368$$

4.





y component of velocity does not change
y direction total time if t

$t = t_1 + t_2$ ...(1)

then $0 = U\sin\theta t - 1/2 gt^2$

at the time of collision if ball have y component of velocity v_y then

$v_y = v_1 - gt_1$ (here t_1 = time for collide)

$v_y = 10\sin 37^o - 10t_1$ ...(2)

for time $t_1 = \frac{\text{relative distance}}{[\text{relative velocity}]_{in\ x-direction}}$

$$= \frac{10}{U+v_2}$$

$t_1 = \frac{10}{U+8}$ ...(3)

from (2) and (3)

$v_y = 10\sin 37^o - \frac{10\times 10}{U+8}$ ...(4)

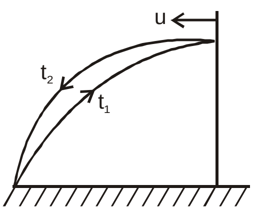
$\because$ Ball again come into his hands so total time t then

$0 = 10\sin 37^o t - 1/2 gt^2$ in y-direction

$$t = \frac{2(10\sin 37^o)}{g} = 1.2 \sec$$

here $t_1 + t_2 = t$ ...(6)

If returning time is t_2 then



$$t_2 = \frac{10 - ut_1}{v_2'}$$

$$= \frac{10-ut_1}{v_2+2u} \text{ from eq. (1)}$$

so $t_2 = \frac{10-ut_1}{8+2u}$ ...(7)

so from eq (3), (5), (6) and (7)
putting values of t_1, t_2 and t

$$\Rightarrow \quad \frac{10}{U+8} + \frac{10 - U\left(\frac{10}{v+8}\right)}{8+2u} = \frac{12}{10}$$

$$\Rightarrow \quad 3u^2 + 114 - 104 = 0$$

so $u = \frac{-11 \pm \sqrt{(11)^2 + 4(3)(104)}}{2\times 3}$

$u = \frac{26}{6}$ (from + sign) $\Rightarrow u = \frac{13}{3}$ m/sec

or $t = \frac{2u\sin\theta}{g} = \frac{2\times 30\times 1/\sqrt{2}}{9.8}$ sec

$t = 4.33$ sec ...(2)

again for time t_1

$t_1 = \frac{50}{u\cos\theta} = \frac{50}{30\times 1/\sqrt{2}}$ sec

$t_1 = 2.36$ sec ...(3)

so from eq. (1), (2) and (3)

$t_2 = t - t_1 = 4.33 - 2.36$

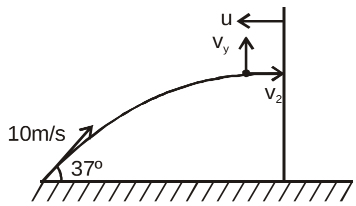
$t_2 = 1.97$ sec

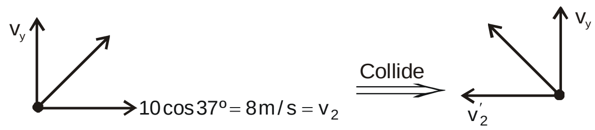
so $x = 50 - (eu\cos\theta)t_2$

$x = 50 - e(41.8)$

(a) e = 0 then x = 50 m
(b) e = 1 then x = 8.2 m
(c) e = 1/2 then x = 2.91 m

5.





then $e = 1 = \frac{v_2' - u}{v_2 + u} \Rightarrow v_2' = v_2 + 2u$ ...(1)

6. (a) Given $m_1 = m_2 = m_3 = m$
when m_1 and m_2 stickes then
Momentum Conservation

$m_1 v = (m_1 + m_2)v'$ ...(A)

$v' = \left(\dfrac{m_1 v}{m_1 - m_2}\right)$ velocity after collision of (m_1 + m_2)

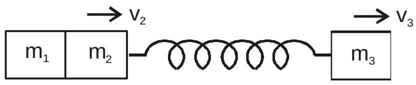
$v' = \dfrac{v}{2}$ ...(B)

and velocity of m_3 = 0 just after impact.

(b) Now just after impact the total energy

$$E = \frac{1}{2}(m_1 + m_2)(v')^2 = \frac{1}{4}mv^2 \quad ...(1)$$

The K.E. of m_3 will max. (K.E. of m_2 will minimum) when spring is in normal position and after some time of impact at this time if velocity are v_2 and v_3 then momentum conservation



$(m_1 + m_2)v' = (m_1 + m_2)v_2 + m_3 v_3$

or $m_1 v = (m_1 + m_2)\sqrt{2} + m_3 v_3$

from (A) $v = 2v_2 + v_3$ ...(2)

$\because$ Total energy after impact remain same for all time so

$E_f = E_i$

$\Rightarrow \dfrac{1}{2}2mv_2^2 + \dfrac{1}{2}mv_3^2 + \dfrac{1}{2}k(o)^2 = \dfrac{1}{4}mv^2$

$v^2 = 4v_2^2 + 2v_3^2$ ...(3)

from (2) and (3) element v_2

$v^2 = 4\left[\dfrac{v - v_3}{2}\right]^2 + 2v_3^2$

$v^2 = v^2 + v_3^2 - 2vv_3 + 2v_3^2$

$v_3 = \dfrac{2v}{3}$ ...(4)

$K.E_{max} = \dfrac{1}{2}mv_3^2 = \dfrac{2}{9}mv^2$

(c) from (4) and (2)

$v = 2v_2 + \dfrac{2v}{3} \Rightarrow v_2 = \dfrac{v}{6}$

so $K.E_{min} = \dfrac{1}{2}m\left(\dfrac{v}{6}\right)^2 = \dfrac{1}{12}mv^2$

(d) When their velocity will same from mome. con.

$m_1 v = (m_1 + m_2 + m_3)v'$

$v' = v/3$

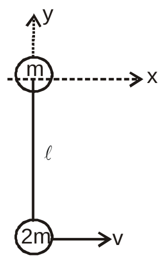
Ene. constant so

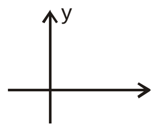
$\dfrac{1}{4}mv^2 = \dfrac{1}{2}3m(v')^2 + \dfrac{1}{2}k(x_m)^2$

$mv^2 = \dfrac{6mv^2}{9} + 2K(x_m)^2$

$\Rightarrow x_{cm} = \sqrt{\dfrac{m}{6k}}.v$ **Ans.**

7.





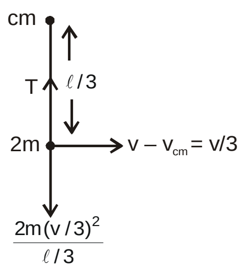
$y_{cm} = \dfrac{2m(-\ell) + m(o)}{(2m + m)} = -\dfrac{2\ell}{3}$ ...(1)

$v_{cm} = \dfrac{2mv + m(o)}{2m + m} = \dfrac{2v}{3}$ ...(2)

the velocity of 2m w.r.t. C.M.

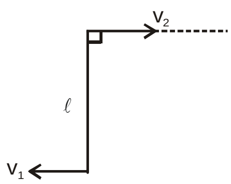
$v - v_{cm} = v/3$

Then $T = \dfrac{2m(v/3)^2}{\ell/3}$



$T = \dfrac{2}{3}m\dfrac{v^2}{\ell}$

8.



$(v_{cm})_x = 0 = m_A v_1 - m_B v_2$

$v_1 = \dfrac{m_B}{m_A}v_2$ ...(1)

E. conservation

$m_A g\ell = \dfrac{1}{2}m_A v_1^2 + \dfrac{1}{2}m_B v_2^2$ ...(2)

(1) and (2)

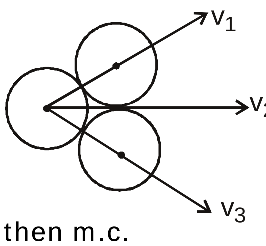
$m_A g\ell = \dfrac{1}{2}m_A\left(\dfrac{m_B}{m_A}v_2\right)^2 + \dfrac{1}{2}m_B(v_2)^2$

$2n_A g\ell = \left(\dfrac{m_B^2}{m_A} + m_B\right)v_2^2$

$$v_2^2 = 2\left(\frac{m_A}{m_B}\right)^2 \frac{g\ell}{\left(1+\frac{m_A}{m_B}\right)} \quad \ldots(3)$$

$$T = \frac{m_A v^2}{\ell} \Rightarrow T = m_A\left(\frac{v_1+v_2}{\ell}\right)^2$$

$$T = 2g\left(\frac{m_A}{m_B}\right)(m_A+m_B)$$

9. 

After collision

then m.c.

$m10 = mv_2 + 2mv_1\dfrac{\sqrt{3}}{2}$

$10 = v_2 + \sqrt{3}v_1 \quad \ldots(1)$

and $\quad e = 1 = \dfrac{v_1 - v_2\sqrt{3}/2}{10\sqrt{3}/2}$

$10\sqrt{3} = 2v_1 - \sqrt{3}v_2 \ldots(2)$

from (1) and (2)

$10 = v_2 + 3\left[\dfrac{10+v_2}{2}\right]$

$20 = 2v_2 + 30 + 3v_2$

$5v_2 = 10 \Rightarrow v_2 = -2\,m/s$

10. (a) v_{cm} remain same

$$v_{cm} = \frac{m_0 v_0\hat{i} + m_2(0)}{(m_1+m_2)} \Rightarrow v_{cm} = \frac{v_0}{3}\hat{i}$$

(b) In x direction

$m_1v_0 = m_1(0) + m_2(v_2\cos\theta)$

$v_0 = 2v_2 \cos\theta \quad \ldots(1)$

In y direction $\quad 0 = m_1v_0/2 - m_2(v_2\sin\theta)$

$v_0 = 4v_2\sin\theta \quad \ldots.(2)$

(c) eq. (2) ÷ 1

$= \dfrac{4v_2}{2v_2}\tan\theta \Rightarrow \tan\theta = \dfrac{1}{2}$

so $\sin\theta = \dfrac{1}{\sqrt{5}}$, so from (2) $v_2 = \dfrac{\sqrt{5}}{4}v_0$

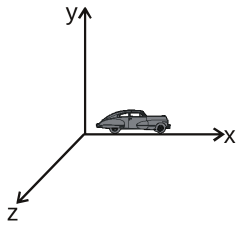
(d) $\Delta k = k_f - k_i$

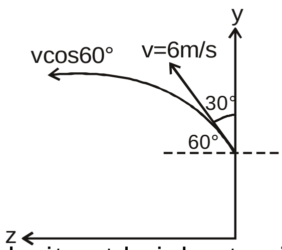
$$\Delta k = \frac{1}{2}m_1\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_1v_0^2$$

$$= \frac{1}{2}m_1\left[\frac{v_0^2}{4} + \frac{2\times5v_0^2}{16} - v_0^2\right]$$

$$\Delta k = -\frac{1}{2}m_1v_0^2\left(\frac{3}{8}\right) = -\frac{1}{16}m_1v_0^2$$

So no elastic

11. 



The ball will have velocity at heighest point

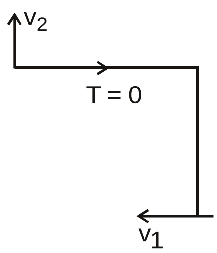
$= 4\hat{i} + v\cos60°(\hat{k})$, $\quad = 4\hat{i} + 6/2(\hat{k})$

when they collide then momentum conservation $\quad m(4\hat{i}+3\hat{k}) = 2m\vec{v}_1$

$\vec{v}_1 = 2\hat{i} + \dfrac{3}{2}\hat{k}$,

so $|\vec{v}_1| = \sqrt{2^2 + \dfrac{9}{4}}$

$= \dfrac{\sqrt{25}}{2} = 2.5\ m/sec$

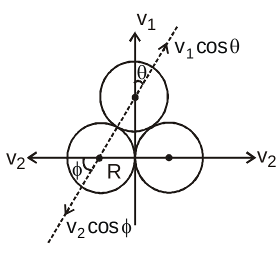


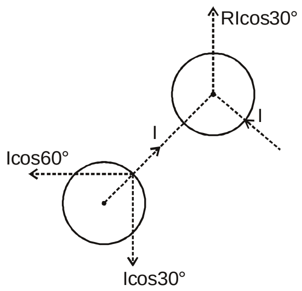
$T = 0$ when $v_2 = 0 \qquad \therefore\ T = \dfrac{mv_2^2}{\ell}$

$\dfrac{1}{2}(2m)v_1^2 = 2mg\ell \Rightarrow v_1^2 = 2g\ell$

$\ell = \dfrac{v_1^2}{2g} \Rightarrow \ell = \dfrac{(2.5)^2}{8\times10}m$

$\ell = 0.3125\ m$

12. 



$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

$\cos\phi = \frac{1}{2} \Rightarrow \phi = 60°$

$$e = \frac{v_1 \cos\theta + v_2 \cos\phi}{v\sqrt{3}/2} = \frac{v_1\sqrt{3}/2 + v_2/2}{v\sqrt{3}/2}$$

$e = 1 \Rightarrow u\sqrt{3} = v_1\sqrt{3} + v_2$ ...(1)

$2I \cos 30° = mv_1 + mu$ ...(2)

$I \cos 60° = mv_2$ ...(3)

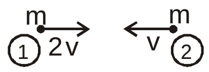
$2\sqrt{3} = \frac{v_1 + u}{v_2}$ ...(5)

$v_1 + u = 2\sqrt{3}[u\sqrt{3} - v_1\sqrt{3}]$

$7v_1 = 5u \Rightarrow v_1 = \frac{5u}{7}$

Exercise-IV   Level - I

1. (C)
Let mass of each body is m. Their motion is represented as shown in figure.



From $\vec{v}_{CM} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

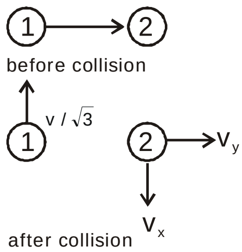
$v_{CM} = \dfrac{m \times 2v - mv}{m + m} = \dfrac{v}{2}$

[The direction of motion of first particle is taken as positive]
So, velocity of centre of mass of the system is $\dfrac{v}{2}$ in the direction of motion of particle having larger speed.

2. (C)
Here B is implying A but A is not implying B, as kinetic energy of a system of particles is zero means speed of each and every Particle is zero. which says that momentum of every particle is zero.
But statement A means linear momentum of a system of particles is zero, which may be true even if particles have equal and opposite momentums and hence, having non-zero kinetic energy.

3. (B)
Since, the acceleration of centre of mass in both the cases is same equal to g, so the centre of mass of the bodies B and C taken together does not shift compared to that of body A.

4. (C)
In x - direction
$mu_1 + 0 = mv_x$
$\Rightarrow\ mv = mv_x$
$\Rightarrow\ v_x = v$
In y - direction



$0 + 0 = m\left(\dfrac{v}{\sqrt{3}}\right) - mv_y$

$v_y = \dfrac{v}{\sqrt{3}}$

Velocity of second mass after collision

$v' = \sqrt{\left(\dfrac{v}{\sqrt{3}}\right)^2 + v^2} = \sqrt{\dfrac{4}{3}v^2}$

or $\qquad v' = \dfrac{2}{\sqrt{3}}v$

5. (C)
To keep the CM at the same position, velocity of CM is zero, so

$\dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = 0$

where $\vec{v}_1$ and $\vec{v}_2$ are velocities of particles 1 and 2 respectively.

$\Rightarrow\ m_1\dfrac{d\vec{r}_1}{dt} + m_1\dfrac{d\vec{r}_2}{dt} = 0$

$\left[\therefore\ \vec{v}_1 = \dfrac{d\vec{r}_1}{dt}\ \&\ \vec{v}_2 = \dfrac{d\vec{r}_2}{dt}\right]$

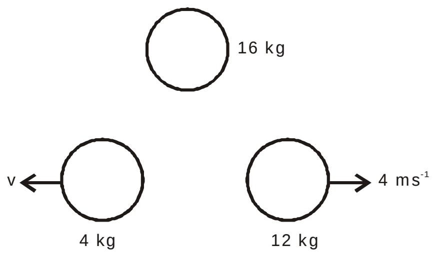
$\Rightarrow\ m_1 d\vec{r}_1 + m_2 d\vec{r}_2 = 0$

$d\vec{r}_1$ and $d\vec{r}_2$ represent the change in displacement of particles.
Let 2nd particle has been displaced by distance x.

$\Rightarrow\ m_1(d) + m_2(x) = 0$

or $\qquad x = -\dfrac{m_1 d}{m_2}$

6. (B)
Here momentum of the system is remaining conserved as no external force is acting on the bomb(system).



Initial momentum (before explosion) = Final momentum (after explosion)
Let velocity of 4 kg mass is v ms^{-1}. From momentum conservation we can say that its direction is opposite to velocity of 12 kg mass.
From $\vec{P}_i = \vec{P}_f \qquad \Rightarrow\ 0 = 12 \times 4 - 4 \times v$
or $\ v = 12$ m/s

KE of 4 kg mass $= \dfrac{4 \times (12)^2}{2} = 288$ J

7. (C)
This is the question based on impulse momentum theorem.

$|F.\Delta t| = |\text{change in momentum}|$

$\Rightarrow\ F \times 0.1 |P_f - P_i|$

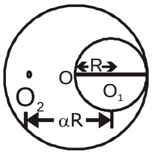
As the ball will stop after catching ;
$P_i = mv_i = 0.15 \times 20 = 3$, $P_f = 0$
$\Rightarrow\ \ F \times 0.1 = 3$
$\Rightarrow\ \ F = 30N$

8. (A)
In this question distance of centre of mass of new disc from the centre of mass of remaining disc is αR.
Mass of remaining disc

$$= M - \frac{M}{4} = \frac{3M}{4}$$

$$\therefore \quad -\frac{3M}{4}\alpha R + \frac{M}{4}R = 0$$

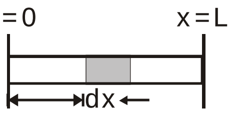
$$\Rightarrow \quad \alpha = \frac{1}{3}$$



Note : In this question the given distance must be αR for real opproach to the solution.

9. (A)

$$X_{CM} = \frac{\int x\,dm}{\int dm}$$



If n = 0,

then $\quad X_{CM} = \frac{L}{2}$

As n increases, the centre of mass shift away from $x = \frac{L}{2}$ which only option (a) is satisfying. Alternately, you can use basic concept.

$$X_{CM} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n \times x\,dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = L\left[\frac{n+1}{n+2}\right]$$

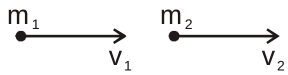
10. (C)
$0.5 \times 2 + 1 \times 0 = 1.5 \times v$
[assumed that 2nd body is at rest]

$$\Rightarrow \quad v = \frac{2}{3} \qquad \Rightarrow \qquad \Delta K = K_f - K_i$$

$$= \frac{1.5 \times \left(\frac{2}{3}\right)^2}{2} - (0.5) \times \frac{2^2}{2} = -\frac{2}{3}\,J = -0.67\ J$$

So, energy lost is 0.67 J.

11. (A)
If it is a completely inelastic collision then
$m_1v_1 + m_2v_2 = m_1v + m_2v$



$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \quad \Rightarrow \quad KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As $\vec{P}_1$ and $\vec{P}_2$ both simultaneously cannot be zero therefore total KE cannot be lost.

12. (B)
Statement–I $\rightarrow$ (False)
For max energy loss, e = 0

$$V = \frac{mv}{m+M}$$

$$\text{loss} = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{m+M}$$

$$= \frac{1}{2}\frac{mv^2(m+M) - m^2v^2}{(m+M)} = \frac{1}{2}\frac{mMv^2}{m+M}$$

Statement–II $\rightarrow$ (True)

Exercise-IV Level - II

1. $V_{com} = \dfrac{M_1V_1 + M_2V_2}{M_1 + M_2} = \dfrac{10 \times 14 + 4 \times 0}{10 + 4}$ = 10 m/s

2. In case of elastic collision, cofficient of restitution e = 1

Magnitude of relative velocity of approach

= Mag of rel velocity of seperation

But Relative speed of approach

$\neq$ Relative speed of seperation

3. Initial Momentum of the system

$\vec{P}_1 + \vec{P}_2 = 0$

Final momentum $\vec{P}'_1 + \vec{P}'_2$ shovia also be zero

Option A : $\vec{P}'_1 + \vec{P}'_2 \neq 0$

C_1 component of $\hat{K}$ will not be Zero

Option (B)

$\vec{P}'_1 + \vec{P}'_2 = 0$ If $C_1 = -C2 \neq 0$

Option (C)

$\vec{P}'_1 + \vec{P}'_2 = 0$

If $a_1 = -a_2 \neq 0$

$b_1 = -b_2 \neq 0$

Option D

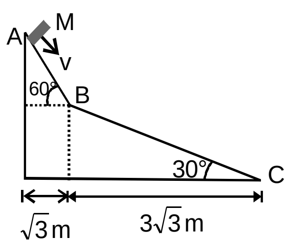
$\vec{P}'_1 + \vec{P}'_2 \neq 0$ b_1 $\hat{j}$ will not be zero

4. Between A and B

h_1 = tan 60° × $\sqrt{3}$

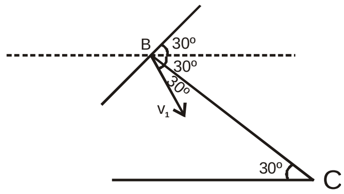
= $\sqrt{3} \times \sqrt{3}$ = 3m

Speed of block just before slriking the second collision



$V_1 = \sqrt{2gh_1} = \sqrt{60}$ m/s

Inelastic collision



Comp. of $V_1 \perp$ BC is Zero

Comp of V_1 II BC is remain chnchaged

v_2 = component of v_1 along BC ....

$\therefore V_2 = \cos 30° = \sqrt{45}$ m/s

5. Height faller by the block from B to C

$h_2 = 3\sqrt{3}$ tan 30° = 3m

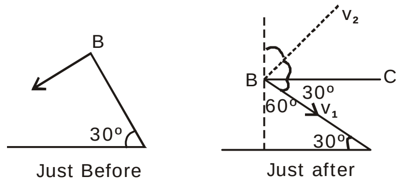
let the required speed V_3

$\therefore V_3 = \sqrt{V_2^2 + 2gh_2}$

$= \sqrt{45 + 2 \times 10 \times 3}$

$= \sqrt{105}$ m/s

6.



Elastic collision

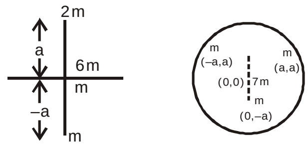
$V_{11} = v_1 \cos 30° = \sqrt{60} \times \sqrt{\dfrac{3}{2}} = \sqrt{45}$ m/s

$v_1 = v_1 \sin 30° = \sqrt{60} \times \dfrac{1}{2} = \sqrt{15}$ m/s

Now, vertical component

$V = V_1 \cos 30° - V_{11} \cos 60°$

7.

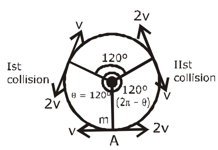


$Y_{com} = \dfrac{2m(+a) + 6m(0) + m(0) + m(-a)}{2m + 6m + m + m}$

$= \dfrac{ma}{10m} = \dfrac{a}{10}$

8. Collision is elastic and mass is same

So ofter collision, velocity of particles will change



$e = \dfrac{v_2 - v_1}{u_1 - u_2}$

At time t, Particles collide

θ = vt ....(1)

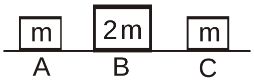
$2\pi - \theta = 2vt$ .....(2)

Now equaltion (1) and (2)

$\frac{\theta}{2\pi - \theta} = \frac{vt}{2vt} \Rightarrow 2\theta = 2\pi - \theta \Rightarrow \theta = \frac{2\pi}{3} = 120^o$

After two collision they will reach at point A.

9. Collision between A & B



$m \times 9 + 2m \times 0 = mV_A + 2mV_B$

$v_A + 2v_B = 9$ .....(1)

$e = \frac{V_B - V_A}{u_A - u_B}$

$\because$ e = 1

$u_A = 9$ $u_B = 0$

$v_B - v_A = 9$ ....(2)

Now, equation (1) & (2)

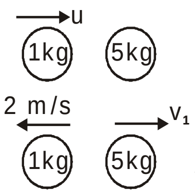
$\Rightarrow$ $v_B = 6$ m/s

Collision between B & C

$2m \times 6 + m \times 0 = (2m + m)\, v_C$

$v_C = \frac{12m}{3m} = 4$ m/s

10.



Before collision

After collision

Now form $e = \frac{v_2 + 2}{u} = 1$

$u = v_2 + 2$ ....(1)

from M.C. $u(1) = -2\,(1) + 5\,v_2$ ......(2)

After solu equa (1) & (2)

$v_2 = 1$ m/s & $u = 5$ m/s

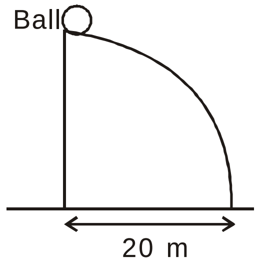
11. Applying momentum conservation

$0.01 \times v + 0.2 \times 0 = 0.01 V_{f_{Bullet}} + 0.2 \times v_{Ball}$

...(1)

for

$X_B = v_B \times \sqrt{\frac{2H}{g}}$

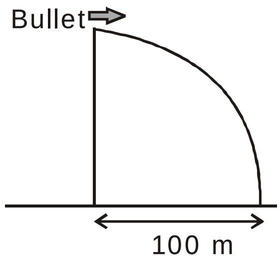


$20 = v_B \times 1$

$v_{B_{Ball}} = 20$ m / sec ...(2)

for Bullet

$X_{Bullet} = V_{Bullet} \times \sqrt{\frac{2H}{g}}$



$\therefore$ $100 = v_{Ball} \times 1$

$v_{Ball} = 100$ m/s ...(3)

from (1) (2) & (3)

we get

v = 500 m/s