MAIN+ADVANCED

TOPIC

OSCILLATIONS

SOLUTIONS

OSCILLATIONS

Exercise-1

1. A
F = ma ⇒ $F \alpha - x$ (for SHM) ⇒ $a\alpha - x$

2 C
The distance moved in T/4 time is A.
Hence in one time period distance travelled is 4 (distance in T/4 time) = 4A

3. B
A particle appears only once at one of the extreme position in entire oscillation.

4. C
In the particle moves from extreme to half of amplitude then let the time taken is t
$y = A\sin(\omega t+\phi)$

$y = A\sin\left(\omega t + \frac{\pi}{2}\right)$ (As particle starts from extreme)

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{4}t + \frac{\pi}{2}\right)$$

$$\sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) = \sin\frac{5\pi}{6}$$

$$\frac{\pi}{2}t + \frac{\pi}{2} = \frac{5\pi}{6}$$

$$\frac{\pi}{2}t + \frac{5\pi}{6} = \frac{\pi}{2}$$

$$\frac{\pi}{2}t + \frac{\pi}{3}$$

$$t = \frac{2}{3}\text{sec}$$

5. C

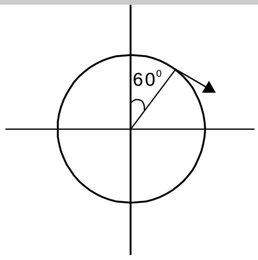
$y = A(1+\cos 2\omega t)$ $y = 2A\sin\left(\omega t + \pi/3\right)$

$y = A + A\cos 2\omega t$ $V_{max} = 2A\omega$

$V_{max} = A \times 2\omega$ Ratio = 1:1

6. A
Velocity is maximum at mean. To come back to mean the particle has to move $\left(\pi - \frac{\pi}{3}\right) = \frac{2\pi}{3}$.

Hence $t = \frac{2\pi}{3.2\pi} = \frac{1}{3}$ sec.

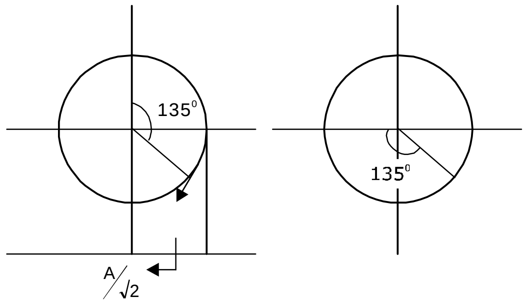


7. A

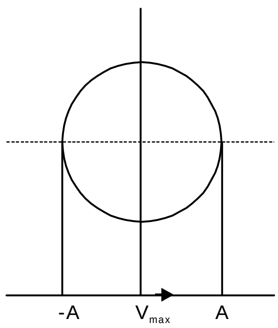
$y = 5\sin\pi(t+4) \Rightarrow y = 5\sin(\pi t + 4\pi)$ ---(1)
The standard equation is $y = A\sin(\omega t+\phi)$---(2)
comparing equation (1) & (2)

A = 5m, $\omega = \pi$ $T = \frac{2\pi}{\omega} = 2$ sec.

8. C
Let particle A be the particle shown with initial phase 135° and B be the particle at extreme. Hence the phase difference between them is 135°.



9. A
A particle has same velocity between 0 & V_{max} and 0 and $-V_{max}$ twice in its motion. Only V_{max} is a velocity which a particle attains once in its one oscillation.



10. D

Total $Acc^n = 0$

$$a_{av.} = \frac{\text{Total change in Velocity in one T.P.}}{\text{Time Period}}$$

$$a_{av} = \frac{0}{T} = 0$$

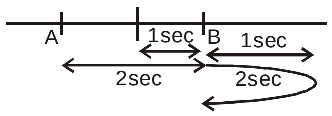
11. B

$v^2 = \omega^2 (A^2 - x^2) = -\omega^2 x^2 + \omega^2 A^2$

$a^2 = \omega^2 x^2$

$\Rightarrow v^2 = -a^2 + \omega^2 a^2$ straight line with a -ve slope.

12. B



Hence $\frac{T}{4} = 2$ sec. $T = 8$ sec.

13. A

We know that

$x = A \sin \omega t$

$$\frac{a}{2} = a \sin \omega t$$

$$\omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega}$$

Now $v = a\omega \cos\omega t$

$$a\omega \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\frac{a2\pi}{T} = \frac{a\pi\sqrt{3}}{T}$$

14. C

$y = 0.45 \sin 2t$

$$\frac{7.5}{100} = 0.45 \sin 2t$$

$\sin 2t = 0.167$

speed $= a\omega\cos 2t$

$$= 0.45 \times 2\sqrt{1 - \sin^2 2t}$$

$$= 0.9 \times 0.98 = 0.87 = \frac{0.5}{\sqrt{3}} \text{ m/s}$$

15. B

$a_{max} = \omega^2 A$

$(1.57)^2 = \omega^2 (1)$

$\omega = 1.57$ rad/sec^2

$$\frac{2\pi}{T} = 1.57$$

$T = 4$ sec.

16. A

$x = A\sin\omega t$ if $t = 1$ is $t=0$

$v = A\omega\cos\omega t$ $\Rightarrow 0.25 = A \times \frac{2\pi}{6}\cos\left(\frac{\pi}{3}(t-1)\right)$

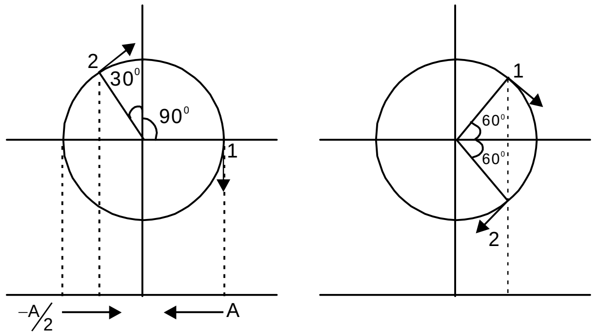
At $t = 2$ sec.

$\Rightarrow 0.25 = A \times \frac{2\pi}{6} \times \frac{1}{2}$ $\Rightarrow A = \frac{3}{2\pi}$

17. C

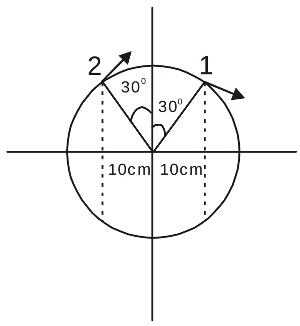
$$\frac{4d^2y}{dt^2} + 9y = 0 \Rightarrow \omega^2 = \frac{9}{4} \Rightarrow \omega = \frac{3}{2}$$

18. D



$t = \frac{2\pi}{3} \times \frac{T}{2\pi}$ $t = T/6$

19. C



Particle 1 and 2 are as shown and their phase difference is 60°.

20. B

Slope of F-x curve gives K

$slope = \frac{13.5}{1.5}$ $F = -Kx \Rightarrow K = 9$

$\omega^2 = \frac{K}{m} = 9$ $\omega = 3$, $T = 2\pi/3$

21. B

$\frac{1}{2}KA^2 = 8 \times 10^{-3} J \Rightarrow$ A = 0.1 m, M=0.1 Kg

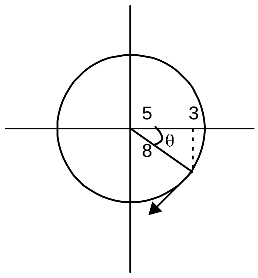
$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1.6}{0.1}} = 4$$

$$\phi = 45^\circ = \frac{\pi}{4} \Rightarrow \quad x = 0.1\sin(4t + \frac{\pi}{4})$$

22. C

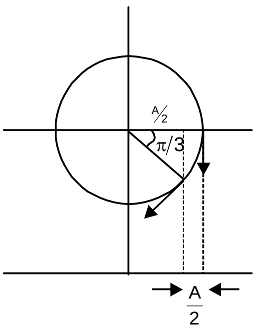
$$\cos\theta = \frac{5}{8} \Rightarrow \qquad \theta = \cos^{-1}(5/8) = \omega t = 51^\circ$$

$t = 0.17\,\text{sec}.$



23. C

$$\theta = \pi/3 = \omega t$$



$$t = \frac{T}{6}$$

$$\therefore V_{avg} = \frac{A/2}{T/6} = \frac{3A}{T}$$

24. C

$T = 2\pi\sqrt{\frac{M}{K}}$ When the rubber ribbon slacks it will not exert any force.

25. C

K will remain unchanged

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \text{ constant}$$

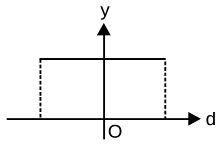
26. D

Initially the COM of spere and water lies at centre of sphere .As water flows out the COM shifts down and length of pendulum increases hence time period increases but when water level becomes half of the sphere the COM again starts shifting up and hence as length decreases time period also decreases.

27. A

Total energy is constant during S.H.M.

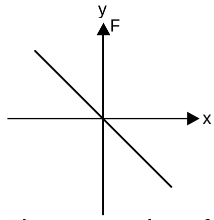
$$\Rightarrow E = \frac{1}{2}KA^2$$



28. D

From mean to extrence the displacement decreases with time whereas from externce to mean displacement increases with time.

29. D



The restoring force always acts towards the mean & increases with increase in displacement.

30. D

Total M.E. = T.E. at M.P. + Total oscillation energy

9 = 5 + 4

Total oscillation energy = $\frac{1}{2}Ka^2$ = 4

$$\Rightarrow K = 8\times 10^4 \Rightarrow \quad T = 2\pi\sqrt{M/K} \quad \Rightarrow T = \pi/100$$

31. B

In series $T_1 = 2\pi\sqrt{\frac{2m}{K}}$

In parallel $T_2 = 2\pi\sqrt{\frac{m}{2K}}$

$$\frac{T_{series}}{T_{parallel}} = 2$$

32. B

$V_{max} = \omega A \qquad \omega_p A_p = \omega_Q A_Q$

$$\frac{A_p}{A_Q} = \frac{\omega_Q}{\omega_p} = \sqrt{\frac{K_2}{K_1}}$$

33. C

$$E_i = \frac{1}{2}KA^2$$

$$E_i = \frac{1}{2}m\omega^2 A^2$$

$$E_f = \frac{1}{2}(2m)\omega'^2 A^2$$

The value of k remains same so $\omega' = \frac{\omega}{\sqrt{2}}$

and hence $E_f = \frac{1}{2} m\omega^2 A^2$

So the energy remains unchanged.

34. A

$V_{max} = A\omega = V$

ω remains unchanged in this case.

Hence $V_{max_2} = 2A\omega$

$V_{max} = 2V$

35. A

$x = 2\sin\omega t$; $\qquad y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$

from Lissajous figures if $\phi = \frac{\pi}{4}$ then the path of particle is an ellipse.

36. C

$a_1 = 1, \quad a_2 = 1 \qquad \theta = \frac{\pi}{3}$

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta} \qquad \Rightarrow a = \sqrt{3}$

37. B

$E = \frac{1}{2}KA_{eq}^2 \qquad A_{eq} = \sqrt{A^2 + A^2}$

$E = \frac{1}{2}m\omega^2\left(\sqrt{2}A\right)^2 \qquad = \sqrt{2}A$

$E = m\omega^2 A^2$

38. A

$T = 2\pi\sqrt{\frac{m}{K}}$

$T_1 = 2\pi\sqrt{\frac{m}{K_1}} \qquad T_2 = 2\pi\sqrt{\frac{m}{K_2}}$

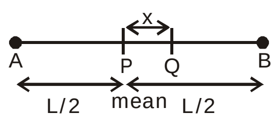
As kl = constant

Hence $k_1l_1 = k_2l_2$

$k_1L = k_2 2L$

$\frac{k_1}{k_2} = 2$; $\qquad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

41. B



Let the particle be displaced a distance x to Q then a quarter of way from A to B means particle is at L/2 from A so time taken is T/6.

$F_{net} = 2\left(L/2 - x\right) - \left(L/2 + x\right) \Rightarrow F_{net} = -4x$

$T = 2\pi$, $\omega = 1$, $T/6 \Rightarrow \therefore \pi/3$

42. C

$T_p = 2\pi\sqrt{\frac{m}{K}} \qquad T_s = 2\pi\sqrt{\frac{\ell}{g+a}}$

T_p = Remain same $\qquad T_s$ decreases

43. A

$n_1T_1 = n_2T_2 \qquad n_1T = \frac{5T}{4} \times n_2$

$\frac{n_1}{n_2} = \frac{5}{4} \Rightarrow n_1 = 5, \qquad n_2 = 4$

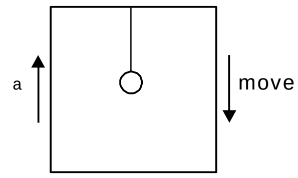
44. A

When the lift is going down with constant velocity the acceleration is zero.

When there is a retardation of 'a' the $g_{effective}$ is g + a.

V = constant $\Rightarrow a = 0$ So there is no effect.

$T_1 = T = 2\pi\sqrt{\frac{\ell}{g}} \qquad g_{eff} = (g + a)$



$T_2 = 2\pi\sqrt{\frac{\ell}{g+a}}$

$T_1 > T_2$

45. C

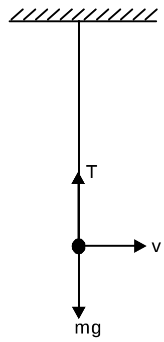
$T_{max} = mg + \frac{mv^2}{\ell}$

$v = A\omega$

$\omega = \sqrt{g/\ell}$

$v^2 = \frac{g}{\ell}A^2$

$T_{max} = mg[1 + (A/\ell)^2]$

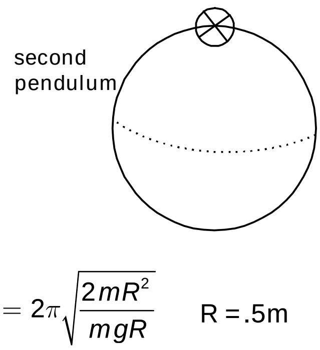


46. A

T = 2 sec.

$T = 2\pi\sqrt{\frac{I}{mgd}}$

Time period of a second's pendulum is two seconds.



$2 = 2\pi\sqrt{\frac{2mR^2}{mgR}} \qquad$ R = .5m

Exercise-2

1. **C**

$2A_i\,\omega_i^2 = A_f\omega_f^2$

$A_i\omega_i = A_f\omega_f$

If $\omega_f = 2\omega_i$ and $A_f = \frac{A_i}{2}$

The condition will be satisfied.

2. **D**

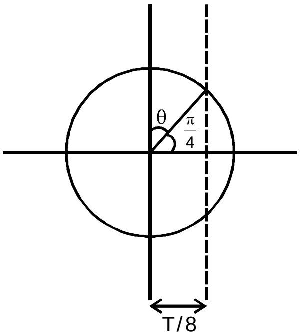
$\theta = \omega t$

$\theta = \frac{2\pi}{T}\cdot\frac{T}{8}$

$\theta = \frac{\pi}{4}$

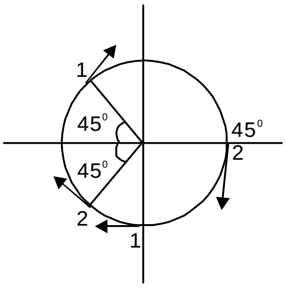
$\cos\frac{\pi}{4} = \frac{x}{A}$

$x = A/\sqrt{2}$



3. **B**

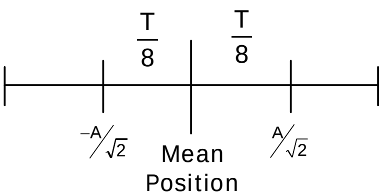
$\frac{135\times\pi}{180} = \frac{3\pi}{4}$



$\therefore t = \frac{3\pi}{4\times 2\pi}T = \frac{3T}{8}$

4. **D**

Max. Average velocity $= \frac{\text{Total max displacement}}{\text{Total time}}$



Max^m displacement will be close to $M.P. = \sqrt{2}A$

$V_{avg} = \frac{4\sqrt{2}}{T}A$

5. **D**

Time period = 8 sec.

In 1st second = $\frac{2\pi}{8}\times 1$

Displacement = $\frac{A}{\sqrt{2}}$

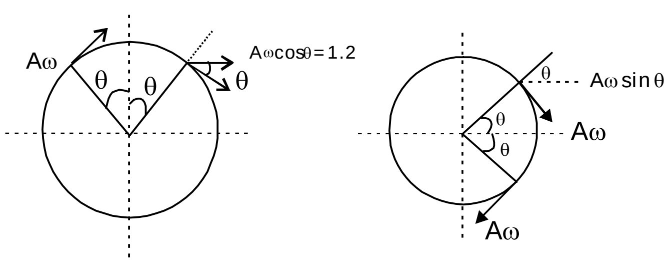
In 1st second = $\frac{2\pi}{8}\times 1$

Displacement = A = $\frac{\pi}{2}$

t = 1

$\frac{A.\sqrt{2}}{A\sqrt{2}\left(\sqrt{2}-1\right)} = \left(\sqrt{2}+1\right)$

6. D

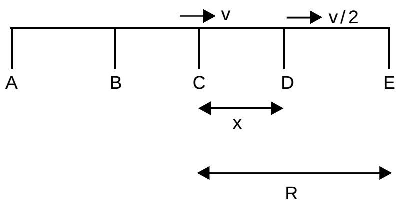


$A\omega\cos\theta = 1.2$ -- (1) $\quad A\omega\sin\theta = 1.6$ -- (2)

$\tan\theta = 4/3 \Rightarrow \quad \theta = 53°$

$\therefore A\omega.\frac{3}{5} = 1.2 \Rightarrow A\omega = 2m/\text{sec}.$

7. **C**



$v/2 = \omega\sqrt{R^2 - x^2} \quad R\omega/2 = \omega\sqrt{R^2 - x^2}$

$R^2/4 = R^2 - x^2 \quad x = \frac{\sqrt{3}}{2}R$

Distance $= 2x = \sqrt{3}R$

8. **B**

$T_1 = 2\pi\sqrt{\frac{m}{K_1}}, \quad T_2 = 2\pi\sqrt{\frac{m}{K_2}} \quad T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$K_{eq}\frac{K_1K_2}{K_1+K_2}$ (for series) $t_1^2 = \frac{2\pi m}{K_1}$,

$t_2^2 = \frac{2\pi m}{K_2}$ and $T^2 = 2\pi m\left(\frac{1}{K_1}+\frac{1}{K_2}\right)$

$\Rightarrow T^2 = t_1^2 + t_2^2$

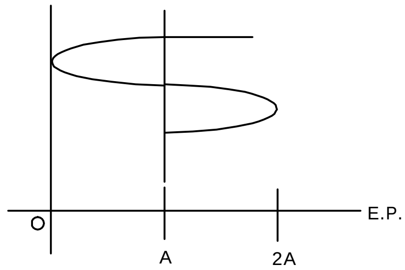
9. **C**

$x = A + A\sin\omega t$

$$t = \frac{2.5\pi}{\omega}$$

$$t = \frac{2\pi}{\omega} + \frac{0.5\pi}{\omega}$$

= 5A



10. C

A = 2cm. $\Rightarrow x = 1cm = \frac{A}{2}$

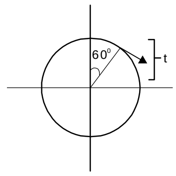
$\Rightarrow \quad v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{3} \Rightarrow \quad a = w^2.1$

$\omega = \sqrt{3} = 2\pi n \quad \Rightarrow \quad n = \frac{\sqrt{3}}{2\pi}$

11. A

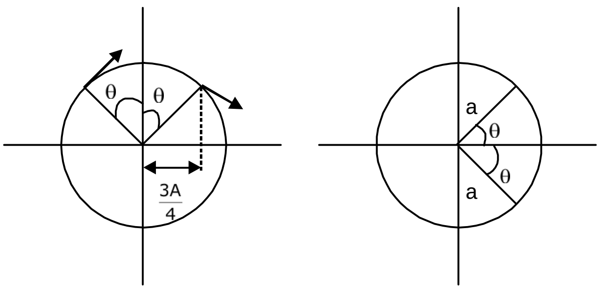
$y = \sin\omega t + \sqrt{3}\cos\omega t$

$y = 2\sin\left(\omega t + \pi/3\right) \qquad A\omega^2 = g$



$\Rightarrow \omega = \sqrt{g/2} \Rightarrow$ t = $\frac{\pi}{6}\sqrt{\frac{2}{g}}$

12. B



$\Rightarrow \cos\theta = \frac{\sqrt{7}}{4}$

$\Rightarrow a\cos\theta = \frac{a\sqrt{7}}{4}$

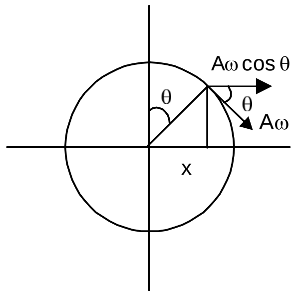
13. B

$\theta = \omega\frac{T}{12} = \frac{\pi}{6}$, $\quad x = A\sin\frac{\pi}{6} = \frac{A}{2}$

$$P.E. = \frac{1}{2}Kx^2 = \frac{1}{2}\frac{KA^2}{4}$$

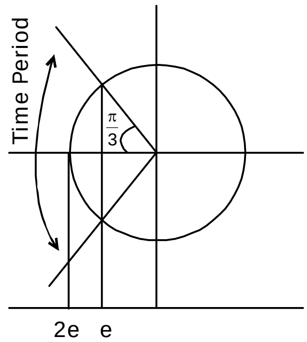
$$v = A\omega\cos\theta = \frac{A\omega\sqrt{3}}{2}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}K\left(\frac{3}{4}A^2\right) \Rightarrow \frac{KE}{PE} = 3$$



14. A

$\omega t = \frac{2\pi}{3} \quad \Rightarrow \frac{2\pi}{T}t = \frac{2\pi}{3} \quad \Rightarrow t = \frac{T}{3} \Rightarrow t = \frac{2\pi}{3}\sqrt{\frac{m}{K}}$

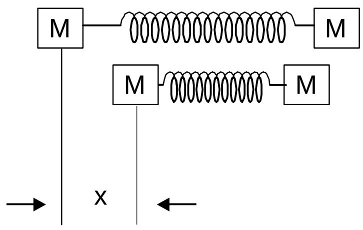


15. C

$\mu = \frac{m_2}{2m} = \frac{m}{2}$

It can be considered as a two block system for $\frac{T}{2}$ time

$t = \frac{T}{2} = \pi\sqrt{\frac{m}{2K}} \quad \Rightarrow t = \pi\sqrt{\frac{2}{2.\pi^2}} = 1s.$



16. A

Elastic Collision

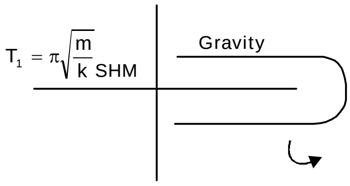
17. C

$$K_{A_{eq}} = K/3,\quad K_B = 3K \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{K_B}{K_A}} = \frac{3}{1}$$

18. C

for x < 0 perform SHM

$$\frac{1}{2}mU^2 = E$$



$$U = \sqrt{\frac{2E}{m}}$$

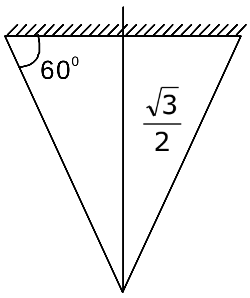
$$\Rightarrow T_2 = \frac{2}{g}\sqrt{\frac{2E}{m}}$$

$$\therefore T = T_1 + T_2 \Rightarrow T = \pi\sqrt{\frac{m}{K}} + \frac{2}{g}\sqrt{\frac{2E}{m}}$$

19. B

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

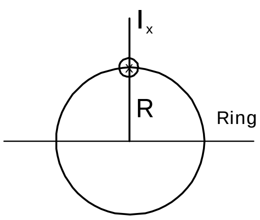
$$= 2\pi\sqrt{\frac{m\left(\frac{\sqrt{3}}{2}\ell\right)^2}{mg\frac{\sqrt{3}}{2}\ell}}$$



$$= 2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$$

20. C

$$I_x = \frac{mR^2}{2} + mR^2$$



$$= \frac{3}{2}mR^2$$

$$T = 2\pi\sqrt{\frac{3}{2}\frac{mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} \Rightarrow \ell = \frac{3}{2}$$

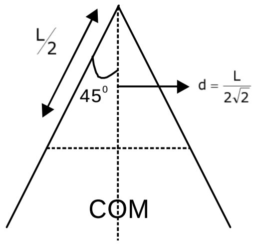
21. $T_1 = 2\pi\sqrt{\ell/g} = T \Rightarrow T_2 = 2\pi\sqrt{\ell/4g} = \frac{T}{2}$

$$T_{eq} = \frac{T_1}{2} + \frac{T_2}{2} \Rightarrow = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

22. B

$$T = 2\pi\sqrt{\frac{I}{mgd}} \quad I = \frac{2md^2}{3}$$

$$T = 2\pi\sqrt{\frac{2md^2}{3}\frac{2\sqrt{2}}{(2m)gd}}$$

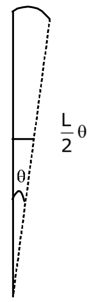


$$T = 2\pi\sqrt{\frac{2\sqrt{2}d}{3g}}$$

23. C

$$KL^2\theta + \frac{KL^2\theta}{4} > mg.\frac{L}{2}.\theta$$

K> 2mg/5L



24. A

$$\tau_{net} = KL^2\theta + \frac{KL^2\theta}{4} - mg.\frac{L\theta}{2} \quad \because K = mg/L$$

$$I = mL^2/3 \Rightarrow \quad \tau_{net} = \frac{3}{4}KL^2\theta \Rightarrow K_{eq} = \frac{3}{4}KL^2$$

$$\omega = \sqrt{\frac{K_{eq}}{I}} = \frac{3}{2}\left(\frac{K}{m}\right)^{1/2}$$

25. D

$$T_A = 2\pi\sqrt{\frac{I_A}{mgx}} \quad \text{----- (1)}$$

$$T_B = 2\pi\sqrt{\frac{I_B}{mg(0.25 - x)}} \quad \text{-----(2)}$$

Eq. (1) & (2)

$$\frac{3}{4} = \sqrt{\frac{I_A(0.25 - x)}{I_B x}} \Rightarrow \frac{3}{4} = \sqrt{\frac{9}{4}\left(\frac{0.25 - x}{x}\right)}$$

$$\Rightarrow \frac{9}{16} = \frac{9}{4}\left(\frac{0.25 - x}{x}\right) \Rightarrow x = 0.20\text{m}$$

26. B,C,D

$$mg = Kx_o \qquad x_o = \frac{10}{500} = 2cm$$

$$\omega = \sqrt{\frac{500}{1}} = 10\sqrt{5}\ \frac{rad}{sec}$$

$\therefore$ Maximum velocity = $A\omega = 3\times10^{-2} \times 10\sqrt{5}$

$= 30\sqrt{5}$ cm/sec. $\Rightarrow$ $Max^m\ Acc^n = A\omega^2 = 15\ m/s^2$

27. A

$$2\times\frac{1}{2}KA^2 = \frac{1}{2}KA'^2 \Rightarrow A' = \sqrt{2}A \Rightarrow T = 2\pi\sqrt{\frac{m}{K}}$$

28. B $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

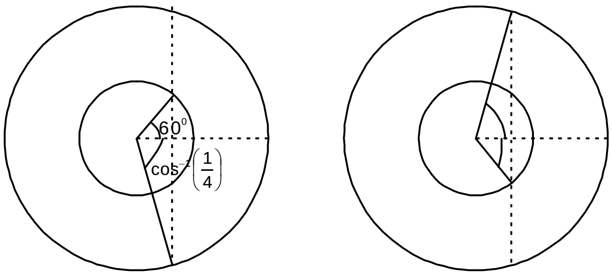
29. B, C

Phase by $\omega = \frac{\omega\times2\pi}{3\omega} = \frac{2\pi}{3}$

Phase by $2\omega = \phi + 2\omega\times{}^{2\pi}\!/_{3\omega}$,

$\phi + \frac{4\pi}{3} - \frac{2\pi}{3} = 0, 2\pi, 4\pi \qquad \phi = \frac{2\pi}{3}, \frac{4\pi}{3}......$

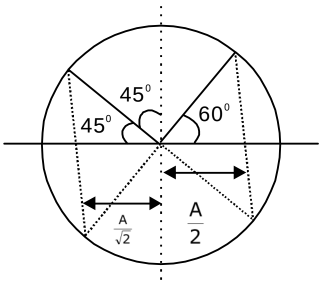
30. A



31. A,B,C

$x = 5\sin\left(4\pi t + \tan^{-1} {}^4\!/_3\right)$

32. C



33. B

$v^2 = 108 - 9x^2 \quad \Rightarrow v^2 = 9[12-x^2]$

$\Rightarrow 2v\frac{dv}{dx} = -2x\times9 \qquad \Rightarrow a = -9x \Rightarrow \omega^2 = 9$

34. A,B,C,D

$A\omega^2 = g \quad \Rightarrow 0.40\ \omega^2 = 10$

b) Negative Extreme

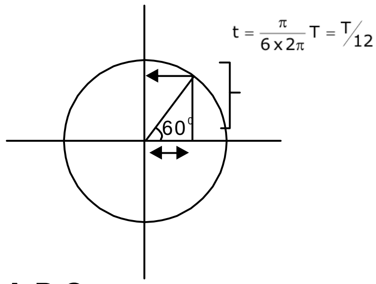
35. B,C,D

$u = 5x^2 - 20x \qquad \Rightarrow F = \frac{-du}{dx} = -10(x-2)$

M.P. at x = 2m

36. A,B

$\omega^2 R\sin60 \qquad = {}^a\!/_2$



37. A,B,C

$\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \qquad = \frac{1}{2}m\omega^2A^2(0.64)$

$\frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2(0.64)$

Put A = 10 cm. $\Rightarrow$ x = 6 cm.

38. B,D

$x = 3\sin100t + 4(1+\cos100t)$

$x = 5\sin\left(100t + \tan^{-1}{}^4\!/_3\right) + 4$

M.P. is at 4 with A = 5

39. A

F = -Kx Slope = -K

40. C,D

$x = a\sin\omega t$ .....(1)

$y - a = -a\cos\omega t$ ....(2)

$(1)^2 + (2)^2 \quad \Rightarrow (y-a)^2 + x^2 = a^2$

41. B,C,D

$v^2 = \omega^2\left(A^2 - x^2\right) \qquad F = -Kx$

$\frac{v^2}{\omega^2} + x^2 = A^2$ ---- Ellipse

$v^2 = \left(A^2 - \omega^2x^2\right)$, $a = -\omega^2x$, $a^2 = \omega^4x^2$

$v^2 = \left(A^2 - \frac{a^2}{\omega^2}\right) \Rightarrow v^2 + \frac{a^2}{\omega^2} = A^2$ ---- Ellipse

42. A,B,C

v = ωA & $\omega = \frac{10}{2.5} = 4$

(a) $T = \frac{2\pi}{\omega} = \frac{\pi}{2} = 1.57$

(b) $a = \omega^2A = 40$

(c) $v = \omega\sqrt{A^2 - x^2} \quad = 2\sqrt{21}$

43. A,C

$v = \sqrt{\frac{900}{3}}\sqrt{\left(2^2 - 1^2\right)} = 10\sqrt{3}\times\sqrt{3}$ = 30 m/s

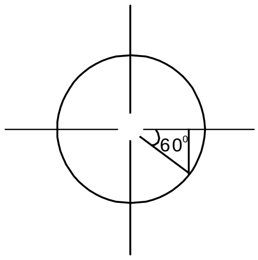
M.C. 3 x 30 = 9 x v $\Rightarrow$ v = 10 m/s

$$10 = \sqrt{\frac{900}{9}}\sqrt{\left(A^2 - 1^2\right)} \Rightarrow A = \sqrt{2}m$$

44. B,D

$$\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6} \Rightarrow \quad x = A\sin\left(\frac{2\pi t}{T} + \frac{5\pi}{6}\right)$$

$$x = A\cos\left(\frac{2\pi}{T}.t + \frac{\pi}{3}\right)$$

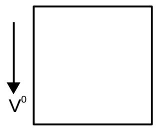


45. B

$T = 2\pi\sqrt{\frac{m}{K}}$ ----> not dependent on q_{eff}.

the velocity of particle

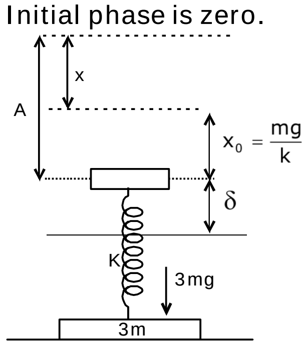
at M.P. = V_0 $\therefore V_0 = A\omega_0$



$$A = \frac{v_0}{\omega_0}$$

Initial phase is zero.

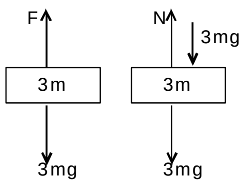
46.



kx ≥ 3mg $\qquad$ δ = $\frac{2mg}{k}$

x ≥ $\frac{3gm}{k}$ $\qquad$ F = kδ + kx = 3mg

A = $\frac{4mg}{k}$



N = 6mg

F ≥ 3mg

47. A,C

$$V_{rms} = \sqrt{\frac{\int v^2.dt}{\int dt}}$$

48. B,C

KE Average = $\frac{1}{4}KA^2 = m\pi^2f^2A^2$

PE Average = $m\pi^2f^2A^2$

49. B,C

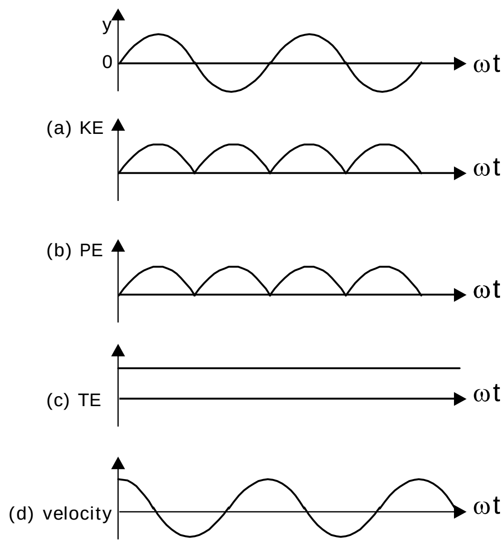
$$\frac{1}{2}KA^2 = \frac{1}{2}\times 2\times 10^6 \times 10^{-4} = 100$$

$\therefore$ 60 J P.E. at M.P.

$Max^m K.E. = 100$ *joule*

$Max^m P.E. = 160$ *joule*

50. B



Exercise-3 Level-I

1. $x = (5m)\sin\left[\left(\pi s^{-1}\right)t + \frac{\pi}{6}\right]$

$\because x = A\sin(\omega t + \phi)$

$\Rightarrow A = 5m, \phi = \frac{\pi}{6}, T = \frac{2\pi}{\omega} = 2\,\text{sec}$. and

$V_{max} = A\omega = 5\pi m/s$

2. x = 2.0 sin$\left[\left(100s^{-1}\right)t + \frac{\pi}{6}\right]$

A=0.2 cm, w=100, M=10gm

$v = (2.0 \times 100)$ cm/s $\cos\left[\left(100s^{-1}\right)t + \frac{\pi}{6}\right]$

$a = 2 \times 10^4 cm/s^2\left[-\sin\left[\left(100s^{-1}\right)t + \frac{\pi}{6}\right]\right]$

$\Rightarrow a = -10^4 x \;\; \because F = -Kx$ and F=100 N/m

$\Rightarrow K = 10^4 \times M = 10^4 \times \frac{1}{100} = 100 N/m$

$\Rightarrow T = \frac{2\pi}{w} = \frac{2\pi}{100} = \frac{\pi}{50}\,\text{sec}.$

a at t=0 $\Rightarrow a = -200\ m/s^2 \sin\frac{\pi}{6}$

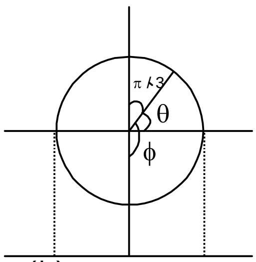
$= -100\ m/s^2$

3. ω= 20

(a) $\theta = \frac{\pi}{6} = \omega t$

$t = \frac{\pi}{120}$

(b) $\phi = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3} \quad t = \frac{2\pi}{3 \times 20} = \frac{\pi}{30}$



(c) Same as (b)

4. $K_1\ell_1 = K_2\ell_2 = K\ell$

$\Rightarrow \frac{K_1}{K_2} = \frac{\ell_2}{\ell_1} = \frac{3}{1} \quad \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}} = \frac{1}{\sqrt{3}}$

5. $\mu = \frac{3 \times 6}{3 + 6} = 2$ Kg. $\omega = \sqrt{\frac{k}{\mu}} = 20$

(a) $T = 2\pi\sqrt{\frac{\mu}{K}} = \frac{\pi}{10}$

(b) $A_1 + A_2 = 6$

$\Rightarrow m_1A_1 = m_2A_2 \Rightarrow A_1 = \frac{m_2}{m_1 + m_2}A$

A_1 = 4 cm, A_2 = 2 cm.

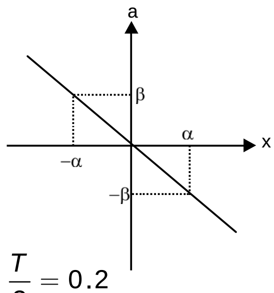
(c) $m_2V_{max} = 6.\omega A_2 = 2.4$

6. $T = \frac{2\pi}{5\pi} = 0.4\,\text{sec}.$

Av. Speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{0.2 \times 7}{0.7}$ = 2m/s

7. F=-Kx

$a = -\frac{K}{m}x \qquad \frac{K}{m} = \tan\theta = \frac{\beta}{\alpha}$



$\omega = 2\pi f$, $f = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

8. $\frac{T}{3} = 0.2$

$T = 0.6 = 2\pi\sqrt{\frac{0.9}{K}} \Rightarrow$ K = 100 N/m

9. F=-10x + 2 = -10(x – 0.2)

x = 0.2 is M.P.

(a) Amplitude = 2 + 0.2 = 2.2 m

(b) $T = 2\pi\sqrt{\frac{0.1}{10}} = \frac{\pi}{5}s$

(c) x = 0.2 - 2.2 cos ωt

10. U=(x^2 -4x + 3)

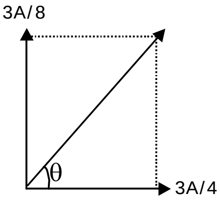
$F = -\frac{du}{dx} = -2x + 4 \quad \Rightarrow$ F = -2(x-2)

(a) For equillibrium F = 0 $\Rightarrow x = 2m$

(b) $\omega = \sqrt{\frac{2}{1}} = \sqrt{2} \quad \Rightarrow T = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

(c) $\frac{1}{2}KA^2 = \frac{1}{2}mv^2 \quad \Rightarrow A = 2\sqrt{3}$

11. $\tan\theta = \frac{1}{2}$



$$A_{net} = \sqrt{\left(\frac{3A}{4}\right)^2 + \left(\frac{3A}{8}\right)^2} = \frac{3A}{4}\sqrt{1+\left(\frac{1}{2}\right)^2}$$

$$= \frac{3\sqrt{5}}{8}A$$

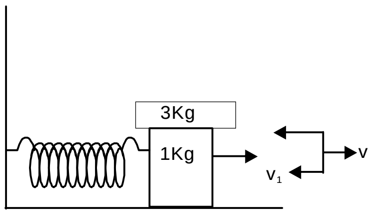
12. F_{max} = KA = 50

$$x = \frac{A}{\sqrt{2}} \quad \therefore F = Kx = \frac{KA}{\sqrt{2}} \quad F = \frac{F_{max}}{\sqrt{2}} = 50\sqrt{2}$$

$F = 25\sqrt{2}$ N

13. Given A=0.1m, K=100 N/m
m_1 = 1Kg, m_2 = 3Kg
Energy of system is
not conserved.

$$\frac{1}{2}KA^2 = \frac{1}{2}m_1v_1^2$$



$$\Rightarrow v_1 = \frac{KA^2}{m_1}$$

$$\Rightarrow v_1 = 1 \text{ m/s}$$

$$m_1v_1 = (m_1+m_2)v \qquad \Rightarrow v = \frac{1}{(m_1+m_2)}$$

$$\Rightarrow v = 0.25 \text{ m/s}$$

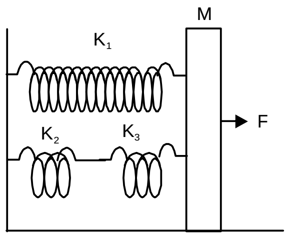
$$\because \text{freq}^n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{100}{4}} = \frac{5}{2\pi}H_2$$

Now amplitude

$$\frac{1}{2}KA'^2 = \frac{1}{2}(m_1+m_2)v^2 \Rightarrow A'^2 = \frac{(0.25)^2 \times 4}{100}$$

$$\Rightarrow A' = 5cm.$$

14. $K_{eq} = K_1 + \frac{K_2K_3}{K_2+K_3}$

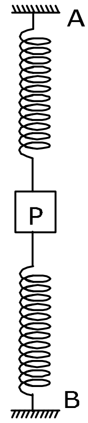


$$\because F = K_{eq}A$$

$$\Rightarrow A = \frac{F}{K_{eq}} = \frac{(K_2+K_3)F}{K_1K_2+K_2K_3+K_3K_1}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M}} = \frac{1}{2\pi}\left[\frac{K_1K_2+K_2K_3+K_3K_1}{M(K_2+K_3)}\right]^{1/2}$$

15. Mg **=** K x 20 - K x 10



$$\Rightarrow K = \frac{mg}{10/100} \text{ N/m}$$

$$T = 2\pi\sqrt{\frac{M}{K}}$$

$$T = 2\pi\sqrt{\frac{1}{49\times 4}} = \frac{\pi}{7}\text{sec}$$

16. General equation

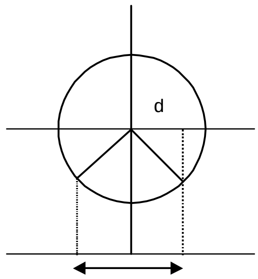
$$x = A\sin(\omega t+\phi) \text{ A=10, } \omega = \frac{2\pi}{T} = \pi$$

At t = 0, x = 5 ⇒ 5=10 sin ϕ

$$\phi = \frac{\pi}{6} \quad \Rightarrow x = 10\sin\left(\pi t + \frac{\pi}{6}\right)$$

17. $d = a\sin\frac{\phi}{2}$

Max distance = 2d



$$2d = 2a\sin\frac{\phi}{2}$$

= 2a x 0.9 = 1.8 a

18. By momentum conservation
5 x 3 + 10 x 2 = 7v
v = 5 m/s
By Energy Conservation

$$\frac{1}{2}.5.3^2 + \frac{1}{2}\times 2\times(10)^2 = \frac{1}{2}\times 7\times 5^2 + \frac{1}{2}Kx^2$$

x = 0.25 m, $T = 2\pi\sqrt{\frac{\mu}{K}} \Rightarrow \mu = \frac{10}{7}$

$$\Rightarrow T = \pi/14$$

first compression occurs $= \frac{3T}{4} = \frac{3\pi}{56}s$

19. $x = 30\sin(\pi t + \pi/6)$

$$\frac{1}{2}Kx^2 = 2\left(\frac{1}{2}m\omega^2(A^2-x^2)\right)$$

$$\Rightarrow \frac{x^2}{2} = \left(0.09 - x^2\right)$$

$$\Rightarrow x^2 = 0.06 \;\Rightarrow x = \frac{\sqrt{6}}{10}\text{ m.}$$

20. $a = 8\pi^2 - 4\pi^2 x$

$F = -4\pi^2 m\left[x - 2\right]$ $\qquad \Rightarrow$ x = 2 M.P.

$$T = 2\pi\sqrt{\frac{m}{4\pi^2 m}} = 1s$$

$\omega = 2\pi$ rad/s

$x = 2 - 4\cos 2\pi t$

21. (a) $T = 2\pi\sqrt{\frac{I}{K}} \quad = 2\pi\sqrt{\frac{10}{10\pi^2}} = 2s$

(b) $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} \;\Rightarrow\; g_{eff} = \sqrt{g^2 + a^2} = 5\sqrt{5}$

$$T = 2\pi\sqrt{\frac{0.5}{5\sqrt{5}}} \;= \frac{2}{(5)^{1/4}}$$

22. $\frac{1}{2}Kx^2 = 0.4 \qquad \omega = 2\pi \times \frac{25}{\pi}$

$x^2 = \frac{0.8}{K}$, ω = 50, K = 500

$\frac{1}{2}m\omega^2\left(A^2 - x^2\right) = 0.5 \Rightarrow A^2 = x^2 + \frac{1}{500}$

A = 0.06 m

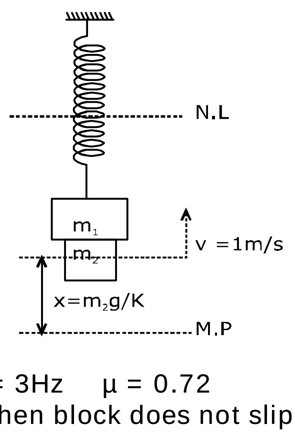
23. Momentum Conservation

0.5(3) = 1.5 v

v = 1m/s

$$\omega = \sqrt{\frac{K}{m_1 + m_2}} = 20$$

$$v = \omega\sqrt{A^2 - x^2} \Rightarrow A^2 = \frac{v^2}{\omega^2} - x^2$$



24. f = 3Hz $\quad \mu = 0.72$

When block does not slip

$ma_{max} < f \Rightarrow ma_{max} < \mu mg$

$\Rightarrow a_{max} < \mu g \qquad \Rightarrow \omega^2 A_{max} < \mu g$

$$A_{max} = \frac{\mu g}{4\pi^2 f^2} = 0.02$$

25. $U(x) = -ax^2 + bx^4$

$F = \frac{-dU}{dx} \Rightarrow F = 2ax - 43bx^3$

for MP F = 0 $\Rightarrow$ x = 0, $x = \sqrt{\frac{a}{2b}}$

$F = 2a(x + x_0) - 4b(x + x_0)^3$

$= 2ax\left(1 + \frac{x_0}{x}\right) - 4bx^2\left(1 + \frac{3x_0}{x}\right)$

$= 2a\sqrt{\frac{a}{2b}}\left[1 + \frac{x_0}{x} - 1 - \frac{3x_0}{x}\right]$

$= 2a\sqrt{\frac{a}{2b}}\left[\frac{-2x_0}{x}\right] \Rightarrow 2a\sqrt{\frac{a}{2b}}\sqrt{\frac{2b}{a}}\left(-2x_0\right)$

$= -4ax_0$

$k_{eq} = 4a \quad \Rightarrow \omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{4a}{m}}$

26. $T = 2\pi\sqrt{\frac{\ell}{g}} = 2 \Rightarrow \;\ell = 1m$

27. $\theta = \frac{\pi}{90}\sin\left(\pi s^{-1}\right)t \Rightarrow \quad \omega = \pi$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\,\text{sec}.$

$T = 2\pi\sqrt{\frac{\ell}{g}} = 2 \Rightarrow \ell = 1m$

28. $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} \;\Rightarrow\; g_{eff} = g - \frac{3g}{4} = \frac{g}{4}$

(i) $T = 2\pi\sqrt{\frac{4\ell}{g}} = 2T_0 \qquad$ if $T' = \frac{T_0}{2}$

(ii) $2\pi\sqrt{\frac{\ell}{g_{eff}}} = \pi\sqrt{\frac{\ell}{g}} \;\Rightarrow\; g_{eff} = 4g$

$g + a = 4g \qquad \Rightarrow \qquad a = 3g \quad \uparrow$

29. (a) $g_{eff} = g + a_0 \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g + a_0}}$

(b) $g_{eff} = g - a_0 \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g - a_0}}$

(c) $g_{eff} = g \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g}}$

30. $T_1 = 2\pi\sqrt{\dfrac{\ell}{g}}$ $\quad T_2 = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2 + a^2}}}$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\left(g^2 + a^2\right)^{1/4}}{\sqrt{g}} \quad \Rightarrow \left(\frac{T_1}{T_2}\right)^4 g^2 = g^2 + a^2$$

$$\Rightarrow g^2 = \left(\frac{4 - 0.01}{4}\right)^4 \left(g^2 + a^2\right)$$

$$\Rightarrow g^2 = (1 - 0.01)\left(g^2 + a^2\right) \Rightarrow a^2 = 0.01g^2$$

$$\Rightarrow a = 0.1g \qquad \Rightarrow a = g/10$$

31. $I = \dfrac{m\ell^2}{3} + \dfrac{m\ell^2}{12} + m\ell^2 = \dfrac{17}{12}m\ell^2$

$$d = \frac{3\ell}{4}$$

$$T = 2\pi\sqrt{\frac{17m\ell^2}{12.2mg.\frac{3}{4}\ell}} \Rightarrow T = 2\pi\sqrt{\frac{17\ell}{18g}}$$

32. $T = 2s$

$$T_1 = \frac{T}{2}$$

$$T_2 = 2\pi\sqrt{\frac{\ell}{4g}} = \frac{T}{2}$$

$$T' = T_1 + \frac{T_2}{2} \qquad \Rightarrow T' = \frac{T}{2} + \frac{T}{4} = \frac{3}{2}s.$$

Exercise-3 Level-II

1. $K.E. = 8\times10^{-3}$ J

$\dfrac{1}{2}m\omega^2A^2 = 8\times10^{-3}$ J

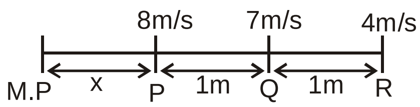
$\dfrac{0.1}{2}\omega^2(.1)^2 = 8\times10^{-3}$

$\omega = 4$

$\phi = 45^o = \pi/4$

$x = (.1)\sin(4t + \pi/4)$

2.



As $V^2 = \omega^2(A^2 - x^2)$

For P, $\quad 64 = \omega^2(A^2 - x^2)$ ...(1)

For Q, $\quad 49 = \omega^2[A^2 - (x+1)^2]$ ...(2)

For, R, $16 = \omega^2[A^2 - (x+2)^2]$ ...(3)

(1) – (2)

$15 = \omega^2(2x + 1)$

(2) – (3)

$33 = \omega^2(2x + 3)$

$\dfrac{15}{33} = \dfrac{(2x+1)}{(2x+3)}$

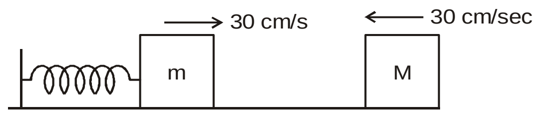
$x = \dfrac{1}{3}$

Putting the value in equation above

$\omega = 3$

$A = \dfrac{\sqrt{65}}{3}$, $\qquad$ Max. Speed $= A\omega = \sqrt{65}$

3.



we know that $\omega^2 = \dfrac{k}{m}$

$k = m\omega^2 = (1)(10)^2 = 100$ N/m

At t = 0 block of mass m is at mean position

x = 10 cm.

velocity of block $m = v_m = \dfrac{dx}{dt} = 30\cos10t$

at $t = 0$ $v_m = 30$ cm/sec.

from momentum conservation

$(M + m)v = M(30) - m(30)$

$v = 15$ cm/sec

Now $\quad \dfrac{1}{2}(M+m)v^2 = \dfrac{1}{2}kA^2$

on solving $A = 3$ cm

(b) New ω of the system having mass (M + m)

$\omega' = \sqrt{\dfrac{K}{M+m}} = \sqrt{\dfrac{100}{4}} = 5$ rad/s

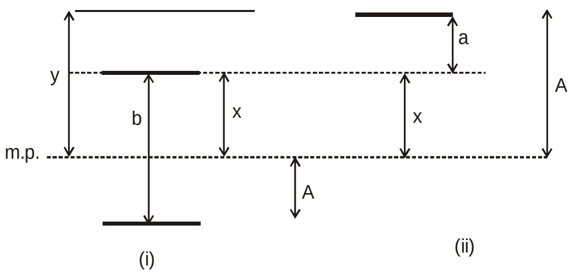
$x' = 10 - 3\sin5t$

(c) Losse of energy during collision = Energy before collision – Energy after collision

$= \dfrac{1}{2}m(0.3)^2 + \dfrac{1}{2}M(0.3)^2 - \dfrac{1}{2}(M+m)(0.15)^2$

$= 0.135$ Jule

4.



(A) from figure (i) b = A + x ...(1)

from figure (ii) A = a + x ...(2)

from eq. (1) & (2)

b = a + 2x $\Rightarrow$ 2x = b – a

and x = mg/k

$$\Rightarrow \quad K = \frac{2mg}{b-a}$$

(B) Oscillation frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{K}{m_{total}}}$

$$= \frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$$

(C) By energy conservation.

5. (a) Both the spring have same force so.

It is parallel equivalent of spring

$$k_{eq} = k_1 + k_2 = 0.2\,N/m$$

Now the problem change in two block system in which reduced mass is

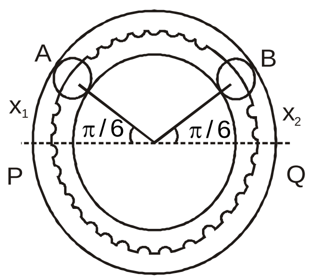
$$m = \frac{m_1 m_2}{m_1 + m_2} = \frac{0.1\times 0.1}{0.1+0.1} = 0.05\,kg$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.20}{0.05}} = \frac{1}{\pi}H_2$$

(b) Balls are at rest in position A & B so

Total energy is in potential energy for

$$E = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$



$$= \frac{1}{2}kx^2 + \frac{1}{2}kx^2 \qquad \left\{\begin{array}{l} x_1 = x_2 \\ R_1 = R_2 \end{array}\right.$$

$$E = kx^2$$

$$x = x_1 + x_2 \qquad = R\,\pi/6 + R\,\pi/6$$

$$= 0.02\,\pi\,m$$

Now $E = kx^2$

$$= (0.1)\,(0.02\pi)^2 = 4\pi^2 \times 10^{-5}\,J$$

(c) At P & Q no stretch in spring so complete energy is in the kinetic form

$$\Rightarrow \quad \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = E$$

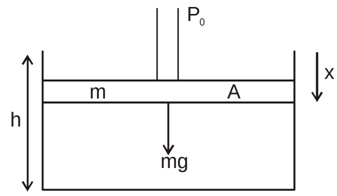
$$m_1 = m_2 = 0.1\,kg$$

$$v_1 = v_2 = v \;\Rightarrow\; 0.1v^2 = 4\pi^2\times 10^{-5}$$

$$v = 2\pi\times 10^{-2}\,m/sec$$

6. $P = P_0 + \dfrac{mg}{A}$

Bulk modulus B = γp



$$\frac{-\Delta P}{\dfrac{\Delta v}{v_0}} = \gamma p$$

$$\Delta P = \frac{-\gamma p\,\Delta V}{V_0}$$

$$F_{net} = A\,\Delta P = \frac{-A\gamma p\,\Delta v}{v_0}$$

$$= -A\gamma P\frac{x\,A}{h\,A} \qquad \left\{\begin{array}{l} \because \quad \Delta v = xA \\ \quad v_0 = hA \end{array}\right\}$$

$$= -A\gamma P\frac{x}{h}$$

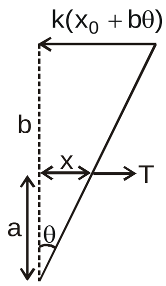
$$F_{net} = -A\gamma\left[P_0 + \frac{mg}{A}\right]\frac{x}{h} \Rightarrow k = mw^2 = A\gamma\left(P_0 + \frac{mg}{A}\right)\frac{1}{h}$$

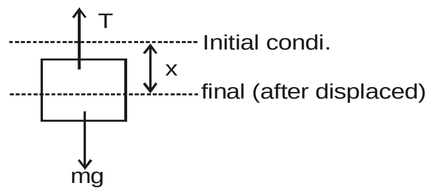
$$\omega^2 = \frac{A\gamma}{mh}\left(P_0 + \frac{mg}{A}\right) \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{A\gamma}{mh}\left(P_0 + \frac{mg}{A}\right)}$$

7. At equilibrium condition we assume elongation is spring is x_0 then

mg(a) = Kx_0b ...(1)

Now rod is moved small angle θ then





$$\Rightarrow \quad T.a = K(x_0 + b\theta).b$$

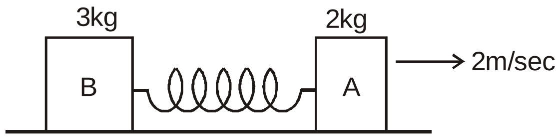
$$T=\frac{k(x_0+b\theta)b}{a}$$

On block of mass m $F_{net}=mg-T$

$$F_{net}=mg-\frac{k(x_0+b\theta)b}{a}$$

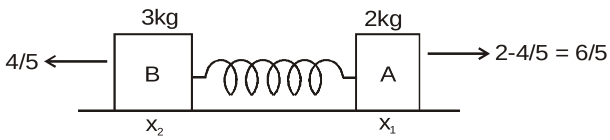
$$=mg-\frac{kx_0b}{a}-\frac{kb^2\theta}{a}=\frac{-kb^2\theta}{a}\quad\left\{\text{and }\theta=\frac{x}{a}\right.$$

$$=\frac{-kb^2x}{a^2}\quad\Rightarrow\quad T=2\pi\sqrt{\frac{ma^2}{kb^2}}$$

8. 

$V_{com}=4/5$

In frame of chita :-



Let us assume that elongation in spring is x then

$x_1+x_2=x$ ....(1)

$2x_1=3x_2$ ....(2)

(Centre of mass is at rest)

from (1) & (2)

$$x_1+\frac{2x_1}{3}=x \qquad ...(3)$$

from energy conservation

$$\frac{1}{2}\times2\times\left(\frac{6}{5}\right)^2+\frac{1}{2}\times3\times\left(\frac{4}{5}\right)^2=\frac{1}{2}kx^2$$

$x=0.2$ ...(3)

from (2) & (3)

$x_1=0.12$ m

Maximum velocity $=A\omega=6/5$

$x_1\omega=6/5$

$(0.12)\,\omega=6/5$

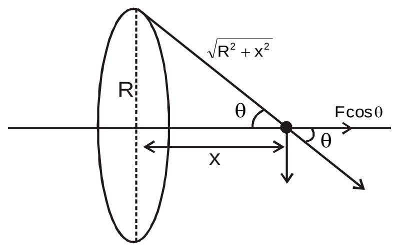
$\omega=10$

then equation of block A

$$x=\left(\frac{4}{5}\right)t+0.12\sin10t$$

9. $F_{net}=f\cos\theta$

$$\cos\theta=\frac{x}{\sqrt{R^2+x^2}}$$



$$\cos\theta=\frac{x}{R}\qquad\because\ x^2\cong0\ \text{(for small distance)}$$

$$F=-\frac{GM_1M_2}{R^3}x$$

$$T=2\pi\sqrt{\frac{M_2}{K}}\qquad\therefore\ M_1=\rho\times2\pi R$$

$$=2\pi\sqrt{\frac{R^3}{G\rho2\pi R}}\Rightarrow T=\frac{2\pi R}{\sqrt{G\rho2\pi}}=\sqrt{\frac{2\pi R^2}{G\rho}}$$

Exercise-4 Level-I

1. B

As we know that spring constant of spring is inversely proportional to length of spring, so new spring constant for each part is given by k' = nk where, k is the spring constant of whole spring. From the theory of spring pendulum, we know that time period of spring pendulum is inversely proportional to square root of spring constant ie,

$$T\propto\frac{1}{\sqrt{k}}\quad T'\propto\frac{1}{\sqrt{nk}}$$

So, $T'=\dfrac{T}{\sqrt{n}}$.

2. C

Kinetic energy of particle of mass m is SHM at any point is,

$$=\frac{1}{2}m\omega^2(a^2-x^2)$$

and potential energy = $=\frac{1}{2}m\omega^2x^2$

where, a is amplitude of particle and x is the distance from mean position.

So, at mean position, x = 0

$$\therefore KE=\frac{1}{2}m\omega^2a^2\,(\text{maximum})$$

$$PE=0\ \ (\text{minimum})$$

3. B

As the child stands up, the effective length of pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T will decrease.

4. A

At x = 0, kinetic energy is maximum and potenial energy is minimum.

5. D

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

Given, $\frac{\Delta l}{l} = 21\%$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2} \times 21\% = 10.5\%$$

6. C

$$v_{max} = a\omega = a\frac{2\pi}{T}$$

$$= \frac{2\pi a}{2\pi\sqrt{\frac{m}{k}}} = a\sqrt{\frac{k}{m}}$$

Hence, $\frac{v_{max_1}}{v_{max_2}} = \frac{a_1}{a_2}\sqrt{\frac{k_1}{k_2}}$

$\because \quad v_{max_1} = v_{max_2}$

$$\Rightarrow \quad \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

7. C

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad \ldots(i)$$

$$T' = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}} \qquad \ldots(ii)$$

Dividing Eq. (i) and (ii), we have

$$\therefore \quad \frac{3}{5} = \sqrt{\frac{M}{M+m}} \quad \text{or} \quad \frac{9}{25} = \frac{M}{M+m}$$

or $\quad 9M + 9m = 25\,M$

or $\quad 16M = 9m$

or $\quad \frac{m}{M} = \frac{16}{9}$

8. A

For amplitude of oscillation and energy to be maximum, frequency of force must be equal to the initial frequency and this is only possible in case of resonance.

In resonance state, $\omega_1 = \omega_2$

9. B

Initial angular velocity of particle = ω_0

and at any instant t, angular velocity = ω

Therefore, for a displacement x, the resultant acceleration

$$f = (\omega_0^2 - \omega^2)x \qquad \ldots(i)$$

External force, $F = m(\omega_0^2 - \omega^2)x \quad \ldots(ii)$

Since, $\quad F \propto \cos\omega t \quad$ (given)

$\therefore$ From Eq. (ii)

$$m(\omega_0^2 - \omega^2)\,x \propto \cos\omega t \qquad \ldots(iii)$$

Now, from equation of simple harmonic motion

$x = A\sin(\omega t + \phi) \qquad \ldots(iv)$

at t = 0, x = A

$A = A\sin(0 + \phi)$

$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

$$\therefore \quad x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos\omega t \quad \ldots(v)$$

Hence, from Eqs. (iii) and (v), we finally get

$$m(\omega_0^2 - \omega^2)A\cos\omega t \propto \cos\omega t$$

$$\Rightarrow \quad A \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

10. C

In simple harmonic motion when a particle is displaced to a position from its mean position then its kinetic energy gets converted into potential energy. Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on displacement x.

11. B

Time period of spring

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Here k, being the force constant of spring.

For first spring

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \quad \ldots(i)$$

For second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \quad \ldots(ii)$$

The effective force constant in their series combination is

$$k = \frac{k_1k_2}{k_1+k_2}$$

Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1+k_2)}{k_1k_2}\right]}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1k_2} \quad \ldots(iii)$$

From Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1}+\frac{m}{k_2}\right)$$

or $$t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$$

or $$t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1k_2}$$

$$\Rightarrow \quad t_1^2 + t_2^2 = T^2 \quad \text{[From Eqs. (iii)]}$$

12. C

The time period of simple pendulum in air

$$T = t_o = 2\pi\sqrt{\left(\frac{l}{g}\right)} \quad \ldots(i)$$

Here l, being the length of simple pendulu.
In water, effective weight of bob
w' = weight of bob in air - upthrust

$$\Rightarrow \rho V g_{eff} = mg - m'g$$

$$= \rho Vg - \rho' Vg$$

$$= (\rho - \rho')Vg$$

where ρ' = density of water
ρ = density of bob

$$\therefore \quad g_{eff} = \left(\frac{\rho-\rho'}{\rho}\right)g = \left(1-\frac{\rho'}{\rho}\right)g$$

$$\therefore \quad t = 2\pi\sqrt{\left[\frac{\ell}{\left(1-\frac{\rho'}{\rho}\right)g}\right]} \quad \ldots.(ii)$$

Thus, $$\frac{t}{t_o} = \sqrt{\left[\frac{1}{\left(1-\frac{\rho}{\rho'}\right)}\right]}$$

$$= \sqrt{\left[\frac{1}{1-\frac{1000}{(4/3)\times 1000}}\right]} = \sqrt{\left(\frac{4}{4-3}\right)}$$

= 2
or $t = 2t_0$

13. B

$$\frac{d^2x}{dt^2} = -\alpha x \quad \ldots(i)$$

We know that

$$a = \frac{d^2x}{dt^2} = -\omega^2 x \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\omega^2 = \alpha$$

or $$\omega = \sqrt{\alpha}$$

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\alpha}$$

or $$T = \frac{2\pi}{\sqrt{\alpha}}$$

14. A

Given, $$y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$$

$$\Rightarrow \quad \frac{dy_1}{dt} = v_1 = 0.1\times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$$

or $$v_1 = 10\pi\sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

or $$v_1 = 10\pi\sin\left(100\pi t + \frac{5\pi}{6}\right)$$

and $y_2 = 0.1\cos\pi t$

$$\Rightarrow \frac{dy_2}{dt} = v_2 = 0.1\sin(\pi t + \pi)$$

Hence, phase difference

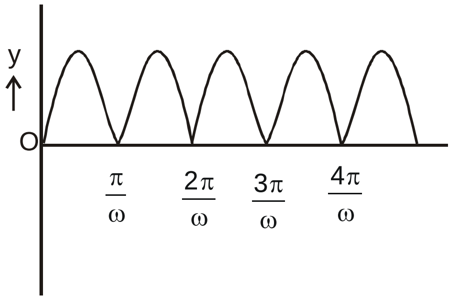
$\Delta\phi = \phi_1 - \phi_2$

$$\left(100\pi t + \frac{5\pi}{6}\right) - (\pi t - \pi)$$

$$= \frac{5\pi}{6} - \pi = -\frac{\pi}{6} \qquad \text{(at t=0)}$$

15. B

Here, $y = \sin^2\omega t$



$$\frac{dy}{dt} = 2\omega\sin\omega t\cos\omega t = \omega\sin 2\omega t$$

$$\frac{d^2y}{dt^2} = 2\omega^2\cos 2\omega t$$

For SHM, $\frac{d^2y}{dt^2} \propto -y$

Hence, function is not SHM, but periodic, From the y-t graph, time period is

$$T = \frac{\pi}{\omega}$$

16. A

KE of a body undergoing SHM is given by

$$KE = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

and $KE_{max} = \frac{m\omega^2A^2}{2}$

[symbols represent standard quantities]

From given information

$$KE = (KE_{max}) \times \frac{75}{100}$$

$$\Rightarrow \frac{m\omega^2A^2}{2}\cos^2\omega t = \frac{m\omega^2A^2}{2} \times \frac{3}{4}$$

or $\qquad \cos\omega t = \pm\frac{\sqrt{3}}{2}$

or $\qquad \omega t = \frac{\pi}{6}$

or $\qquad \frac{2\pi}{T} \times t = \frac{\pi}{6}$

or $\qquad t = \frac{T}{12} = \frac{1}{6}s$

17. A

The maximum velocity of a particle performing SHM is given by $v = A\omega$, where A is the amplitude and ω is the angular frequency of oscillation.

$$\therefore \quad 4.4 = (7 \times 10^{-3}) \times 2\pi / T$$

or $\qquad T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01s$

18. A

$x = (2 \times 10^{-2})\cos\pi t$

Here, $a = 2 \times 10^{-2}m = 2cm$

at t = 0, x = 2 cm, ie, the object is at positive extreme, so to acquire maximum speed (ie, to reach mean postion) it takes $\frac{1}{4}$th of time period.

$$\therefore \quad \text{Required time} = \frac{T}{4}$$

where $\omega = \frac{2\pi}{T} = \pi$

$$\Rightarrow T = 2s$$

So, required time $= \frac{T}{4} = \frac{2}{4} = 0.5s$

19. A

Average kinetic energy of particle

$$= \frac{1}{4}ma^2\omega^2$$

$$= \frac{1}{4}ma^2(2\pi v)^2$$

$$= \pi^2v^2ma^2$$

20. D

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

and $f' = \frac{1}{2\pi}.2\sqrt{\frac{k_1 + k_2}{m}} = 2f$

21. D

$$x = x_o\cos\left(\omega t - \frac{\pi}{4}\right)$$

Acceleration, $a = \frac{d^2x}{dt^2}$

$= -\omega^2 x_o \cos\left(\omega t - \frac{\pi}{4}\right)$

$= \omega^2 x_o \cos\left(\omega t - \frac{3\pi}{4}\right)$

So, A= $\omega^2 x_0$

and $\delta = \frac{3\pi}{4}$

22. B

$\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T^2} \times T = \frac{4\pi^2}{T} = \text{constant}$

23. C

At mean position $F_{net} = 0$

$\therefore$ By conservation of linear momentum

$Mv_1 = (M+m)v_2$.

$M\omega_1 A_1 = (M+m)\omega_2 A_2$

But $\omega_1 = \sqrt{\frac{k}{M}}$,

and $\omega_2 = \sqrt{\frac{K}{m+M}}$

On solving $\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$.

24. A

Let $x_1 = A\sin(\omega t+\phi_1)$ and $x_2 = A\sin(wt+\phi_2)$ $x_2 - x_1$ $= A[\sin(\omega t+\phi_2)-\sin(\omega t-\phi_1)]$

$= 2A\cos\left(\frac{2\omega t + \phi_1 + \phi_2}{2}\right)\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

The resultant motion can be treated as a simple harmonic motion with amplitude

$2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

Given, maximum distance between the particles $= X_0 + A$

$\therefore$ Amplitude of resultant SHM

$= X_0 + A - X_0 = A$

$\therefore \quad 2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A$

$\Rightarrow \quad \phi_2 - \phi_1 = \pi / 3$

25. A

For spring $k \propto \frac{1}{l}$

$\therefore \quad \frac{k_A}{k} = \frac{l}{l_A}$

$k_A = \frac{l_A + l_B}{l_A} k_A = \frac{5}{2}k$

26. A

As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about statement 1. But as in statement 2, $k_1 < k_2$

Then, for same force

$W = F.x = F.\frac{F}{k} = \frac{F^2}{k}$

$\Rightarrow \quad W \propto \frac{1}{k}$ i.e., $W_1 > W_2$

But for same displacement

$W = F.x = \frac{1}{2}kx.x = \frac{1}{2}kx^2$

$\Rightarrow \quad W \propto k$, i.e., $W_1 < W_2$

Thus, in the light of statement 2, statement 1 is false.

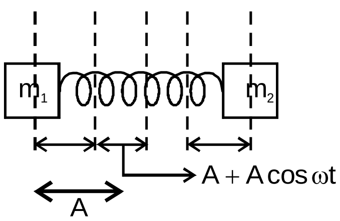
Exercise-4 Level-II

1. Potential Energy = $\frac{1}{2}ky^2$

$= \frac{1}{2}ka^2\cos^2\omega t$

$\Rightarrow$ III and I

2.



w.r.t. = C.O.M. it perform S.H.M.

$x_1(t) = v_0 t - A + A\cos\omega t$

$m_1 . A = m_2 . A_2$

$A_2 = \frac{m_1}{m_2}A$

$A + A_2 = \ell_0$

$A_2 = (\ell_0 - A)$

$= V_0 t + \frac{m_1}{m_2}A(1 - \cos\omega t)$

3. At E.P.

$a = Aw^2 = \frac{Ak}{2m}$

Force = f = $\frac{Ak}{2m}$. m = $\frac{Ak}{2}$

4. $y = kt^2 \quad \Rightarrow \frac{d^2y}{dt^2} = 2m/s^2$

$\therefore T_1 = 2\pi\sqrt{\frac{\ell}{g}}$; $T_2 = 2\pi\sqrt{\frac{\ell}{(g+2)}}$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{g+2}{g}\right) = \frac{12}{10} = \frac{6}{5}$

5. $V^2 = w^2 (a^2 - y^2)$

$V^2 = \frac{k}{m}(a^2 - y^2)$, Now $H = \frac{V^2}{2g} + y$

$H = \frac{k}{m}\frac{(a^2 - y^2)}{2g} + y$

$\frac{dx}{dy} = -\frac{-2yk}{2gm} + 1 = 0$, $y = \frac{m}{2k} \times 2g = \frac{mg}{k}$

6. B

7. $dg = \frac{4\pi^2}{T^2} d\ell - \frac{4\pi^2\ell \times 2.dT}{T^2}$

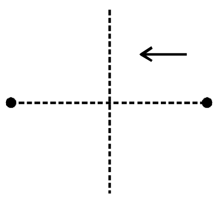
$\frac{dg}{g} = \frac{d\ell}{\ell} - 2\frac{dT}{T}$

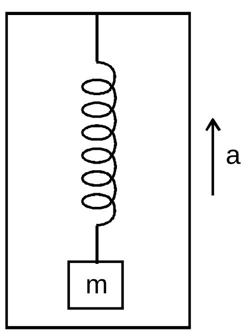
$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{nT}$

8. $V = C_1\sqrt{C_2 - x^2}$

$V^2 = C_1^2 (C_2 - x^2)$

$a = V\frac{dv}{dx} = -C_1^2 x$ (S.H.M.)





V = – kx

It will move towards s

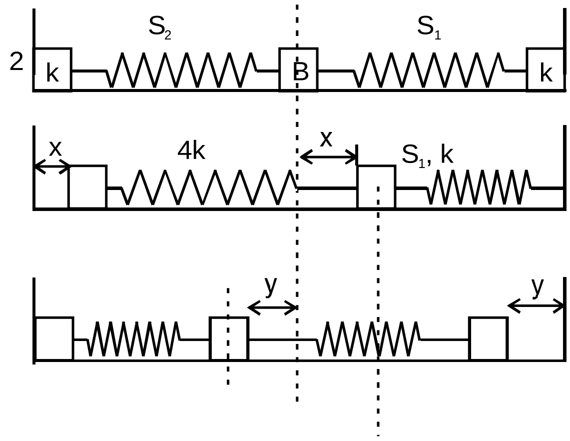
Origin and will stop at origin

$v = \sqrt{4gRe}$

$v > \sqrt{2gRe}$

9. $\frac{1}{2}kx^2 = \frac{1}{2} \times 4(y)^2$

$\frac{y}{x} = \frac{1}{2}$



$V_{avg} = \frac{4\sqrt{2}}{T}A$

Maximum displacement will be close to

M.P. = $\sqrt{2}$ A

$v_{avg} = \frac{4\sqrt{2}}{T}A$

10. D

$y = A \sin \omega t$

$\omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$y = \sin\frac{\pi}{4}t$

at $t = \frac{4}{3}$ sec $\Rightarrow \quad y < \frac{\sqrt{3}}{2}$

$a = -\omega^2 y = -\frac{\pi^2}{16} \times \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\pi^2}{32}$

11. $\frac{4d^2y}{dt^2} + 9y = 0$

$\omega^2 = \frac{9}{4} \Rightarrow \qquad \omega = \frac{3}{2}$

12. $\tau = I\alpha$

$k \times \left(\frac{L}{2}\right) + kx\left(\frac{L}{2}\right) = \frac{ML^2}{12}\alpha$

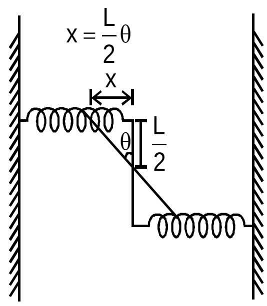
$kxL = \frac{ML^2}{12}\omega^2\theta$

$x = \frac{L}{2\theta}$

$\frac{12 \times L\theta}{ML \times 2} = \omega^2\theta$

$\omega = \sqrt{\frac{6k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$



13. A,D

$\frac{\omega_A}{\omega_B} = \sqrt{\frac{I_B}{I_A}} \Rightarrow \omega_A < \omega_B$

14. D

$V^2 = u^2 - 2gs$

$p = mv = \pm m\sqrt{u^2 - 2gs}$

so its parabola

Initially position = 0

Momentum is +ve

15. C

$\omega_1 = \omega_2$

$E_1 = \frac{1}{2} m\omega^2 (2a)^2 = 4\left(\frac{1}{2} m\omega^2 a^2\right)$ ...... (1)

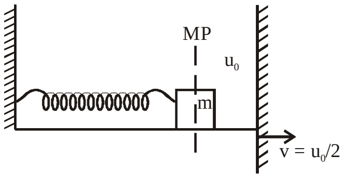
$E_2 = \frac{1}{2} m\omega^2 a^2$ ...... (2)

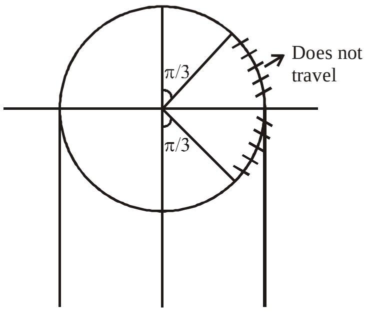
from equation (1) and (2)

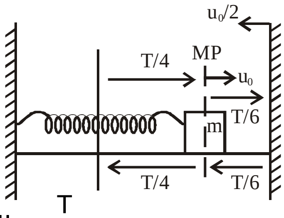
$E_1 = 4E_2$

16. B

17. AD







$x = A \sin \omega t$

$v = A\,\omega \cos \omega t$

$\frac{A\omega}{2} = A\,\omega \cos \omega t$

$\omega t = \frac{\pi}{3}$

Time to reach the wall = $\frac{T}{6}$

Time to return to mean position = $\frac{T}{6} + \frac{T}{6}$

$= \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

Time for maximum compression = $\frac{T}{3} + \frac{T}{4}$

$= \frac{7T}{12} = 14\pi\sqrt{\frac{m}{k}}$

II time = $\frac{T}{3} + \frac{T}{2} = \frac{5T}{6}$

Equilibrium position = $\frac{5T}{6} = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$