

MAIN+ADVANCED

SOLUTIONS

TOPIC

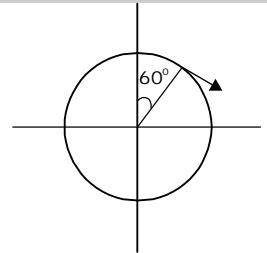
OSCILLATIONS

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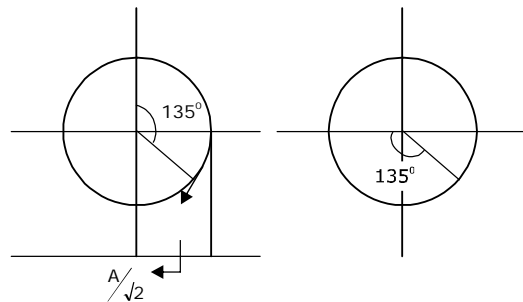
Exercise-1

1. **A**
 $F = ma \Rightarrow F \propto -x$ (for SHM) $\Rightarrow a \propto -x$
2. **C**
 The distance moved in $T/4$ time is A .
 Hence in one time period distance travelled is 4 (distance in $T/4$ time) = $4A$
3. **B**
 A particle appears only once at one of the extreme position in entire oscillation.
4. **C**
 In the particle moves from extreme to half of amplitude then let the time taken is t
 $y = A \sin(\omega t + \phi)$
 $y = A \sin\left(\omega t + \frac{\pi}{2}\right)$ (As particle starts from extreme)
 $\frac{A}{2} = A \sin\left(\frac{2\pi}{4}t + \frac{\pi}{2}\right)$
 $\sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) = \sin\frac{5\pi}{6}$
 $\frac{\pi}{2}t + \frac{\pi}{2} = \frac{5\pi}{6}$
 $\frac{\pi}{2}t + \frac{5\pi}{6} = \frac{\pi}{2}$
 $\frac{\pi}{2}t + \frac{\pi}{3}$
 $t = \frac{2}{3}$ sec
5. **C**
 $y = A(1 + \cos 2\omega t)$ $y = 2A \sin\left(\omega t + \frac{\pi}{3}\right)$
 $y = A + A \cos 2\omega t$ $V_{\max} = 2A\omega$
 $V_{\max} = A \times 2\omega$ Ratio = 1:1
6. **A**
 Velocity is maximum at mean. To come back to mean the particle has to move $\left(\pi - \frac{\pi}{3}\right) = \frac{2\pi}{3}$.

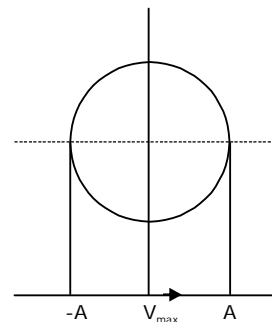
Hence $t = \frac{2\pi}{3.2\pi} = \frac{1}{3}$ sec.



7. **A**
 $y = 5 \sin \pi(t + 4) \Rightarrow y = 5 \sin(\pi t + 4\pi)$ --- (1)
 The standard equation is $y = A \sin(\omega t + \phi)$ --- (2)
 comparing equation (1) & (2)
 $A = 5m, \quad \omega = \pi \quad T = \frac{2\pi}{\omega} = 2$ sec.
8. **C**
 Let particle A be the particle shown with initial phase 135° and B be the particle at extreme. Hence the phase difference between them is 135° .



9. **A**
 A particle has same velocity between 0 & V_{\max} and 0 and $-V_{\max}$ twice in its motion. Only V_{\max} is a velocity which a particle attains once in its one oscillation.



10. D

Total Accⁿ = 0

$$a_{av.} = \frac{\text{Total change in Velocity in one T.P.}}{\text{Time Period}}$$

$$a_{av.} = \frac{0}{T} = 0$$

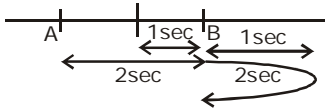
11. B

$$v^2 = \omega^2 (A^2 - x^2) = -\omega^2 x^2 + \omega^2 A^2$$

$$a^2 = \omega^2 x^2$$

⇒ v² = -a² + ω²a² straight line with a -ve slope.

12. B



Hence $\frac{T}{4} = 2 \text{ sec.} \quad T = 8 \text{ sec.}$

13. A

We know that

$$x = A \sin \omega t$$

$$\frac{a}{2} = a \sin \omega t$$

$$\omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega}$$

Now v = aω cos ωt

$$a\omega \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} a \frac{2\pi}{T} = \frac{a\pi\sqrt{3}}{T}$$

14. C

$$y = 0.45 \sin 2t$$

$$\frac{7.5}{100} = 0.45 \sin 2t$$

$$\sin 2t = 0.167$$

$$\text{speed} = a\omega \cos 2t$$

$$= 0.45 \times 2 \sqrt{1 - \sin^2 2t}$$

$$= 0.9 \times 0.98 = 0.87 = \frac{0.5}{\sqrt{3}} \text{ m/s}$$

15. B

$$a_{\max} = \omega^2 A$$

$$(1.57)^2 = \omega^2 (1)$$

$$\omega = 1.57 \text{ rad/sec}^2$$

$$\frac{2\pi}{T} = 1.57$$

$$T = 4 \text{ sec.}$$

16. A

$$x = A \sin \omega t \quad \text{if } t = 1 \text{ is } t = 0$$

$$v = A\omega \cos \omega t \quad \Rightarrow 0.25 = A \times \frac{2\pi}{6} \cos \left(\frac{\pi}{3}(t-1) \right)$$

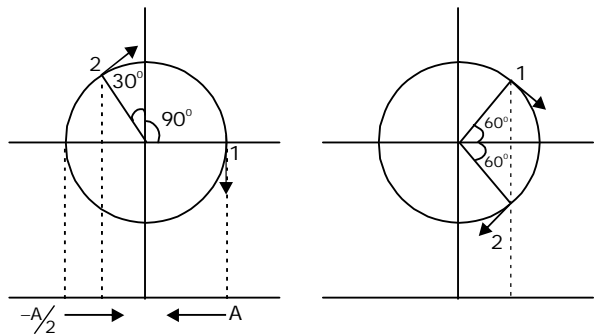
At t = 2 sec.

$$\Rightarrow 0.25 = A \times \frac{2\pi}{6} \times \frac{1}{2} \quad \Rightarrow A = \frac{3}{2\pi}$$

17. C

$$\frac{4d^2y}{dt^2} + 9y = 0 \Rightarrow \omega^2 = \frac{9}{4} \Rightarrow \omega = \frac{3}{2}$$

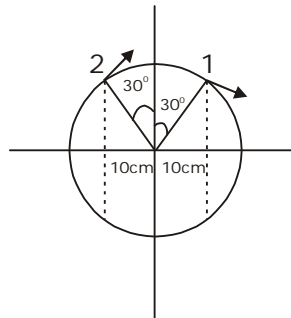
18. D



$$t = \frac{2\pi}{3} \times \frac{T}{2\pi}$$

$$t = \frac{T}{6}$$

19. C



Particle 1 and 2 are as shown and their phase difference is 60°.

20. B

Slope of F-x curve gives K

$$\text{slope} = \frac{13.5}{1.5} \quad F = -Kx \Rightarrow K = 9$$

$$\omega^2 = \frac{K}{m} = 9 \quad \omega = 3, \quad T = \frac{2\pi}{3}$$

21. B

$$\frac{1}{2} KA^2 = 8 \times 10^{-3} \text{ J} \Rightarrow A = 0.1 \text{ m, } M = 0.1 \text{ Kg}$$

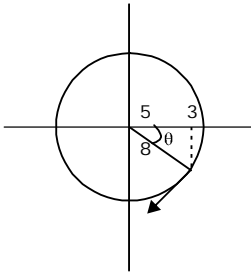
$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1.6}{0.1}} = 4$$

$$\phi = 45^\circ = \frac{\pi}{4} \Rightarrow x = 0.1 \sin(4t + \frac{\pi}{4})$$

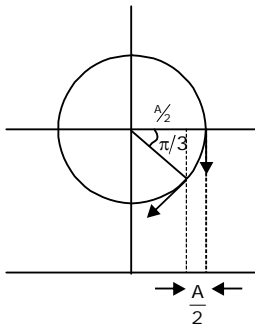
22. C

$$\cos \theta = \frac{5}{8} \Rightarrow \theta = \cos^{-1}(5/8) = \omega t = 51^\circ$$

$$t = 0.17 \text{ sec.}$$



23. C



$$\theta = \frac{\pi}{3} = \omega t$$

$$t = \frac{T}{6}$$

$$\therefore V_{avg} = \frac{A/2}{T/6} = \frac{3A}{T}$$

24. C

$T = 2\pi\sqrt{\frac{M}{K}}$ When the rubber ribbon slacks it will not exert any force.

25. C

K will remain unchanged

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \text{constant}$$

26. D

Initially the COM of sphere and water lies at centre of sphere. As water flows out the COM shifts down and length of pendulum increases hence time period increases but when water level becomes half of the sphere the COM again starts shifting up and hence as length decreases time period also decreases.

27. A

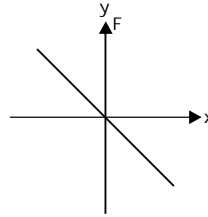
Total energy is constant during S.H.M.

$$\Rightarrow E = \frac{1}{2} KA^2$$

28. D

From mean to extreme the displacement decreases with time whereas from extreme to mean displacement increases with time.

29. D



The restoring force always acts towards the mean & increases with increase in displacement.

30. D

Total M.E. = T.E. at M.P. + Total oscillation energy

$$9 = 5 + 4$$

$$\text{Total oscillation energy} = \frac{1}{2} Ka^2 = 4$$

$$\Rightarrow K = 8 \times 10^4 \Rightarrow T = 2\pi\sqrt{\frac{M}{K}} \Rightarrow T = \frac{\pi}{100}$$

31. B

$$\text{In series } T_1 = 2\pi\sqrt{\frac{2m}{K}}$$

$$\text{In parallel } T_2 = 2\pi\sqrt{\frac{m}{2K}}$$

$$\frac{T_{series}}{T_{parallel}} = 2$$

32. B

$$V_{max} = \omega A \quad \omega_p A_p = \omega_o A_o$$

$$\frac{A_p}{A_o} = \frac{\omega_o}{\omega_p} = \sqrt{\frac{K_2}{K_1}}$$

33. C

$$E_i = \frac{1}{2} KA^2$$

$$E_i = \frac{1}{2} m\omega^2 A^2$$

$$E_f = \frac{1}{2} (2m)\omega'^2 A^2$$

The value of k remains same so $\omega' = \frac{\omega}{\sqrt{2}}$

and hence $E_f = \frac{1}{2} m \omega^2 A^2$

So the energy remains unchanged.

34. A

$$V_{\max} = A\omega = V$$

ω remains unchanged in this case.

Hence $V_{\max_2} = 2A\omega$

$$V_{\max} = 2V$$

35. A

$$x = 2 \sin \omega t ; \quad y = 2 \sin \left(\omega t + \frac{\pi}{4} \right)$$

from Lissajous figures if $\phi = \frac{\pi}{4}$ then the path of particle is an ellipse.

36. C

$$a_1 = 1, \quad a_2 = 1 \quad \theta = \frac{\pi}{3}$$

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \theta} \Rightarrow a = \sqrt{3}$$

37. B

$$E = \frac{1}{2} K A_{eq}^2 \quad A_{eq} = \sqrt{A^2 + A^2}$$

$$E = \frac{1}{2} m \omega^2 (\sqrt{2} A)^2 = \sqrt{2} A$$

$$E = m \omega^2 A^2$$

38. A

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$T_1 = 2\pi \sqrt{\frac{m}{K_1}} \quad T_2 = 2\pi \sqrt{\frac{m}{K_2}}$$

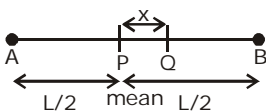
As $kl = \text{constant}$

Hence $k_1 l_1 = k_2 l_2$

$$k_1 L = k_2 2L$$

$$\frac{k_1}{k_2} = 2; \quad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

41. B



Let the particle be displaced a distance x to Q then a quarter of way from A to B means particle is at $L/2$ from A so time taken is $T/6$.

$$F_{net} = 2 \left(\frac{L}{2} - x \right) - \left(\frac{L}{2} + x \right) \Rightarrow F_{net} = -4x$$

$$T = 2\pi, \quad \omega = 1, \quad \frac{T}{6} \Rightarrow \therefore \frac{\pi}{3}$$

42. C

$$T_p = 2\pi \sqrt{\frac{m}{K}} \quad T_s = 2\pi \sqrt{\frac{\ell}{g+a}}$$

T_p = Remain same T_s decreases

43. A

$$n_1 T_1 = n_2 T_2 \quad n_1 T = \frac{5T}{4} \times n_2$$

$$\frac{n_1}{n_2} = \frac{5}{4} \Rightarrow n_1 = 5, \quad n_2 = 4$$

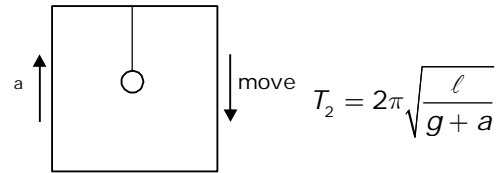
44. A

When the lift is going down with constant velocity the acceleration is zero.

When there is a retardation of 'a' the $g_{\text{effective}}$ is $g + a$.

$V = \text{constant} \Rightarrow a = 0$ So there is no effect.

$$T_1 = T = 2\pi \sqrt{\frac{\ell}{g}} \quad g_{\text{eff}} = (g + a)$$



45. C

$$T_{\max} = mg + \frac{mv^2}{\ell}$$

$$v = A\omega$$

$$\omega = \sqrt{g/\ell}$$

$$v^2 = \frac{g}{\ell} A^2$$

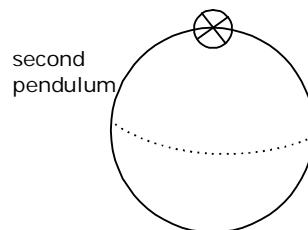
$$T_{\max} = mg [1 + (A/\ell)^2]$$

46. A

$$T = 2 \text{ sec.}$$

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

Time period of a second's pendulum is two seconds.



$$2 = 2\pi \sqrt{\frac{2mR^2}{mgR}} \quad R = .5m$$

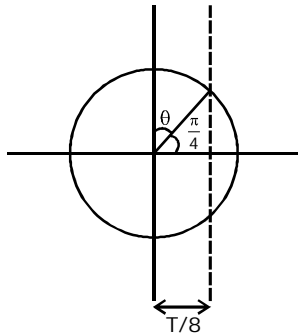
Exercise-2

1. **C**
 $2A_1 \omega_1^2 = A_2 \omega_2^2$
 $A_1 \omega_1 = A_2 \omega_2$

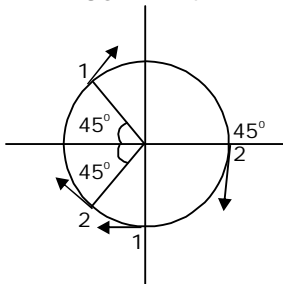
If $\omega_2 = 2\omega_1$ and $A_2 = \frac{A_1}{2}$
 The condition will be satisfied.

2. **D**
 $\theta = \omega t$
 $\theta = \frac{2\pi}{T} \cdot \frac{T}{8}$

$\theta = \frac{\pi}{4}$
 $\cos \frac{\pi}{4} = \frac{x}{A}$
 $x = A / \sqrt{2}$

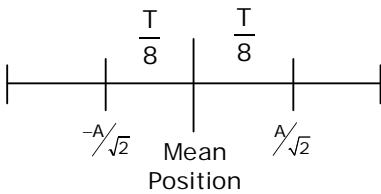


3. **B**
 $\frac{135 \times \pi}{180} = \frac{3\pi}{4}$



$\therefore t = \frac{3\pi}{4 \times 2\pi} T = \frac{3T}{8}$

4. **D**
 Max. Average velocity = $\frac{\text{Total max displacement}}{\text{Total time}}$



Max^m displacement will be close to $M.P. = \sqrt{2}A$

$V_{avg} = \frac{4\sqrt{2}}{T} A$

5. **D**
 Time period = 8 sec.
 In 1st second = $\frac{2\pi}{8} \times 1$
 Displacement = $\frac{A}{\sqrt{2}}$

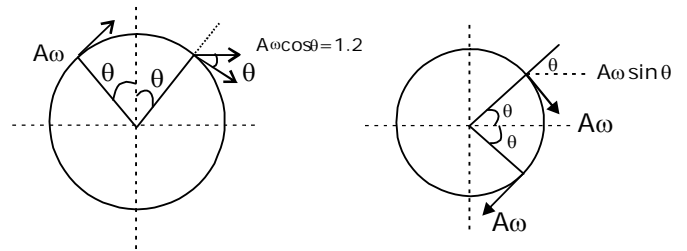
In 1st second = $\frac{2\pi}{8} \times 1$

Displacement = $A = \frac{\pi}{2}$

t = 1

$\frac{A\sqrt{2}}{A\sqrt{2}(\sqrt{2}-1)} = (\sqrt{2}+1)$

6. **D**

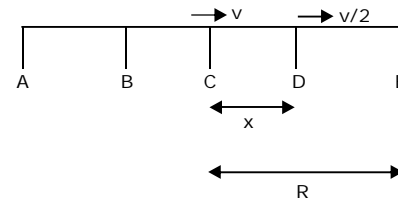


$A\omega \cos \theta = 1.2 \dots (1)$ $A\omega \sin \theta = 1.6 \dots (2)$

$\tan \theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$

$\therefore A\omega \cdot \frac{3}{5} = 1.2 \Rightarrow A\omega = 2 \text{ m/sec.}$

7. **C**



$v/2 = \omega\sqrt{R^2 - x^2}$ $R\omega/2 = \omega\sqrt{R^2 - x^2}$

$R^2/4 = R^2 - x^2$ $x = \frac{\sqrt{3}}{2} R$

Distance = $2x = \sqrt{3}R$

8. **B**

$T_1 = 2\pi\sqrt{\frac{m}{K_1}}$, $T_2 = 2\pi\sqrt{\frac{m}{K_2}}$ $T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$K_{eq} \frac{K_1 K_2}{K_1 + K_2}$ (for series) $t_1^2 = \frac{2\pi m}{K_1}$

$t_2^2 = \frac{2\pi m}{K_2}$ and $T^2 = 2\pi m \left(\frac{1}{K_1} + \frac{1}{K_2} \right)$

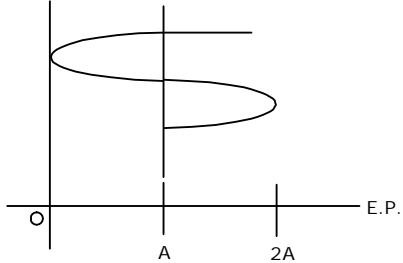
$\Rightarrow T^2 = t_1^2 + t_2^2$

9. **C**

$x = A + A \sin \omega t$

$$t = \frac{2.5\pi}{\omega}$$

$$t = \frac{2\pi}{\omega} + \frac{0.5\pi}{\omega} = 5A$$



10. C

$$A = 2\text{cm} \Rightarrow x = 1\text{cm} = \frac{A}{2}$$

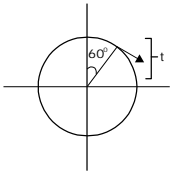
$$\Rightarrow v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{3} \Rightarrow a = \omega^2 \cdot 1$$

$$\omega = \sqrt{3} = 2\pi n \Rightarrow n = \frac{\sqrt{3}}{2\pi}$$

11. A

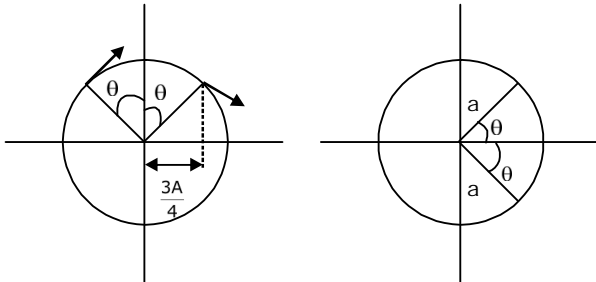
$$y = \sin \omega t + \sqrt{3} \cos \omega t$$

$$y = 2 \sin\left(\omega t + \frac{\pi}{3}\right) \quad A\omega^2 = g$$



$$\Rightarrow \omega = \sqrt{\frac{g}{2}} \Rightarrow t = \frac{\pi}{6} \sqrt{\frac{2}{g}}$$

12. B



$$\Rightarrow \cos \theta = \frac{\sqrt{7}}{4}$$

$$\Rightarrow a \cos \theta = \frac{a\sqrt{7}}{4}$$

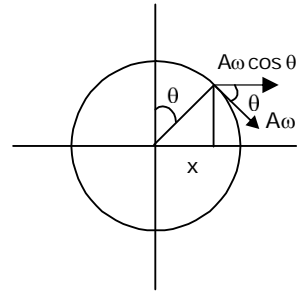
13. B

$$\theta = \omega \frac{T}{12} = \frac{\pi}{6}, \quad x = A \sin \frac{\pi}{6} = \frac{A}{2}$$

$$P.E. = \frac{1}{2} Kx^2 = \frac{1}{2} \frac{KA^2}{4}$$

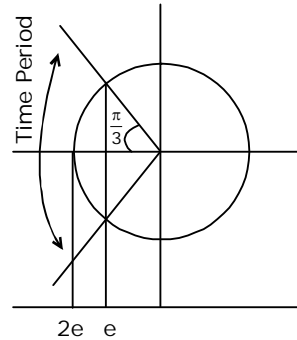
$$v = A\omega \cos \theta = \frac{A\omega\sqrt{3}}{2}$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} K \left(\frac{3}{4} A^2\right) \Rightarrow \frac{KE}{PE} = 3$$



14. A

$$\omega t = \frac{2\pi}{3} \Rightarrow \frac{2\pi}{T} t = \frac{2\pi}{3} \Rightarrow t = \frac{T}{3} \Rightarrow t = \frac{2\pi}{3} \sqrt{\frac{m}{K}}$$



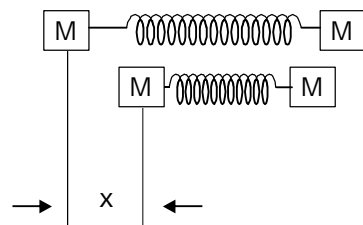
15. C

$$\mu = \frac{m_2}{2m} = \frac{m}{2}$$

It can be considered as a two block system for

$\frac{T}{2}$ time

$$t = \frac{T}{2} = \pi \sqrt{\frac{m}{2K}} \Rightarrow t = \pi \sqrt{\frac{2}{2 \cdot \pi^2}} = 1\text{s}$$



16. A

Elastic Collision

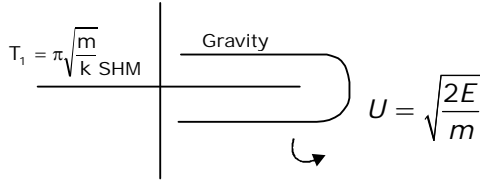
17. C

$$K_{Aeq} = K/3, \quad K_B = 3K \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{K_B}{K_A}} = \frac{3}{1}$$

18. C

for $x < 0$ perform SHM

$$\frac{1}{2} mU^2 = E$$



$$\Rightarrow T_2 = \frac{2}{g} \sqrt{\frac{2E}{m}}$$

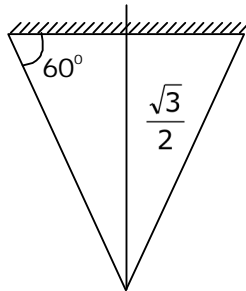
$$\therefore T = T_1 + T_2 \Rightarrow T = \pi \sqrt{\frac{m}{K}} + \frac{2}{g} \sqrt{\frac{2E}{m}}$$

19. B

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

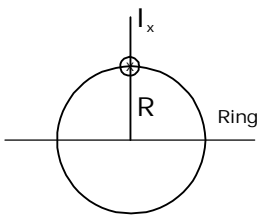
$$= 2\pi \sqrt{\frac{m \left(\frac{\sqrt{3}}{2} \ell \right)^2}{mg \frac{\sqrt{3}}{2} \ell}}$$

$$= 2\pi \sqrt{\frac{\sqrt{3}L}{2g}}$$



20. C

$$I_x = \frac{mR^2}{2} + mR^2$$



$$= \frac{3}{2} mR^2$$

$$T = 2\pi \sqrt{\frac{3}{2} \frac{mR^2}{mgR}} = 2\pi \sqrt{\frac{3R}{2g}} \Rightarrow \ell = \frac{3}{2}$$

21. $T_1 = 2\pi \sqrt{\ell/g} = T \Rightarrow T_2 = 2\pi \sqrt{\ell/4g} = \frac{T}{2}$

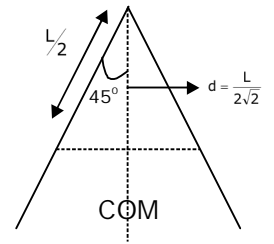
$$T_{eq} = \frac{T_1}{2} + \frac{T_2}{2} \Rightarrow = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

22. B

$$T = 2\pi \sqrt{\frac{I}{mgd}} \quad I = \frac{2md^2}{3}$$

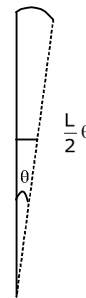
$$T = 2\pi \sqrt{\frac{2md^2}{3} \frac{2\sqrt{2}}{(2m)gd}}$$

$$T = 2\pi \sqrt{\frac{2\sqrt{2}d}{3g}}$$



23. C

$$KL^2\theta + \frac{KL^2\theta}{4} > mg \cdot \frac{L}{2} \cdot \theta$$



$$K > 2mg/5L$$

24. A

$$\tau_{net} = KL^2\theta + \frac{KL^2\theta}{4} - mg \cdot \frac{L\theta}{2} \quad \therefore K = mg/L$$

$$I = mL^2/3 \Rightarrow \tau_{net} = \frac{3}{4} KL^2\theta \Rightarrow K_{eq} = \frac{3}{4} KL^2$$

$$\omega = \sqrt{\frac{K_{eq}}{I}} = \frac{3}{2} \left(\frac{K}{m} \right)^{1/2}$$

25. D

$$T_A = 2\pi \sqrt{\frac{I_A}{mgx}} \quad \text{----- (1)}$$

$$T_B = 2\pi \sqrt{\frac{I_B}{mg(0.25-x)}} \quad \text{----- (2)}$$

Eq. (1) & (2)

$$\frac{3}{4} = \sqrt{\frac{I_A(0.25-x)}{I_B x}} \Rightarrow \frac{3}{4} = \sqrt{\frac{9(0.25-x)}{x}}$$

$$\Rightarrow \frac{9}{16} = \frac{9}{4} \left(\frac{0.25-x}{x} \right) \Rightarrow x = 0.20m$$

26. B,C,D

$$mg = Kx_0 \quad x_0 = \frac{10}{500} = 2cm$$

$$\omega = \sqrt{\frac{500}{1}} = 10\sqrt{5} \frac{rad}{sec}$$

∴ Maximum velocity = $A\omega = 3 \times 10^{-2} \times 10\sqrt{5}$
 = $30\sqrt{5}$ cm/sec. \Rightarrow Max^m Accⁿ = $A\omega^2 = 15$ m/s²

27. A

$2 \times \frac{1}{2} KA^2 = \frac{1}{2} KA'^2 \Rightarrow A' = \sqrt{2}A \Rightarrow T = 2\pi\sqrt{\frac{m}{K}}$

28. B $y = 2 \sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

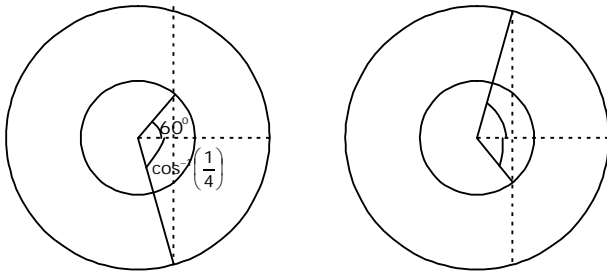
29. B, C

Phase by $\omega = \frac{\omega \times 2\pi}{3\omega} = \frac{2\pi}{3}$

Phase by $2\omega = \phi + 2\omega \times \frac{2\pi}{3\omega}$

$\phi + \frac{4\pi}{3} - \frac{2\pi}{3} = 0, 2\pi, 4\pi \quad \phi = \frac{2\pi}{3}, \frac{4\pi}{3}, \dots$

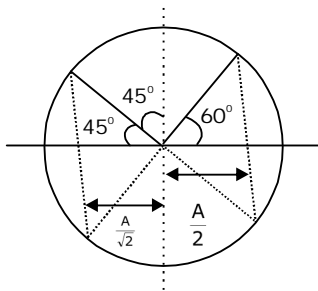
30. A



31. A, B, C

$x = 5 \sin\left(4\pi t + \tan^{-1} \frac{4}{3}\right)$

32. C



33. B

$v^2 = 108 - 9x^2 \Rightarrow v^2 = 9[12 - x^2]$
 $\Rightarrow 2v \frac{dv}{dx} = -2x \times 9 \Rightarrow a = -9x \Rightarrow \omega^2 = 9$

34. A, B, C, D

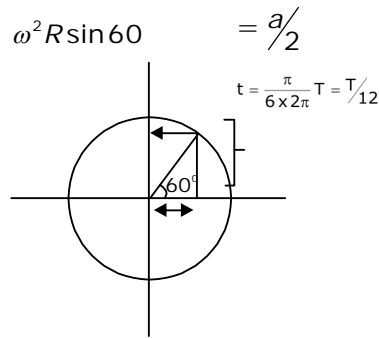
$A\omega^2 = g \Rightarrow 0.40 \omega^2 = 10$
 b) Negative Extreme

35. B, C, D

$u = 5x^2 - 20x \Rightarrow F = \frac{-du}{dx} = -10(x - 2)$

M.P. at $x = 2$ m

36. A, B



37. A, B, C

$\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 A^2 (0.64)$

$\frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 A^2 (0.64)$

Put $A = 10$ cm. $\Rightarrow x = 6$ cm.

38. B, D

$x = 3 \sin 100t + 4(1 + \cos 100t)$

$x = 5 \sin\left(100t + \tan^{-1} \frac{4}{3}\right) + 4$

M.P. is at 4 with $A = 5$

39. A

$F = -Kx$ Slope = $-K$

40. C, D

$x = a \sin \omega t \dots (1)$

$y - a = -a \cos \omega t \dots (2)$

$(1)^2 + (2)^2 \Rightarrow (y - a)^2 + x^2 = a^2$

41. B, C, D

$v^2 = \omega^2 (A^2 - x^2) \quad F = -Kx$

$\frac{v^2}{\omega^2} + x^2 = A^2 \dots \text{Ellipse}$

$v^2 = (A^2 - \omega^2 x^2), a = -\omega^2 x, a^2 = \omega^4 x^2$

$v^2 = \left(A^2 - \frac{a^2}{\omega^2}\right) \Rightarrow v^2 + \frac{a^2}{\omega^2} = A^2 \dots \text{Ellipse}$

42. A, B, C

$v = \omega A \quad \& \quad \omega = \frac{10}{2.5} = 4$

(a) $T = \frac{2\pi}{\omega} = \frac{\pi}{2} = 1.57$

(b) $a = \omega^2 A = 40$

(c) $v = \omega \sqrt{A^2 - x^2} = 2\sqrt{21}$

43. A, C

$v = \sqrt{\frac{900}{3}} \sqrt{(2^2 - 1^2)} = 10\sqrt{3} \times \sqrt{3} = 30$ m/s

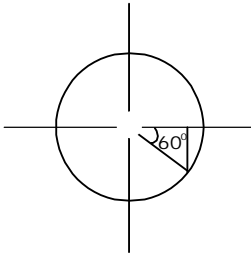
M.C. $3 \times 30 = 9 \times v \Rightarrow v = 10 \text{ m/s}$

$10 = \sqrt{\frac{900}{9}} \sqrt{(A^2 - 1^2)} \Rightarrow A = \sqrt{2} \text{ m}$

44. B,D

$\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6} \Rightarrow x = A \sin\left(\frac{2\pi t}{T} + \frac{5\pi}{6}\right)$

$x = A \cos\left(\frac{2\pi}{T} \cdot t + \frac{\pi}{3}\right)$

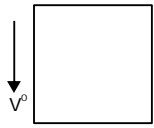


45. B

$T = 2\pi \sqrt{\frac{m}{K}}$ ----> not dependent on q_{eff}

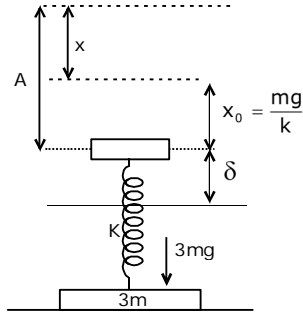
the velocity of particle

at M.P. = $V_0 \therefore V_0 = A\omega_0$



$A = \frac{V_0}{\omega_0}$

Initial phase is zero.

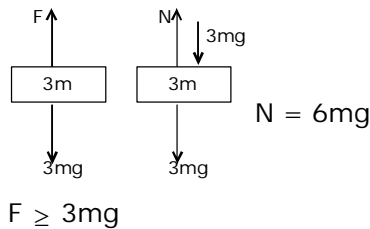


46.

$kx \geq 3mg \quad \delta = \frac{2mg}{k}$

$x \geq \frac{3gm}{k} \quad F = k\delta + kx = 3mg$

$A = \frac{4mg}{k}$



47. A,C

$V_{\text{rms}} = \sqrt{\frac{\int v^2 \cdot dt}{\int dt}}$

48. B,C

KE Average = $\frac{1}{4} KA^2 = m\pi^2 f^2 A^2$

PE Average = $m\pi^2 f^2 A^2$

49. B,C

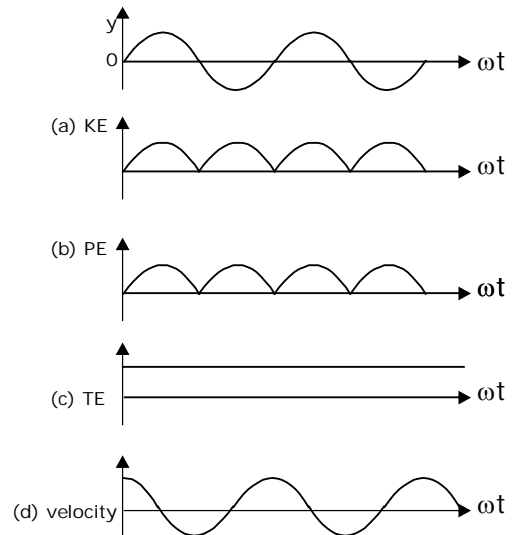
$\frac{1}{2} KA^2 = \frac{1}{2} \times 2 \times 10^6 \times 10^{-4} = 100$

$\therefore 60 \text{ J P.E. at M.P.}$

$\text{Max}^m \text{K.E.} = 100 \text{ joule}$

$\text{Max}^m \text{P.E.} = 160 \text{ joule}$

50. B



Exercise-3

Level-I

1. $x = (5m) \sin\left[\left(\pi s^{-1}\right)t + \frac{\pi}{6}\right]$
 $\therefore x = A \sin(\omega t + \phi)$
 $\Rightarrow A = 5m, \phi = \frac{\pi}{6}, T = \frac{2\pi}{\omega} = 2 \text{ sec. and}$
 $V_{\max} = A\omega = 5\pi m/s$

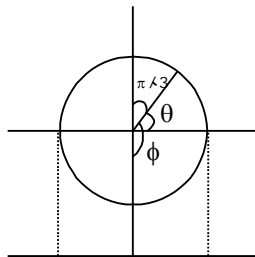
2. $x = 2.0 \sin\left[\left(100s^{-1}\right)t + \frac{\pi}{6}\right]$
 $A=0.2 \text{ cm, } w=100, M=10\text{gm}$
 $v = (2.0 \times 100) \text{ cm/s } \cos\left[\left(100s^{-1}\right)t + \frac{\pi}{6}\right]$
 $a = 2 \times 10^4 \text{ cm/s}^2 \left[-\sin\left[\left(100s^{-1}\right)t + \frac{\pi}{6}\right]\right]$
 $\Rightarrow a = -10^4 x \therefore F = -Kx \text{ and } F=100 \text{ N/m}$
 $\Rightarrow K = 10^4 \times M = 10^4 \times \frac{1}{100} = 100 \text{ N/m}$
 $\Rightarrow T = \frac{2\pi}{w} = \frac{2\pi}{100} = \frac{\pi}{50} \text{ sec.}$

a at t=0 $\Rightarrow a = -200 \text{ m/s}^2 \sin \frac{\pi}{6}$
 $= -100 \text{ m/s}^2$

3. $\omega = 20$
 (a) $\theta = \frac{\pi}{6} = \omega t$

$t = \frac{\pi}{120}$

(b) $\phi = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3} \quad t = \frac{2\pi}{3 \times 20} = \frac{\pi}{30}$



(c) Same as (b)

4. $K_1 l_1 = K_2 l_2 = Kl$
 $\Rightarrow \frac{K_1}{K_2} = \frac{l_2}{l_1} = \frac{3}{1} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}} = \frac{1}{\sqrt{3}}$

5. $\mu = \frac{3 \times 6}{3 + 6} = 2 \text{ Kg. } \omega = \sqrt{\frac{k}{\mu}} = 20$

(a) $T = 2\pi \sqrt{\frac{\mu}{K}} = \frac{\pi}{10}$

(b) $A_1 + A_2 = 6$

$\Rightarrow m_1 A_1 = m_2 A_2 \Rightarrow A_1 = \frac{m_2}{m_1 + m_2} A$

$A_1 = 4 \text{ cm, } A_2 = 2 \text{ cm.}$

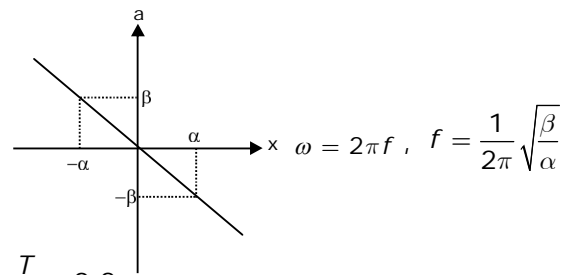
(c) $m_2 V_{\max} = 6. \omega A_2 = 2.4$

6. $T = \frac{2\pi}{5\pi} = 0.4 \text{ sec.}$

Av. Speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{0.2 \times 7}{0.7} = 2 \text{ m/s}$

7. $F = -Kx$

$a = -\frac{K}{m} x \quad \frac{K}{m} = \tan \theta = \frac{\beta}{\alpha}$



8. $\frac{T}{3} = 0.2$

$T = 0.6 = 2\pi \sqrt{\frac{0.9}{K}} \Rightarrow K = 100 \text{ N/m}$

9. $F = -10x + 2 = -10(x - 0.2)$
 $x = 0.2 \text{ is M.P.}$

(a) Amplitude = $2 + 0.2 = 2.2 \text{ m}$

(b) $T = 2\pi \sqrt{\frac{0.1}{10}} = \frac{\pi}{5} \text{ s}$

(c) $x = 0.2 - 2.2 \cos \omega t$

10. $U = (x^2 - 4x + 3)$

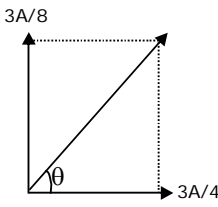
$F = -\frac{du}{dx} = -2x + 4 \Rightarrow F = -2(x-2)$

(a) For equilibrium $F = 0 \Rightarrow x = 2 \text{ m}$

(b) $\omega = \sqrt{\frac{2}{1}} = \sqrt{2} \Rightarrow T = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

(c) $\frac{1}{2} KA^2 = \frac{1}{2} mv^2 \Rightarrow A = 2\sqrt{3}$

11. $\tan \theta = \frac{1}{2}$



$$A_{net} = \sqrt{\left(\frac{3A}{4}\right)^2 + \left(\frac{3A}{8}\right)^2} = \frac{3A}{4} \sqrt{1 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{3\sqrt{5}}{8} A$$

12. $F_{max} = KA = 50$

$$x = \frac{A}{\sqrt{2}} \therefore F = Kx = \frac{KA}{\sqrt{2}} \quad F = \frac{F_{max}}{\sqrt{2}} = 50\sqrt{2}$$

$$F = 25\sqrt{2} \text{ N}$$

13. Given $A=0.1\text{m}$, $K=100 \text{ N/m}$
 $m_1 = 1\text{Kg}$, $m_2 = 3\text{Kg}$
 Energy of system is not conserved.

$$\frac{1}{2} KA^2 = \frac{1}{2} m_1 v_1^2$$

$$\Rightarrow v_1 = \frac{KA^2}{m_1}$$

$$\Rightarrow v_1 = 1 \text{ m/s}$$

$$m_1 v_1 = (m_1 + m_2) v \quad \Rightarrow v = \frac{1}{(m_1 + m_2)}$$

$$\Rightarrow v = 0.25 \text{ m/s}$$

$$\therefore \text{freq}^n = \frac{1}{2\pi} \sqrt{\frac{K}{m}} = \frac{1}{2\pi} \sqrt{\frac{100}{4}} = \frac{5}{2\pi} \text{ Hz}$$

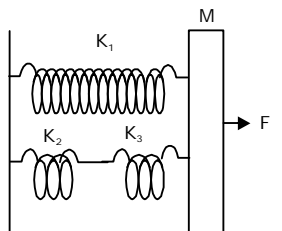
Now amplitude

$$\frac{1}{2} KA'^2 = \frac{1}{2} (m_1 + m_2) v^2 \Rightarrow A'^2 = \frac{(0.25)^2 \times 4}{100}$$

$$\Rightarrow A' = 5\text{cm}$$

14. $K_{eq} = K_1 + \frac{K_2 K_3}{K_2 + K_3}$

$$\therefore F = K_{eq} A$$



$$\Rightarrow A = \frac{F}{K_{eq}} = \frac{(K_2 + K_3) F}{K_1 K_2 + K_2 K_3 + K_3 K_1}$$

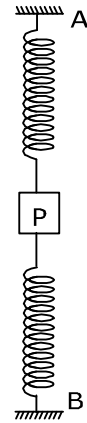
$$f = \frac{1}{2\pi} \sqrt{\frac{K_{eq}}{M}} = \frac{1}{2\pi} \left[\frac{K_1 K_2 + K_2 K_3 + K_3 K_1}{M(K_2 + K_3)} \right]^{1/2}$$

15. $Mg = Kx - Kx - 10$

$$\Rightarrow K = \frac{mg}{10/100} \text{ N/m}$$

$$T = 2\pi \sqrt{\frac{M}{K}}$$

$$T = 2\pi \sqrt{\frac{1}{49 \times 4}} = \frac{\pi}{7} \text{ sec}$$



16. General equation

$$x = A \sin(\omega t + \phi) \quad A=10, \quad \omega = \frac{2\pi}{T} = \pi$$

$$\text{At } t = 0, x = 5 \Rightarrow 5 = 10 \sin \phi$$

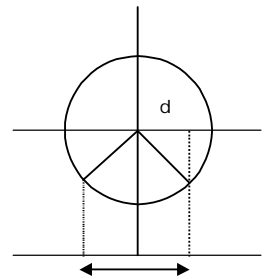
$$\phi = \frac{\pi}{6} \Rightarrow x = 10 \sin\left(\pi t + \frac{\pi}{6}\right)$$

17. $d = a \sin \frac{\phi}{2}$

$$\text{Max distance} = 2d$$

$$2d = 2a \sin \frac{\phi}{2}$$

$$= 2a \times 0.9 = 1.8 a$$



18. By momentum conservation

$$5 \times 3 + 10 \times 2 = 7v$$

$$v = 5 \text{ m/s}$$

By Energy Conservation

$$\frac{1}{2} \cdot 5 \cdot 3^2 + \frac{1}{2} \times 2 \times (10)^2 = \frac{1}{2} \times 7 \times 5^2 + \frac{1}{2} Kx^2$$

$$x = 0.25 \text{ m}, \quad T = 2\pi \sqrt{\frac{\mu}{K}} \Rightarrow \mu = \frac{10}{7}$$

$$\Rightarrow T = \frac{\pi}{14}$$

$$\text{first compression occurs} = \frac{3T}{4} = \frac{3\pi}{56} \text{ s}$$

19. $x = 30 \sin(\pi t + \pi/6)$

$$\frac{1}{2} Kx^2 = 2 \left(\frac{1}{2} m\omega^2 (A^2 - x^2) \right)$$

$$\Rightarrow \frac{x^2}{2} = (0.09 - x^2)$$

$$\Rightarrow x^2 = 0.06 \Rightarrow x = \frac{\sqrt{6}}{10} \text{ m.}$$

20. $a = 8\pi^2 - 4\pi^2 x$

$$F = -4\pi^2 m[x - 2] \Rightarrow x = 2 \text{ M.P.}$$

$$T = 2\pi \sqrt{\frac{m}{4\pi^2 m}} = 1 \text{ s}$$

$$\omega = 2\pi \text{ rad/s}$$

$$x = 2 - 4\cos 2\pi t$$

21. (a) $T = 2\pi \sqrt{\frac{I}{K}} = 2\pi \sqrt{\frac{10}{10\pi^2}} = 2 \text{ s}$

(b) $T = 2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}} \Rightarrow g_{\text{eff}} = \sqrt{g^2 + a^2} = 5\sqrt{5}$

$$T = 2\pi \sqrt{\frac{0.5}{5\sqrt{5}}} = \frac{2}{(5)^{1/4}}$$

22. $\frac{1}{2} Kx^2 = 0.4 \quad \omega = 2\pi \times \frac{25}{\pi}$

$$x^2 = \frac{0.8}{K}, \omega = 50, K = 500$$

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = 0.5 \Rightarrow A^2 = x^2 + \frac{1}{500}$$

$$A = 0.06 \text{ m}$$

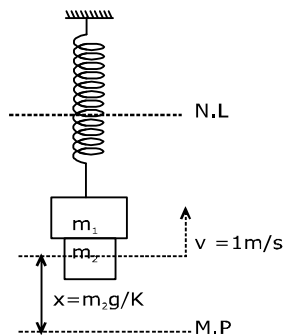
23. Momentum Conservation

$$0.5(3) = 1.5 v$$

$$v = 1 \text{ m/s}$$

$$\omega = \sqrt{\frac{K}{m_1 + m_2}} = 20$$

$$v = \omega \sqrt{A^2 - x^2} \Rightarrow A^2 = \frac{v^2}{\omega^2} + x^2$$



24. $f = 3 \text{ Hz} \quad \mu = 0.72$

When block does not slip

$$ma_{\text{max}} < f \Rightarrow ma_{\text{max}} < \mu mg$$

$$\Rightarrow a_{\text{max}} < \mu g \Rightarrow \omega^2 A_{\text{max}} < \mu g$$

$$A_{\text{max}} = \frac{\mu g}{4\pi^2 f^2} = 0.02$$

25. $U(x) = -ax^2 + bx^4$

$$F = \frac{-dU}{dx} \Rightarrow F = 2ax - 4bx^3$$

for MP $F = 0 \Rightarrow x = 0, x = \sqrt{\frac{a}{2b}}$

$$F = 2a(x + x_0) - 4b(x + x_0)^3$$

$$= 2ax \left(1 + \frac{x_0}{x}\right) - 4bx^2 \left(1 + \frac{3x_0}{x}\right)$$

$$= 2a\sqrt{\frac{a}{2b}} \left[1 + \frac{x_0}{x} - 1 - \frac{3x_0}{x}\right]$$

$$= 2a\sqrt{\frac{a}{2b}} \left[\frac{-2x_0}{x}\right] \Rightarrow 2a\sqrt{\frac{a}{2b}} \sqrt{\frac{2b}{a}} (-2x_0)$$

$$= -4ax_0$$

$$k_{\text{eq}} = 4a \Rightarrow \omega = \sqrt{\frac{k_{\text{eq}}}{m}} = \sqrt{\frac{4a}{m}}$$

26. $T = 2\pi \sqrt{\frac{\ell}{g}} = 2 \Rightarrow \ell = 1 \text{ m}$

27. $\theta = \frac{\pi}{90} \sin(\pi s^{-1}) t \Rightarrow \omega = \pi$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2 \text{ sec.}$$

$$T = 2\pi \sqrt{\frac{\ell}{g}} = 2 \Rightarrow \ell = 1 \text{ m}$$

28. $T = 2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}} \Rightarrow g_{\text{eff}} = g - \frac{3g}{4} = \frac{g}{4}$

(i) $T = 2\pi \sqrt{\frac{4\ell}{g}} = 2T_0 \quad \text{if } T' = \frac{T_0}{2}$

(ii) $2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}} = \pi \sqrt{\frac{\ell}{g}} \Rightarrow g_{\text{eff}} = 4g$

$$g + a = 4g \Rightarrow a = 3g \uparrow$$

29. (a) $g_{\text{eff}} = g + a_0 \Rightarrow T = 2\pi \sqrt{\frac{\ell}{g + a_0}}$

(b) $g_{\text{eff}} = g - a_0 \Rightarrow T = 2\pi \sqrt{\frac{\ell}{g - a_0}}$

(c) $g_{\text{eff}} = g \Rightarrow T = 2\pi \sqrt{\frac{\ell}{g}}$

30. $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$ $T_2 = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + a^2}}}$

$$\Rightarrow \frac{T_1}{T_2} = \frac{(g^2 + a^2)^{1/4}}{\sqrt{g}} \Rightarrow \left(\frac{T_1}{T_2}\right)^4 g^2 = g^2 + a^2$$

$$\Rightarrow g^2 = \left(\frac{4 - 0.01}{4}\right)^4 (g^2 + a^2)$$

$$\Rightarrow g^2 = (1 - 0.01)(g^2 + a^2) \Rightarrow a^2 = 0.01g^2$$

$$\Rightarrow a = 0.1g \quad \Rightarrow a = g/10$$

31. $I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + m\ell^2 = \frac{17}{12}m\ell^2$

$$d = \frac{3\ell}{4}$$

$$T = 2\pi\sqrt{\frac{17m\ell^2}{12.2mg \cdot \frac{3}{4}\ell}} \Rightarrow T = 2\pi\sqrt{\frac{17\ell}{18g}}$$

32. $T = 2s$

$$T_1 = \frac{T}{2}$$

$$T_2 = 2\pi\sqrt{\frac{\ell}{4g}} = \frac{T}{2}$$

$$T' = T_1 + \frac{T_2}{2} \Rightarrow T' = \frac{T}{2} + \frac{T}{4} = \frac{3}{2}s$$

Exercise-3

Level-II

1. K.E. = 8×10^{-3} J

$$\frac{1}{2}m\omega^2 A^2 = 8 \times 10^{-3} \text{ J}$$

$$\frac{0.1}{2}\omega^2 (1)^2 = 8 \times 10^{-3}$$

$$\omega = 4$$

$$\phi = 45^\circ = \pi/4$$

$$x = (1) \sin(4t + \pi/4)$$

2.

As $V^2 = \omega^2(A^2 - x^2)$

For P, $64 = \omega^2(A^2 - x^2)$... (1)

For Q, $49 = \omega^2[A^2 - (x+1)^2]$... (2)

For R, $16 = \omega^2[A^2 - (x+2)^2]$... (3)

(1) - (2)

$$15 = \omega^2(2x + 1)$$

(2) - (3)

$$33 = \omega^2(2x + 3)$$

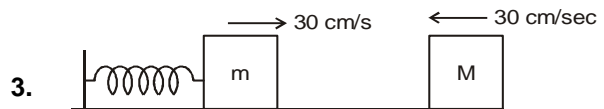
$$\frac{15}{33} = \frac{(2x + 1)}{(2x + 3)}$$

$$x = \frac{1}{3}$$

Putting the value in equation above

$$\omega = 3$$

$$A = \frac{\sqrt{65}}{3}, \quad \text{Max. Speed} = A\omega = \sqrt{65}$$



we know that $\omega^2 = \frac{k}{m}$

$$k = m\omega^2 = (1)(10)^2 = 100 \text{ N/m}$$

At $t = 0$ block of mass m is at mean position $x = 10 \text{ cm}$.

velocity of block $m = v_m = \frac{dx}{dt} = 30 \cos 10t$

at $t = 0$ $v_m = 30 \text{ cm/sec}$.

from momentum conservation

$$(M + m)v = M(30) - m(30)$$

$$v = 15 \text{ cm/sec}$$

Now $\frac{1}{2}(M + m)v^2 = \frac{1}{2}kA^2$

on solving $A = 3 \text{ cm}$

(b) New ω of the system having mass $(M + m)$

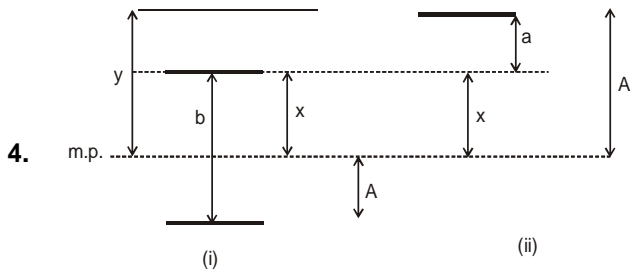
$$\omega' = \sqrt{\frac{K}{M + m}} = \sqrt{\frac{100}{4}} = 5 \text{ rad/s}$$

$$x' = 10 - 3 \sin 5t$$

(c) Loss of energy during collision = Energy before collision - Energy after collision

$$= \frac{1}{2}m(0.3)^2 + \frac{1}{2}M(0.3)^2 - \frac{1}{2}(M + m)(0.15)^2$$

$$= 0.135 \text{ Jule}$$



4. (A) from figure (i) $b = A + x$... (1)
 from figure (ii) $A = a + x$... (2)
 from eq. (1) & (2)
 $b = a + 2x \Rightarrow 2x = b - a$
 and $x = mg/k$
 $\Rightarrow K = \frac{2mg}{b - a}$

(B) Oscillation frequency = $\frac{1}{2\pi} \sqrt{\frac{K}{m_{total}}}$
 $= \frac{1}{2\pi} \sqrt{\frac{2mg}{(b - a)(M + m)}}$

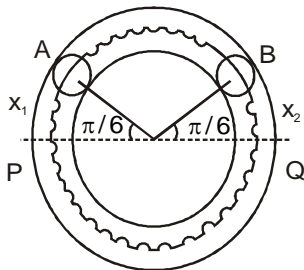
(C) By energy conservation.
 5. (a) Both the spring have same force so. It is parallel equivalent of spring
 $k_{eq} = k_1 + k_2 = 0.2N/m$
 Now the problem change in two block system in which reduced mass is

$m = \frac{m_1 m_2}{m_1 + m_2} = \frac{0.1 \times 0.1}{0.1 + 0.1} = 0.05kg$

$f = \frac{1}{2\pi} \sqrt{\frac{K_{eq}}{m}} = \frac{1}{2\pi} \sqrt{\frac{0.20}{0.05}} = \frac{1}{\pi} Hz$

(b) Balls are at rest in position A & B so Total energy is in potential energy for

$E = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$



$= \frac{1}{2} kx^2 + \frac{1}{2} kx^2$ $\begin{cases} x_1 = x_2 \\ R_1 = R_2 \end{cases}$

$E = kx^2$
 $x = x_1 + x_2 = R \pi/6 + R \pi/6$
 $= 0.02\pi m$

Now $E = kx^2$
 $= (0.1) (0.02\pi)^2 = 4\pi^2 \times 10^{-5} J$

(c) At P & Q no stretch in spring so complete energy is in the kinetic form

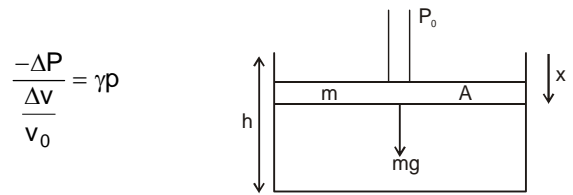
$\Rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = E$

$m_1 = m_2 = 0.1 kg$

$v_1 = v_2 = v \Rightarrow 0.1v^2 = 4\pi^2 \times 10^{-5}$

$v = 2\pi \times 10^{-2} m/sec$

6. $P = P_0 + \frac{mg}{A}$
 Bulk modulus $B = \gamma p$



$\frac{-\Delta P}{\frac{\Delta V}{V_0}} = \gamma p$

$\Delta P = \frac{-\gamma p \Delta V}{V_0}$

$F_{net} = A \Delta P = \frac{-A \gamma p \Delta V}{V_0}$

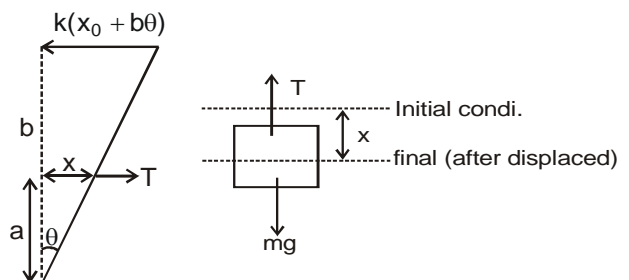
$= -A \gamma p \frac{x A}{h A}$ $\left\{ \begin{array}{l} \Delta V = xA \\ V_0 = hA \end{array} \right.$

$= -A \gamma p \frac{x}{h}$

$F_{net} = -A \gamma \left[P_0 + \frac{mg}{A} \right] \frac{x}{h} \Rightarrow k = m\omega^2 = A \gamma \left(P_0 + \frac{mg}{A} \right) \frac{1}{h}$

$\omega^2 = \frac{A \gamma}{mh} \left(P_0 + \frac{mg}{A} \right) \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{A \gamma}{mh} \left(P_0 + \frac{mg}{A} \right)}$

7. At equilibrium condition we assume elongation is spring is x_0 then
 $mg(a) = Kx_0 b$... (1)
 Now rod is moved small angle θ then



$\Rightarrow T.a = K(x_0 + b\theta).b$

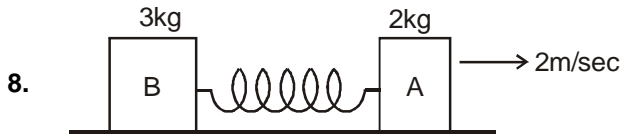
$$T = \frac{k(x_0 + b\theta)b}{a}$$

On block of mass m $F_{net} = mg - T$

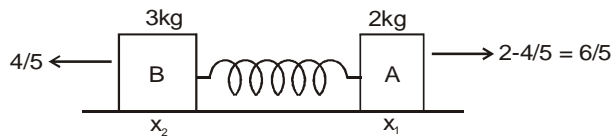
$$F_{net} = mg - \frac{k(x_0 + b\theta)b}{a}$$

$$t = mg - \frac{kx_0b}{a} - \frac{kb^2\theta}{a} \quad t = \frac{-kb^2\theta}{a} \quad \left\{ \text{and } \theta = \frac{x}{a} \right.$$

$$= \frac{-kb^2x}{a^2} \Rightarrow T = 2\pi\sqrt{\frac{ma^2}{kb^2}}$$



$V_{com} = 4/5$
In frame of chita :-



Let us assume that elongation in spring is x then

$$x_1 + x_2 = x \quad \dots(1)$$

$$2x_1 = 3x_2 \quad \dots(2)$$

(Centre of mass is at rest)

from (1) & (2)

$$x_1 + \frac{2x_1}{3} = x \quad \dots(3)$$

from energy conservation

$$\frac{1}{2} \times 2 \times \left(\frac{6}{5}\right)^2 + \frac{1}{2} \times 3 \times \left(\frac{4}{5}\right)^2 = \frac{1}{2} kx^2$$

$$x = 0.2 \quad \dots(3)$$

from (2) & (3)

$$x_1 = 0.12 \text{ m}$$

Maximum velocity = $A\omega = 6/5$

$$x_1\omega = 6/5$$

$$(0.12)\omega = 6/5$$

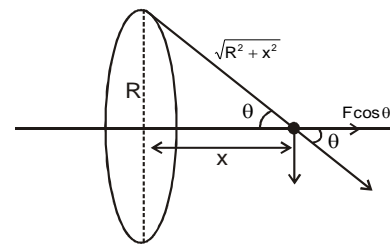
$$\omega = 10$$

then equation of block A

$$x = \left(\frac{4}{5}\right)t + 0.12\sin 10t$$

9. $F_{net} = f \cos\theta$

$$\cos\theta = \frac{x}{\sqrt{R^2 + x^2}}$$



$$\cos\theta = \frac{x}{R} \quad \because x^2 \cong 0 \text{ (for small distance)}$$

$$F = -\frac{GM_1M_2}{R^3}x$$

$$T = 2\pi\sqrt{\frac{M_2}{K}} \quad \because M_1 = \rho \times 2\pi R$$

$$= 2\pi\sqrt{\frac{R^3}{G\rho 2\pi R}} \Rightarrow T = \frac{2\pi R}{\sqrt{G\rho 2\pi}} = \sqrt{\frac{2\pi R^2}{G\rho}}$$

Exercise-4

Level-I

1. B

As we know that spring constant of spring is inversely proportional to length of spring, so new spring constant for each part is given by $k' = nk$ where, k is the spring constant of whole spring. From the theory of spring pendulum, we know that time period of spring pendulum is inversely proportional to square root of spring constant ie,

$$T \propto \frac{1}{\sqrt{k}} \quad T' \propto \frac{1}{\sqrt{nk}}$$

$$\text{So, } T' = \frac{T}{\sqrt{n}}$$

2. C

Kinetic energy of particle of mass m is SHM at

any point is,

$$= \frac{1}{2} m\omega^2(a^2 - x^2)$$

$$\text{and potential energy} = \frac{1}{2} m\omega^2x^2$$

where, a is amplitude of particle and x is the distance from mean position.

So, at mean position, $x = 0$

$$\therefore KE = \frac{1}{2} m\omega^2a^2 \text{ (maximum)}$$

$$PE = 0 \text{ (minimum)}$$

3. B

As the child stands up, the effective length of

pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T will decrease.

4. A

At $x = 0$, kinetic energy is maximum and potential energy is minimum.

5. D

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta l}{l}$$

Given, $\frac{\Delta l}{l} = 21\%$

$$\therefore \frac{\Delta T}{T} = \frac{1}{2} \times 21\% = 10.5\%$$

6. C

$$v_{\max} = a\omega = a \frac{2\pi}{T}$$

$$= \frac{2\pi a}{2\pi\sqrt{\frac{m}{k}}} = a\sqrt{\frac{k}{m}}$$

Hence, $\frac{v_{\max_1}}{v_{\max_2}} = \frac{a_1}{a_2} \sqrt{\frac{k_1}{k_2}}$

$$\therefore v_{\max_1} = v_{\max_2}$$

$$\Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

7. C

$$T = 2\pi\sqrt{\frac{M}{k}} \quad \dots(i)$$

$$T' = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}} \quad \dots(ii)$$

Dividing Eq. (i) and (ii), we have

$$\therefore \frac{3}{5} = \sqrt{\frac{M}{M+m}} \quad \text{or} \quad \frac{9}{25} = \frac{M}{M+m}$$

or $9M + 9m = 25M$

or $16M = 9m$

or $\frac{m}{M} = \frac{16}{9}$

8. A

For amplitude of oscillation and energy to be maximum, frequency of force must be equal to the initial frequency and this is only possible in case of resonance.

In resonance state, $\omega_1 = \omega_2$

9. B

Initial angular velocity of particle = ω_0

and at any instant t , angular velocity = ω

Therefore, for a displacement x , the resultant acceleration

$$f = (\omega_0^2 - \omega^2)x \quad \dots(i)$$

$$\text{External force, } F = m(\omega_0^2 - \omega^2)x \quad \dots(ii)$$

Since, $F \propto \cos \omega t$ (given)

\therefore From Eq. (ii)

$$m(\omega_0^2 - \omega^2)x \propto \cos \omega t \quad \dots(iii)$$

Now, from equation of simple harmonic motion

$$x = A \sin(\omega t + \phi) \quad \dots(iv)$$

at $t = 0$, $x = A$

$$A = A \sin(0 + \phi)$$

$$\Rightarrow \phi = \frac{\pi}{2}$$

$$\therefore x = A \sin\left(\omega t + \frac{\pi}{2}\right) = A \cos \omega t \quad \dots(v)$$

Hence, from Eqs. (iii) and (v), we finally get

$$m(\omega_0^2 - \omega^2)A \cos \omega t \propto \cos \omega t$$

$$\Rightarrow A \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

10. C

In simple harmonic motion when a particle is displaced to a position from its mean position then its kinetic energy gets converted into potential energy. Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on displacement x .

11. B

Time period of spring

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Here k , being the force constant of spring.

For first spring

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \quad \dots(i)$$

For second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \quad \dots(ii)$$

The effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1 + k_2)}{k_1 k_2}\right]}$$

$$\Rightarrow T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \quad \dots(iii)$$

From Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

$$\text{or} \quad t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\text{or} \quad t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow t_1^2 + t_2^2 = T^2 \quad [\text{From Eqs. (iii)}]$$

12. C

The time period of simple pendulum in air

$$T = t_o = 2\pi\sqrt{\left(\frac{l}{g}\right)} \quad \dots(i)$$

Here l , being the length of simple pendulum.

In water, effective weight of bob
 $w' = \text{weight of bob in air} - \text{upthrust}$

$$\Rightarrow \rho V g_{\text{eff}} = mg - m'g$$

$$= \rho V g - \rho' V g$$

$$= (\rho - \rho') V g$$

where $\rho' = \text{density of water}$

$\rho = \text{density of bob}$

$$\therefore g_{\text{eff}} = \left(\frac{\rho - \rho'}{\rho}\right)g = \left(1 - \frac{\rho'}{\rho}\right)g$$

$$\therefore t = 2\pi\sqrt{\left[\frac{l}{\left(1 - \frac{\rho'}{\rho}\right)g}\right]} \quad \dots(ii)$$

$$\text{Thus, } \frac{t}{t_o} = \sqrt{\left[\frac{1}{\left(1 - \frac{\rho'}{\rho}\right)}\right]}$$

$$= \sqrt{\left[\frac{1}{1 - \frac{1000}{(4/3) \times 1000}}\right]} = \sqrt{\left(\frac{4}{4-3}\right)}$$

$$= 2$$

$$\text{or } t = 2t_o$$

13. B

$$\frac{d^2x}{dt^2} = -\alpha x \quad \dots(i)$$

We know that

$$a = \frac{d^2x}{dt^2} = -\omega^2 x \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\omega^2 = \alpha$$

$$\text{or} \quad \omega = \sqrt{\alpha}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\alpha}$$

$$\text{or} \quad T = \frac{2\pi}{\sqrt{\alpha}}$$

14. A

$$\text{Given, } y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$$

$$\Rightarrow \frac{dy_1}{dt} = v_1 = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$$

$$\text{or} \quad v_1 = 10\pi \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$\text{or} \quad v_1 = 10\pi \sin\left(100\pi t + \frac{5\pi}{6}\right)$$

$$\text{and} \quad y_2 = 0.1 \cos\pi t$$

$$\Rightarrow \frac{dy_2}{dt} = v_2 = 0.1 \sin(\pi t + \pi)$$

Hence, phase difference

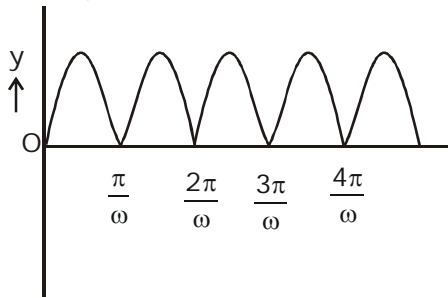
$$\Delta\phi = \phi_1 - \phi_2$$

$$\left(100\pi t + \frac{5\pi}{6}\right) - (\pi t - \pi)$$

$$= \frac{5\pi}{6} - \pi = -\frac{\pi}{6} \quad (\text{at } t=0)$$

15. B

Here, $y = \sin^2\omega t$



$$\frac{dy}{dt} = 2\omega \sin\omega t \cos\omega t = \omega \sin 2\omega t$$

$$\frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$$

For SHM, $\frac{d^2y}{dt^2} \propto -y$

Hence, function is not SHM, but periodic, From the y-t graph, time period is

$$T = \frac{\pi}{\omega}$$

16. A

KE of a body undergoing SHM is given by

$$KE = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

$$\text{and } KE_{\max} = \frac{m\omega^2 A^2}{2}$$

[symbols represent standard quantities]

From given information

$$KE = (KE_{\max}) \times \frac{75}{100}$$

$$\Rightarrow \frac{m\omega^2 A^2}{2} \cos^2 \omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4}$$

$$\text{or } \cos \omega t = \pm \frac{\sqrt{3}}{2}$$

$$\text{or } \omega t = \frac{\pi}{6}$$

$$\text{or } \frac{2\pi}{T} \times t = \frac{\pi}{6}$$

$$\text{or } t = \frac{T}{12} = \frac{1}{6} \text{ s}$$

17. A

The maximum velocity of a particle performing SHM is given by $v = A\omega$, where A is the amplitude and ω is the angular frequency of oscillation.

$$\therefore 4.4 = (7 \times 10^{-3}) \times 2\pi / T$$

$$\text{or } T = \frac{7 \times 10^{-3} \times 2 \times 22}{4.4 \times 7} = 0.01 \text{ s}$$

18. A

$$x = (2 \times 10^{-2}) \cos \pi t$$

Here, $a = 2 \times 10^{-2} \text{ m} = 2 \text{ cm}$

at $t = 0$, $x = 2 \text{ cm}$, ie, the object is at positive extreme, so to acquire maximum speed (ie, to

reach mean position) it takes $\frac{1}{4}$ th of time period.

$$\therefore \text{Required time} = \frac{T}{4}$$

$$\text{where } \omega = \frac{2\pi}{T} = \pi$$

$$\Rightarrow T = 2 \text{ s}$$

$$\text{So, required time} = \frac{T}{4} = \frac{2}{4} = 0.5 \text{ s}$$

19. A

Average kinetic energy of particle

$$= \frac{1}{4} m a^2 \omega^2$$

$$= \frac{1}{4} m a^2 (2\pi v)^2$$

$$= \pi^2 v^2 m a^2$$

20. D

$$f = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}$$

$$\text{and } f' = \frac{1}{2\pi} \cdot 2 \sqrt{\frac{k_1 + k_2}{m}} = 2f$$

21. D

$$x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$$

Acceleration, $a = \frac{d^2x}{dt^2}$

$= -\omega^2 x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$

$= \omega^2 x_0 \cos\left(\omega t - \frac{3\pi}{4}\right)$

So, $A = \omega^2 x_0$

and $\delta = \frac{3\pi}{4}$

22. B

$\frac{aT}{x} = \frac{\omega^2 x T}{x} = \frac{4\pi^2}{T^2} \times T = \frac{4\pi^2}{T} = \text{constant}$

23. C

At mean position $F_{\text{net}} = 0$

\therefore By conservation of linear momentum

$Mv_1 = (M+m)v_2$

$M\omega_1 A_1 = (M+m)\omega_2 A_2$

But $\omega_1 = \sqrt{\frac{k}{M}}$,

and $\omega_2 = \sqrt{\frac{K}{m+M}}$

On solving $\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$

24. A

Let $x_1 = A \sin(\omega t + \phi_1)$ and $x_2 = A \sin(\omega t + \phi_2)$
 $x_2 - x_1 = A[\sin(\omega t + \phi_2) - \sin(\omega t + \phi_1)]$

$= 2A \cos\left(\frac{2\omega t + \phi_1 + \phi_2}{2}\right) \sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

The resultant motion can be treated as a simple harmonic motion with amplitude

$2A \sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

Given, maximum distance between the particles

$= X_0 + A$

\therefore Amplitude of resultant SHM

$= X_0 + A - X_0 = A$

$\therefore 2A \sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A$

$\Rightarrow \phi_2 - \phi_1 = \pi/3$

25. A

For spring $k \propto \frac{1}{l}$

$\therefore \frac{k_A}{k} = \frac{l}{l_A}$

$k_A = \frac{l_A + l_B}{l_A} k_A = \frac{5}{2} k$

26. A

As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about statement 1. But as in statement 2, $k_1 < k_2$ Then, for same force

$W = F \cdot x = F \cdot \frac{F}{k} = \frac{F^2}{k}$

$\Rightarrow W \propto \frac{1}{k}$ i.e., $W_1 > W_2$

But for same displacement

$W = F \cdot x = \frac{1}{2} kx \cdot x = \frac{1}{2} kx^2$

$\Rightarrow W \propto k$, i.e., $W_1 < W_2$

Thus, in the light of statement 2, statement 1 is false.

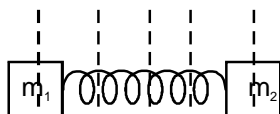
Exercise-4

Level-II

1. Potential Energy $= \frac{1}{2} ky^2$

$= \frac{1}{2} ka^2 \cos^2 \omega t$

\Rightarrow III and I



2.

w.r.t. = C.O.M. it perform S.H.M.

$x_1(t) = v_0 t - A + A \cos \omega t$

$m_1 \cdot A = m_2 \cdot A_2$

$A_2 = \frac{m_1}{m_2} A$

$A + A_2 = \ell_0$

$A_2 = (\ell_0 - A)$

$= v_0 t + \frac{m_1}{m_2} A (1 - \cos \omega t)$

3. At E.P.

$$a = Aw^2 = \frac{Ak}{2m}$$

$$\text{Force} = f = \frac{Ak}{2m} \cdot m = \frac{Ak}{2}$$

4. $y = kt^2 \Rightarrow \frac{d^2y}{dt^2} = 2m/s^2$

$$\therefore T_1 = 2\pi\sqrt{\frac{l}{g}}; T_2 = 2\pi\sqrt{\frac{l}{(g+2)}}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{g+2}{g}\right) = \frac{12}{10} = \frac{6}{5}$$

5. $V^2 = w^2 (a^2 - y^2)$

$$V^2 = \frac{k}{m} (a^2 - y^2), \text{ Now } H = \frac{V^2}{2g} + y$$

$$H = \frac{k}{m} \frac{(a^2 - y^2)}{2g} + y$$

$$\frac{dx}{dy} = -\frac{-2yk}{2gm} + 1 = 0, y = \frac{m}{2k} \times 2g = \frac{mg}{k}$$

6. B

7. $dg = \frac{4\pi^2}{T^2} dl - \frac{4\pi^2 l \times 2 \cdot dT}{T^2}$

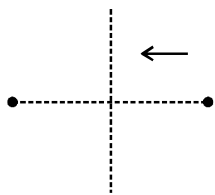
$$\frac{dg}{g} = \frac{dl}{l} - 2 \frac{dT}{T}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{nT}$$

8. $V = C_1 \sqrt{C_2 - x^2}$

$$V^2 = C_1^2 (C_2 - x^2)$$

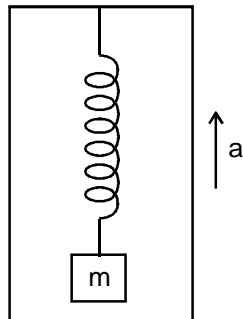
$$a = V \frac{dv}{dx} = -C_1^2 x \text{ (S.H.M.)}$$



$V = -kx$
It will move towards s
Origin and will stop at origin

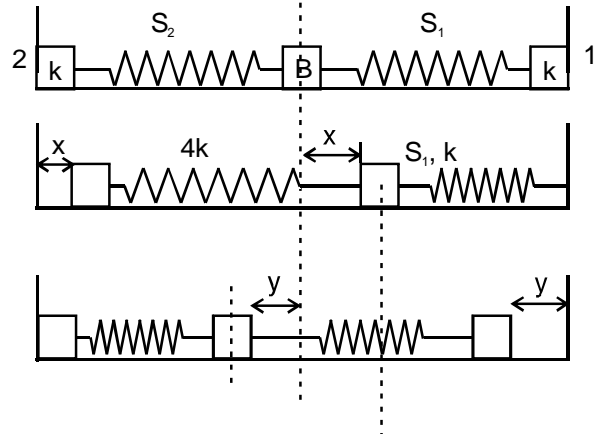
$$v = \sqrt{4gRe}$$

$$v > \sqrt{2gRe}$$



9. $\frac{1}{2} kx^2 = \frac{1}{2} \times 4(y)^2$

$$\frac{y}{x} = \frac{1}{2}$$



$$V_{avg} = \frac{4\sqrt{2}}{T} A$$

Maximum displacement will be close to

$$\text{M.P.} = \sqrt{2} A$$

$$v_{avg} = \frac{4\sqrt{2}}{T} A$$

10. D

$$y = A \sin \omega t$$

$$\omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$y = \sin \frac{\pi}{4} t$$

$$\text{at } t = \frac{4}{3} \text{ sec} \Rightarrow y < \frac{\sqrt{3}}{2}$$

$$a = -\omega^2 y = -\frac{\pi^2}{16} \times \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\pi^2}{32}$$

11. $\frac{4d^2y}{dt^2} + 9y = 0$

$$\omega^2 = \frac{9}{4} \Rightarrow \omega = \frac{3}{2}$$

12. $\tau = I\alpha$

$$k \times \left(\frac{L}{2}\right) + kx \left(\frac{L}{2}\right) = \frac{ML^2}{12} \alpha$$

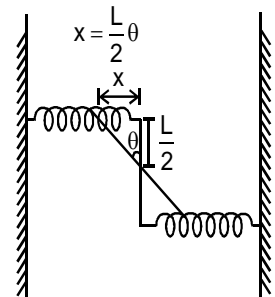
$$kxL = \frac{ML^2}{12} \omega^2 \theta$$

$$x = \frac{L}{2\theta}$$

$$\frac{12 \times L\theta}{ML \times 2} = \omega^2 \theta$$

$$\omega = \sqrt{\frac{6k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{6k}{m}}$$



13. A, D

$$\frac{\omega_A}{\omega_B} = \sqrt{\frac{l_B}{l_A}} \Rightarrow \omega_A < \omega_B$$

14. D

$$V^2 = u^2 - 2gs$$

$p = mv = \pm m\sqrt{u^2 - 2gs}$
 so its parabola
 Initially position = 0
 Momentum is +ve

15. C

$\omega_1 = \omega_2$

$E_1 = \frac{1}{2} m\omega^2 (2a)^2 = 4\left(\frac{1}{2}m\omega^2 a^2\right)$ (1)

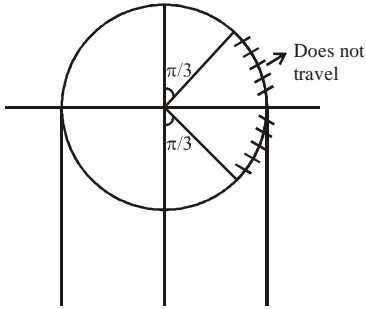
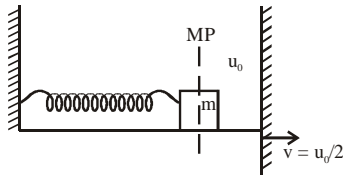
$E_2 = \frac{1}{2}m\omega^2 a^2$ (2)

from equation (1) and (2)

$E_1 = 4E_2$

16. B

17. AD



$x = A \sin \omega t$
 $v = A \omega \cos \omega t$

$\frac{A\omega}{2} = A \omega \cos \omega t$

$\omega t = \frac{\pi}{3}$

Time to reach the wall = $\frac{T}{6}$

Time to return to mean position = $\frac{T}{6} + \frac{T}{6}$

$= \frac{T}{3} = \frac{2\pi}{3} \sqrt{\frac{m}{k}}$

Time for maximum compression = $\frac{T}{3} + \frac{T}{4}$

$= \frac{7T}{12} = 14\pi \sqrt{\frac{m}{k}}$

II time = $\frac{T}{3} + \frac{T}{2} = \frac{5T}{6}$

Equilibrium position = $\frac{5T}{6} = \frac{5\pi}{3} \sqrt{\frac{m}{k}}$

