PHYSICS

For JEE MAIN + JEE ADVANCED

SOLUTIONS BOOKLET

1. HEAT
2. HEAT & THERMODYNAMICS
3. OSCILLATION

HEAD OFFICE : World Tower, 99-Rajpur Road, Dehradun (U.K)
BRANCH OFFICE : GMS Road, Near Ballupur Chowk, Dehradun (U.K)
FOUNDATION WING : Imperial Tower, 101-A, Rajpur Road, Dehradun (U.K)
Give Missed Call : 7669-400-400 **Call at : 8881-400-400**

www.aviral.ac.in
www.aviral.co.in
www.aviralstore.com

JEE-SERIES

CONTENTS

(SOLUTIONS - EXERCISE)

MAIN+ADVANCED

TOPIC

HEAT

SOLUTIONS

HEAT

Exercise-1

1. **D**
Heat is required to raise temperature of (Calorimeter + Ice to vapour) $_{\text{at 0 to 100°C}}$
$= (10\times100+\{10\times80+10\times1\times100+10\times540\})$
= 8200 Cal.

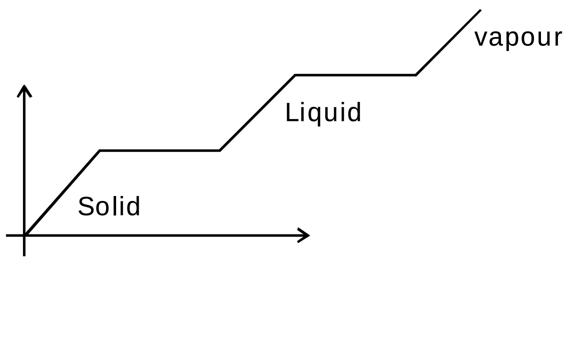
2. **A**
Required heat/sec = 0.1×80 cal/gm = 8 cal/sec
Produced mass = 0.1×100 = 10 gm ice or water
[now $Q = ms\Delta T$]
In unit time rise of temperature will be
$\Delta T = Q/ms = 8/(10\times1) = 0.8$°C/s
R = 0.1 × 80 = 8 cal/sec.

3. **C**
Water flow rate = 20 gm/sec
for 1 sec
$Q = P\times t = 2100\times1 = 2100$ J
$Q = 2100 = 20 \times 4.2\,(t-10)$
t = 35°C

4. **B**
For 1 sec we can say that
$P_c \times 80\ \% = (\rho v)\, s\,(t-10)$
$2\times10^3\times\frac{80}{100} = (1000).100\times(10^{-2})^3.4200\ (t\text{-}10).$
On solving
t = 13.8 °C

5. **A**
$(m_w + w_f)\,(1)\,(70-40) = m_{ice}L_f + m_{ice}\,(1)\,(40\text{-}0)$
$(200+w_f)\,(70-40) = 500\,L_f + 50\times40$ ..(1)
$(m_w + w_f + m_{ice})\,(40-10) = m_{ice}L_f + m'_{ice}\,(1)\,(10-0)$
$(200+w+50)\,30 = 80\,L_f + 80\times10$ ...(2)
from eq. (1) & (2)
$50\times30 = 30\,L_f - 30\times40$
$L_f = 90$ cal/gm $= 3.78\times10^5$ J/kg

6. **D**
$\therefore dQ = msdT \Rightarrow \frac{dT}{dQ} = \frac{1}{ms}$



7. **A**
Using Energy conservation
The energy loss due to potential energy goes into increasing the temperature of ice.
$\frac{m}{5}(L) = mgh$
$\Rightarrow h = \frac{L}{5g}$

8. **B**
At a temperature T
$dQ = SdT = aT^3dT$
$Q = a\int_1^2 T^3dT = \frac{a[T^4]_1^2}{4} = \frac{15a}{4}$

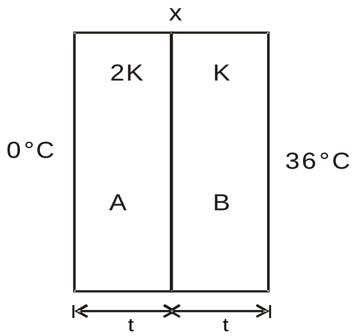
9. **C**
For vapourization the total time required is
= (30 – 20) min = 10 min
Total Heat Given = 42 KtJ × 10 = 420 KJ
so mL = 420 kJ
5L = 420 $\Rightarrow$ L = 84 KJ/kg

10. **D**
From the data given
$S_A\rho_A(8V) = (12V)\,\rho_B s_B$
$\frac{s_A}{s_B} = \frac{12\rho_B}{8\rho_A} = \frac{3}{2}\times\frac{2000}{1500} = 2$

11. **C**
Ice Changes to water hence volume decreases but mass remains same hence
$V_w P_w = V_{ice} P_{ice}$
$V_w = \frac{V_{ice}P_{ice}}{P_w}$
Let volume (V_{ice}) change to water
$(0.9\,\rho_w V_{ice})L = H$ ...(1)
$\Delta v = v_{ice} - v_w = \left(v_{ice} - \frac{v_{ice}\,\rho_{ice}}{\rho_w}\right)$
$= v_{ice}\,(1-0.9) = 0.1\,v_{ice} = 1\ cm^3$
$v_{ice} = 10\ cm^3$
So from eq. (1)
[0.9 × 1 × 10] × 80 = H
H = 720 cal.

12. **A**
Let m is the mass
$mL_v + ms_w\,(100-80) = (1.1 + 0.02)s_w(80-15)$
$m(540 + 20) = (1.12)\,65 \Rightarrow m = 0.130$ kg

13. B



$$\frac{d\theta_A}{dt} = \frac{d\theta_B}{dt}$$

$$2KA\left(\frac{x-0}{t}\right) = KA\left(\frac{36-x}{t}\right)$$

x = 12°C

14. (a) A (b) D

$i_{net} = i_{A\ell} + i_{cu}$

$= K_{A\ell}A\left(\frac{100-20}{3\times10^{-2}}\right) + K_{Cu}A\frac{100-20}{3\times10^{-2}}$

$= (209 + 385)(3 \times 10^{-2}) = \frac{80}{3\times10^{-2}}$

$= 1.43 \times 10^3$ W

(b) $\frac{K_{cu}}{K_{A\ell}} = \frac{385}{209} = 1.84$

15. B

in (a) $\frac{dQ}{dt} = \frac{10}{2}$ cal/min $= AK\left(\frac{100-0}{29}\right)$ ..(1)

in (b) $\frac{d\theta'}{dt} = (2A)K\left(\frac{100-0}{a}\right)$ ...(2)

so $10 = \frac{d\theta'}{dt}.t = 2 \times 10 \times t$

t = 0.5 min

16. B

Here the thermal resistances are in parallel as temperature difference across all the rods is same.

$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)3 \Rightarrow \frac{K_{eq}(6A)}{d} = 3\left(\frac{k_1A}{d} + \frac{k_2A}{d}\right)$

$K_{eq} = \frac{K_1 + K_2}{2}$

17. A

The heat current in the bottom of pot is due to temperature difference at the lower & upper surface.

$i_p = K_{steel}A.\frac{dT}{dx} = \frac{m}{t}.L_v$

$50.2\times0.15\times\frac{(x-100)}{1.2\times10^{-2}} = \frac{0.44}{5\times60}\times2.25\times10^6$

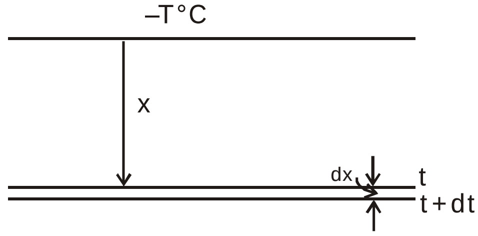
[Let x be temperature of surface in contact with stove]

x = 105.25°C

18. C

The heat current is equal to heat released to formation of ice at the surface in dt time.

In the first case, where w_f is watere quivalent of flask.



$i_t = \frac{KAT}{x} = \frac{dx\,A\,\rho_{ice}}{dt}.L_f$

$\Rightarrow KAT\int_0^{3600} dt = A\rho_{ice}L\int_2^4 x\,dx$

T = 30°C

19. C

The heat current is equal to the heat required for fusion of ice per dt time.

$i = \frac{dm}{dt}.L_f = KA\left(\frac{20-0}{2.35}\right)$

$\frac{dm}{dt} = 2.4\pi\times10^{-6}$

20. A

We know that

$i = K(\pi R^2)\frac{dT}{dx}$, $\quad i \propto \frac{R^2}{\ell}$

21. C

The resistance formed by two cylinders R_1 & R_2 are in parallel as they are kept between same temperature difference.

$A_1 = \pi R^2 \quad A_2 = \pi 4R^2 - \pi R^2 = 3\pi R^2$

Now $R_{eq} = \frac{R_1R_2}{R_1 + R_2} \Rightarrow \frac{\ell}{K_1\pi R^2} + \frac{\ell}{K_2 3\pi R^2}$

$\frac{\ell}{\pi R^2}\left[\frac{1}{K_1} + \frac{1}{3K_2}\right] = \frac{\ell}{K_{eq}(\pi 4R^2)}$

$\frac{1}{4K_{eq}} = \frac{1}{3k_2 + k_2} \Rightarrow k_{eq} = \frac{1}{4}(k_1 + 3k_2)$

22. B

We know that

i = – kAdT / dx

And slope of the curve but dT/dx = – i/kA

i is constant (steady state), A is constant but since k is decreasing from 2k to k, hence slope is –ve but less – ve to more –ve.

23. A

From the given condition as the plates are in series so heat current is same.

$i_1 = i_2 \Rightarrow k_1A\frac{T_B - T_A}{d} = \frac{k_2A(T_c - T_B)}{2d}$

$\frac{k_1}{k_2} = \frac{T_C - T_B}{2(T_B - T_{A)}} = \frac{1}{2}\left(\frac{4T_A - 2T_A}{2T_A - T_A}\right) = 1$

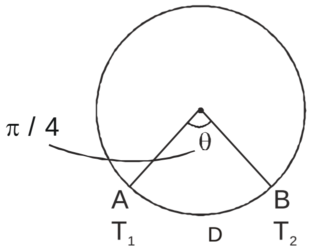
24. D

$i = kA\frac{dT}{dx} \Rightarrow \frac{dT}{dx} \propto \frac{1}{K}$

$\therefore$ i and A are same for both the layers.

i = – kA (dT/dx)
i and A are constant hence slope
dT/dx = –i/(kA) is – ve but
Slope $\propto$ (1/k)
Hence in air slope will be more – ve due to very less conductivity.

25. A



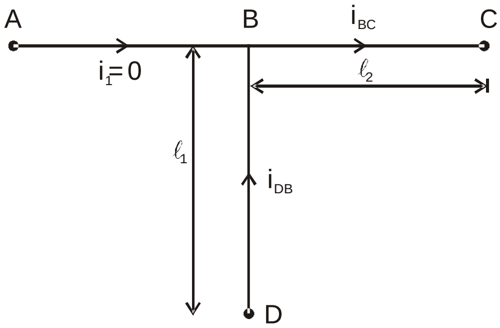
Initially

$H = \frac{kA(T_2 - T_1)}{(2\pi - \theta)R} + \frac{kA(T_2 - T_1)}{\theta R}$ ...(1)

finally $2H = \frac{kA(T_2 - T_1)}{(2\pi - \theta)R} + \frac{k'A(T_2 - T_1)}{\theta R}$ ...(2)

from (1) & (2) $k' = \frac{4k}{3} + k = \frac{7k}{3}$

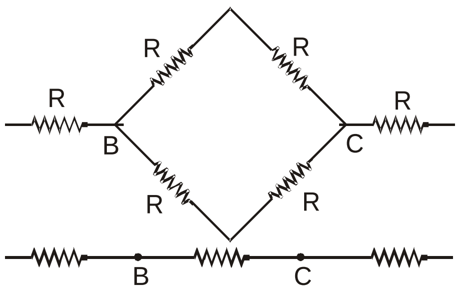
26. B



$i_{BC} = i_{DB} \Rightarrow \frac{kA(90-20)}{\ell_1} = \frac{kA(20-0)}{\ell_2}$

$\frac{\ell_1}{\ell_2} = \frac{7}{2}$

27. C



$T_C - 20 = T_B - T_C = T_A - T_B = \frac{200-20}{3} = 60$

$T_C = 80$

So $T_B = 80 + 60 = 140$ °C

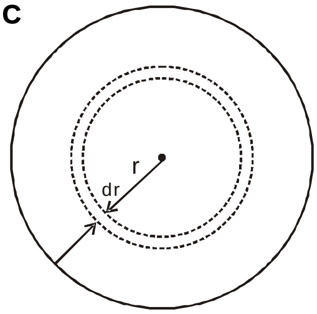
28. B

The heat current is equal to required latent heat of fusion per unit time.

$i = \frac{dm_{ice}}{dt} . L_f = \frac{kA(100)}{\ell}$

$k = \frac{dm_{ice}}{dt} \cdot \frac{\ell L_f}{A(100)} = 60\ Wm^{-1} k^{-1}$

29. C



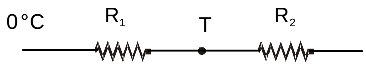
$\int dR = \int_{r_1}^{r_2} \frac{dr}{k(4\pi r^2)}$

$R_{eq} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Now R_1 (when $r_1 = R$, $r_2 = 2R$) $= \frac{1}{8\pi kR}$

and R_2 (when $r_1 = 2R$, $r_2 = 3R$) $= \frac{1}{4\pi kR}\left[\frac{1}{2} - \frac{1}{3}\right]$

$= \frac{1}{24\pi kR}$



$T = \frac{R_1}{R_1 + R_2} \times 100 = 75°$ C

30. C

i = –kA dT / dx
Slope dT/dx = – i/kA is – ve but due to radiation loss because of not lagged, as we move ahead current i will be less. Hence slope wil be more – ve to less – ve.

31. A

$T_p = \frac{100+0}{2} = 50°$

As $T_P > T_Q$ so flow is from P to Q.

$T_Q = \frac{30+60}{2} = 45°$

32. A

Slope dT/dX = – i/kA is less – ve for 1st layer
Hence 1st layer should have larger k.
So $k_1 > k_2$

33. A

Consider the two sections like two resistance R_1 & R_2.

$R_A = \frac{\ell_1}{k_1 A}$ $R_B = \frac{2\ell_1}{\frac{k_1}{2} A}$

So $\theta = \left[\frac{R_B}{R_A + R_B}\right][100 - 0]$

$\theta = 80°$ C

34. A

Thermal resistance is given as

$R_A = \frac{\ell}{3kA}$ $R_B = \frac{\ell}{kA}$

$\frac{R_A}{R_B} = \frac{1}{3}$

35. B

As the rods are in series so that current is same.

$i = \frac{3k_A T_A}{\ell} = \frac{kAT_B}{\ell}$ $\qquad \frac{T_A}{T_B} = \frac{1}{3}$

36. B

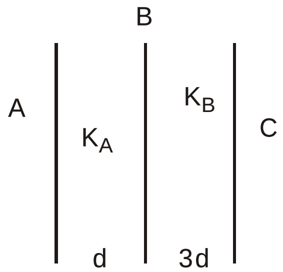
For temperature gradient comparing $\frac{dT}{dx}$ for A & B.

$i_A = i_B \Rightarrow 3kA\left(\frac{dT}{dx}\right)_A = kA\left(\frac{dT}{dx}\right)_B$

$3kAG_A = kAG_B$

$\frac{G_A}{G_B} = \frac{1}{3}$

37. A



38. C

Initially $i = \frac{dm}{dt} . L_f = k\pi R^2 . \frac{100}{\ell}$

Hence $\frac{dm}{dt} \propto \frac{kR^2}{\ell}$

From given condition

$$\frac{\frac{dm_2}{dt}}{\frac{dm_1}{dt}} = \frac{\frac{k}{4}\left(\frac{(2R)^2}{\ell/2}\right)}{\frac{kR^2}{\ell}}$$

$$\frac{\frac{dm_2}{dt}}{0.1} = 2 \Rightarrow \frac{dm_2}{dt} = 0.2$$

39. A

As the heat current through all the rods is same. So all the resistance are in series.

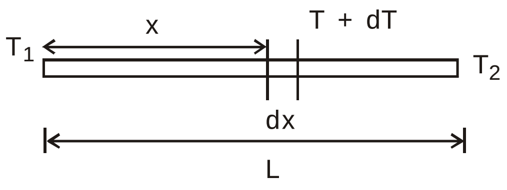
$R_{eq} = R_1 + R_2 + R_3$

$$\frac{3\ell}{k_{eq}A} = \frac{\ell}{\frac{k}{2}A} + \frac{\ell}{5kA} + \frac{\ell}{kA}$$

$$\frac{3}{k_{eq}A} = \frac{2}{k} + \frac{1}{5k} + \frac{1}{k} = \frac{16}{5k}$$

$$k_{eq} = \frac{15}{16}k$$

40. A



Taking an element at a distance x of length dx and having at temperature difference dT.

$i = \frac{\alpha}{T} A \frac{dT}{dx} = C$ (const.)

$\Rightarrow \alpha A[\ell nT]_{T_1}^{T} = Cx$

$\alpha \ell n\left(\frac{T}{T_1}\right) = \left(\frac{C}{A}\right)x$

at x = L, T = $T_2 \Rightarrow \frac{C}{A} = \frac{\alpha}{L} \ln \frac{T_2}{T_1}$

So $T = T_1\left(\frac{T_2}{T_1}\right)^{x/L}$

41. B

Req. $= \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Let at R then

$\frac{1}{R_1} - \frac{1}{R} = \frac{1}{R} - \frac{1}{R_2}$

$R = \frac{2R_1R_2}{R_1 + R_2}$

42 A

Req. is same for both the rods and same temperature same difference so $i_1 = i_2$

43. D

$i = ms\frac{d\theta}{dt} = msk\,(50° - 20°) = 10$ W ..(1)

and $\frac{35.1 - 34.9}{60} = k\,(35 - 20)$ ...(2)

from (1) & (2)

$\frac{0.2}{60} = \frac{10}{ms(30)} \times 15$

ms = 1500 J/°C

44. B

Let at time t radius be r

Then $\frac{dQ}{dt} = CA = 4C\pi r^2 = -\frac{dm}{dt} . L_f$

$m = \rho_{ice} \frac{4}{3} \pi r^3 \Rightarrow dm = C_0 r^2\, dr$

So. $(4\, C\pi)\, r^2 = -L_f\, C_0\, r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \text{const}$

45. D

Power

$P = \frac{dQ}{dt} = A\sigma T^4 = A\sigma\left(\frac{b}{\lambda}\right)^4$

$\frac{P_2}{P_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^4 = \left(\frac{\lambda_0}{3/4\,\lambda_0}\right)^4 = \frac{256}{81}$

46. B

Let $I = \frac{P'}{4\pi d^2}$ or $I = \frac{e\sigma AT^4}{4\pi d^2}$

and I A_f = P (Given)

Now $P_{new} = I_{new} A_f = \frac{e\sigma A(2T)^4}{4\pi(2d)^2}.A_f$

$= \frac{16}{4}\left[\frac{e\sigma AT^4}{4\pi d^2}.A_f\right] = \frac{16}{4}P$

47. A

We know that

$\lambda_{max} \propto \frac{1}{T}$

$\frac{\lambda_{1\,max}}{\lambda_{2\,max}} = \frac{T_2}{T_1}$

$\Rightarrow \frac{T_2}{T_1} = \frac{3}{4}$

48. C

$-\frac{dT_p}{dt} = x\left(-\frac{dT_Q}{dt}\right)$

$\Rightarrow \frac{eA_p\sigma(T^4 - T_0^4)}{m_pS} = \frac{xe\sigma A_Q(T^4 - T_0^4)}{m_QS}$

$\Rightarrow x = \frac{A_p m_Q}{A_Q m_p} = \left(\frac{r}{3r}\right)^2 \times \left(\frac{3r}{r}\right)^3$

$\Rightarrow x = 3$

49. B

Initially the temperature of the substance increases and then phase change from ice to water occurs & this process continues.

50. D

Area = $\int y dx = \int \frac{dE}{d\lambda} \times d\lambda = \int dE$

Area (A) = E = $\sigma T^4 = \sigma\left(\frac{b}{\lambda}\right)^4$

$\frac{Area_1}{Area_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4 \Rightarrow \frac{1}{9} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{3}$

51. B

Using relation $\lambda_{max} \propto \frac{1}{T}$

$\frac{T_S}{T_{NS}} = \frac{\lambda_{NS_{max}}}{\lambda_{S_{max}}} = \frac{350}{510} = 0.69$

52. B

Using formula

$P = \sigma eAT^4$

$P_P = \varepsilon_P \sigma (A) \theta_P^4$ and $P_Q = \varepsilon_Q \sigma A \theta_Q^4$

Now $P_P = P_Q$

$\left(\frac{\varepsilon_Q}{\varepsilon_p}\right)^{1/4} \theta_Q = \theta_P$

53. B

If the body cools from θ_1 to θ_2 then using formula

$\frac{\theta_1 - \theta_2}{t} = \alpha\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$

$\frac{60 - 50}{4} = k\left(\frac{60 + 50}{2} - \theta_0\right)$

$\frac{5}{2} = k(55 - \theta_0)$ ...(1)

and $\frac{40 - 30}{8} = k\left(\frac{40 + 30}{2} - \theta_0\right)$

$\frac{5}{4} = k(35 - \theta_0)$ ..(2)

from (1) & (2)

$2 = \frac{55 - \theta_0}{35 - \theta_0}$

$\theta_0 = 70 - 55 = 15°C$

54. A

$E_{273} = eA(273 + 273)^4$

= E(Given)

$E_0 = eA(273 + 0)^4$

$E_0 = \frac{E}{16}$

55. A

If the body cools from θ_1 to θ_2 then using formula

$\frac{\theta_1 - \theta_2}{t} = \alpha\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$

$\frac{75 - 65}{5} = k\left(\frac{75 + 65}{2} - 25\right)$

$2 = K(70 - 25) \Rightarrow K = \frac{2}{45}$

Now $\frac{65 - x}{5} = k\left(\frac{65 + x}{2} - 25\right)$

2 (65 – x) = 5k (65 + x – 50)

$130 - 2x = 5 \times \frac{2}{45}(15 + x)$

x = 57°c

56. C

$\frac{40 - 36}{5} = k\left(\frac{40 + 36}{2} - 16\right)$

$\frac{4}{5} = K(38 - 16)$

$\Rightarrow k = \frac{2}{55}$ ..(1)

$$\frac{36-32}{t} = \frac{2}{55}\left(\frac{36+32}{2}-16\right)$$

$$\frac{2\times 55}{t} = (34-16)$$

t = 6.1 min

Exercise-2

1. C,D
Not Reflected and Not Refracted.

2. A,B,C
Good Absorbers are good emitters.

3. A,B,D
$\frac{dQ}{dt} = e A \sigma T^4$ So, $\frac{dQ}{dt} \propto A$
$\propto$ e (nature of surface)
$\propto$ T (temprature)
But independent of mass.

4. A,B
(A) $\frac{dQ}{dt} = e A \sigma T^4$
(Rate of emission is same initially)
(B) $\frac{dQ_a}{dt} = e A \sigma T_0^4$
(Rate of obsorption is same always)
(C) $\frac{-dT}{dt} = \frac{eA\sigma(T^4 - T_0^4)}{ms}$
(Due to lesser mass of hollow sphere it cools fast.) (wrong)
(D) Since hollow sphere cools fast ; hollow will have smaller temperature at any moment.(wrong)

5. A,B,C,D
$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}\left(T^4 - T_0^4\right)$$

6. C,D
(A) Heat absorption is surface phenomenon hence wooden (Black surface) absorbs more.(wrong)
(B) After long time both will have temperature of surroundings.(wrong)
(C) Because metal is better conductor it feels hotter.
(D) Because emission depend on surface (i.e. more for black surface)

7. D
Loss(copper) = gain (water + beaker)
$m_{CH}s_{CH}(T_{CH}-T) = m_w s_w (T-T_W) + m_b s_b (T-T_W)$
Hence final temperature can be calculated.

8. D
Rate of melting is doubled if Rate of heat flow is doubled and Rate $\frac{dQ}{dt} = \frac{KA(T-0)}{\ell}$
in (D) T is doubled (50 to 100°C)
and area and length are also doubled hence $\frac{dQ}{dt}$ doubles.

9. AC
$m_A = 4m_B$, $\rho \times \frac{4}{3}\pi r_A^3 = \rho \times \frac{4}{3}\pi r_B^3 \times 4$

$\Rightarrow \frac{r_A}{r_B} = 4^{1/3} = 2^{2/3}$

Rate of heat loss $= \frac{dQ}{dt} = eA\sigma(T^4 - T_0^4)$

Ratio $\frac{(dQ/dt)_A}{(dQ/dt)_B} = \frac{A_A}{A_B} = \left(\frac{r_A}{r_B}\right)^2 = 2^{4/3}$

Rate of cooling $\frac{-dT}{dt} = \frac{dQ/dt}{ms}$

Ratio $\frac{(-dT/dt)_A}{(-dT/dt)_B} = \frac{(dQ/dt)_A}{(dQ/dt)_B} \times \frac{m_B}{m_A}$

$= 2^{4/3} \times \frac{1}{4} = 2^{-2/3}$

10. A,B
$\frac{dQ}{dt} = eA\sigma T^4 = \text{same}$
$\Rightarrow e_A T_A^4 = e_B T_B^4$
$\Rightarrow 0.01 \times (5802)^4 = 0.81 \times (T_B)^4$
$\Rightarrow T_B = 1934$ K
$\lambda_A T_A = \lambda_B T_B$
$\Rightarrow \frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = \frac{5802}{1934} = 3$
$\lambda_B - \lambda_A = 1\ \mu m$
$\Rightarrow \lambda_B - \frac{\lambda_B}{3} = 1$ $\Rightarrow \lambda_B = 1.5\ \mu m$.

11. D
$e_A : e_B : e_C = 1 : \frac{1}{2} : \frac{1}{4}$
Rate of emission : $\frac{dQ}{dt} = eA\sigma T^4$ is same
So, eT^4 is same
$\Rightarrow T_A^4 : T_B^4 : T_C^4 = \frac{1}{e_A} : \frac{1}{e_B} : \frac{1}{e_C}$
$= 1 : 2 : 4$
as $\lambda T = b$ = constant
So, $\lambda_A^4 : \lambda_B^4 : \lambda_C^4 = \frac{1}{T_A^4} : \frac{1}{T_B^4} : \frac{1}{T_C^4}$
$= 1 : \frac{1}{2} : \frac{1}{4}$
On solving
$\sqrt{e_A\lambda_A T_A \times e_B\lambda_B T_B} = e_C\lambda_C T_C$

12. A,B
(A) Emitted energy is very less for longer and shorter wavelength.
(B) From fig. at λ_m intensity is maximum
(C) Area under the curve shows amount of energy emitted.

13. A,B,C,D
When T↑ curve shifts towards shorter wavelength hence curve spreads i.e. Area increases.

14. B

$\lambda_m \propto \frac{1}{T}$, T' > T So, option B is correct.

Exercise-3 Level-I

1. H = Energy/heat required to change 100°C Water in 200°C Vapour
$= 1000\,L_V + 1000 \times 0.5 \times (200-100)$
= 590 Kcal.

2. Let mass = m
So, $\frac{1}{4}[mgh] = mL_s$
$h = \frac{4L_s}{g} = \frac{4 \times 3.4 \times 10^5}{10} = 13.6 \times 10^4$ m

3. From energy conservation
$mgh + \Delta K = ms\Delta T$
$(200\times10^{-3})\times10\times(60\times10^{-2}\sin 37°) + 0$
$= 200 \times 10^{-3} \times 420 \times \Delta T$
$\Delta T = 86 \times 10^{-3}$°C

4. Let all are at 0°C water then heat given is
$\Delta Q_{ice} = -[(10\times0.5\times 10) + (10 \times 80)] = -850$ cal
$\Delta Q_{water} = 10 \times 1 \times 20 = 200$ cal
$\Delta Q_{steam} = [2\times540+2\times1\times100] + 1280$ cal.
So, at 0°C water now have (1280+200–850) cal.
As the heat is extra so it will increase temperature to t
(10 + 10 + 2) (1) (t – 0)
= (1280 + 200 – 850)
t = 28.636°C

5. $A \Rightarrow 200\text{ J} = 4 \times L_A \Rightarrow L_A = 50$
$B \Rightarrow 300\text{ J} = 5 \times L_B \Rightarrow L_B = 60$
$C \Rightarrow 300\text{ J} = 6 \times L_C \Rightarrow L_C = 50$

6. $m_A S_A[\Delta T_A] = -mS_B\Delta T_{B, C, D}$
$\Rightarrow S_A\Delta T_A = -S_B\Delta T_{B, C, D}$ ....(1)
So,
$1 \Rightarrow S_A = S_B$
$2 \Rightarrow S_A > S_C$
$3 \Rightarrow S_A >> S_D$

7. Total energy released
= 10 × 5 × 4200
$= 21 \times 10^4$ J
Energy released/min
$= \frac{0.2\times60}{1000} \times 2.27 \times 10^6$
$= 27.24 \times 10^3$
Time required
$= \frac{21\times10^4}{27.24\times10^3} = 7.7$ min

8. $\Delta Q_0 = 100 \times 4 \times 600$
= 24000 Cal.
For 0°C water
$\Delta Q_1 = (100\times0.2\times20)+(200\times0.5\times20)+(200\times80)$
= 18400 cal.
So, let temperature is t then
24000 – 18400
= (200 × 1 + 100 × 0.2) t
t = 25.5°C

9. Density of drink $= \frac{2m}{120}$
$= 0.833\ \frac{gm}{cc}$ (given)
$m \approx 50$ gm.
$ms_{AS}\,\Delta t + ms_W\Delta t = m_{ice}L_b + m_{ice}S_w\,(t-0)$
$\Rightarrow 50\times0.6\times(25-t)+50\times1\times(25-t) = 20\times80+20\times t$
$t = \frac{50\times25(1+0.6)-20\times80}{50\times(1+0.6)+20}$ = t = 4°C

10. A
20°C W
5 gm X = 40°C
eq = 22°C
B
20°C W
5 gm Y = 40°C
eq = 23°C
For A $W(22-20) = 5 \times S_X \times (40 - 22)$
W = 9 gm
For B $W(23-20) = 5 \times S_Y(40-23)$
$S_Y = \frac{27}{85}$

11. If ice completely melts then heat released
$\Delta Q_{ice} = (2 \times 50)(0.5)(15) + (2 \times 50) \times 80$
= 8750 cal.
for water at 25°C
$\Delta Q_W = 250 \times 1 \times 25$ cal = 6250 cal
Let m gram ice then $mL_S = (8750 - 6250)$ cal
$m = \frac{125}{4}$ g

12. (i) Let m
$\left(\frac{4}{5}\times1000\right) = ms \times \Delta T$
= m 0.5 × 80
m = 20 gm.

(ii) $mL = \left[\frac{1000}{5}\right] \times 4$
$L = \frac{1000\times4}{5\times20} = 40\ \frac{cal}{C\ gm}$

(iii) $ms_\ell(120-80) = \frac{1000}{5}\times3$
$S_\ell = \frac{1000\times3}{5\times40\times20} = 0.75\ \frac{cal}{C\ gm}$

13. $\frac{dQ}{dt} = KA\frac{dT}{dx}$
$= 0.8 \times (10 \times 10^{-2}) \times \left(\frac{90-10}{1\times10^{-2}}\right)$
= 64 J

14. $i = mL_s = KA\dfrac{dT}{dx}$

$m = \dfrac{42 \times (0.04 \times 10^{-4})100}{3.36 \times 10^5 \times 1}$

$= 50 \times 10^{-9}$ Kg

$= 5 \times 10^{-5}$ g/s

15. (T-100)/2.5 + (T–0)/2.5 + (T–25)/5=0

So, T = 45°C

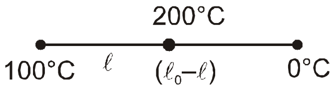
Hence,

I(CD) = V/R

= (45–25)/5 = 4 W

16. $\dfrac{i_c}{i_\ell} = \dfrac{KA\dfrac{dT}{\pi R}}{KA\dfrac{dT}{2R}} = \dfrac{2}{\pi}$

17.



$\dfrac{dm_v}{dt} L_v = \dfrac{KA(200-100)}{\ell}$ ..... (1)

$\dfrac{dm_{ice}}{dt} L_s = KA\left(\dfrac{200-0}{\ell_0-\ell}\right)$ .... (2)

(1) ÷ (2)

$\dfrac{L_V}{L_S} = \dfrac{100}{\ell}\left[\dfrac{\ell_0-\ell}{200}\right]$

$\dfrac{540}{80 \times 40} = \dfrac{1}{2}\left(\dfrac{1.45-\ell}{\ell}\right)$

$27\ell = 2 \times 1.45 - 2\ell$

$\boxed{\ell = 0.1\text{m}}$

18. Req = $R_1 + R_2 + R_3$

$\dfrac{t_1 + t_2 + t_3}{Req\,A}$

$= \dfrac{t_1}{K_1A} + \dfrac{t_2}{K_2A} + \dfrac{t_3}{K_3A}$

19. $KA\dfrac{dT}{dx} = 1 \times 10^3$ W

$0.2 \times 5\ 5\ \dfrac{(T-25)}{4 \times 10^{-2}} = 10^3$

$T = 4 \times 10^{-2} \times 10^3 + 25$

$\boxed{T = 65°C}$

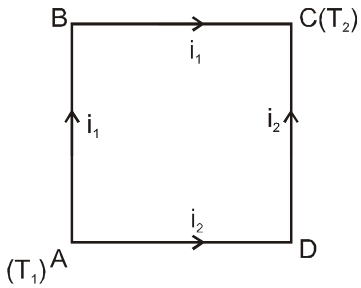
20. $\dfrac{dQ_1}{dt} = KA\dfrac{10}{d}$ ⇒ $\theta = 5°C$

$\dfrac{dQ_2}{dt} = 2KA\left(\dfrac{10-\theta}{d}\right)$ ⇒ $\theta = 5°C$

$\dfrac{dQ_3}{dt} = KA\dfrac{(\theta+5)}{d}$ ⇒ $\theta = 5°C$

$\dfrac{dQ_4}{dt} = 2K\dfrac{(5)}{d}$ ⇒ $\theta = 5°C$

21. Initially



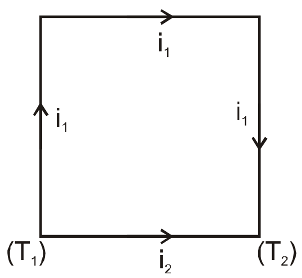
Given

W = $i_1 + i_2$

$\Rightarrow w = \left[KA\left(\dfrac{T_2-T_1}{2\ell}\right)\right] \times 2$

$\boxed{W = \dfrac{KA(T_2-T_1)}{\ell}}$ .... (1)

Now,



Now, w' = $i_1' + i_2'$

$= \dfrac{KA(T_2-T_1)}{\ell} + \dfrac{KA(T_2-T_1)}{3\ell}$

Or

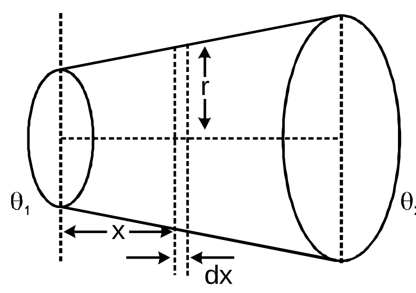
$\boxed{W' = W + \dfrac{W}{3} = \dfrac{4}{3}W}$ From eq-1

22. $160\pi = \dfrac{50-10}{Req}$

$160\pi = \dfrac{50 \times 10}{\dfrac{1}{4\pi K}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]}$

$\dfrac{160\pi}{4\pi} = \dfrac{40K}{\left(\dfrac{1}{5} - \dfrac{1}{20}\right) \times \dfrac{1}{10^{-2}}}$ ⇒ K = 15

23.



$$\frac{r_2 - r_1}{L} = \frac{r - r_1}{x}$$

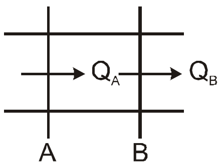
$$r = r_1 + \frac{x}{L}(r_2 - r_1)$$

$$\int dR = \int \frac{dx}{K\pi r^2}$$

$$R = \frac{1}{K\pi}\int \frac{dx}{\left[r_1 + \frac{x}{L}(r_2 - r_1)\right]^2}$$

$$R = \frac{L}{K\pi(r_1 r_2)}$$

$$i = \frac{Q_2 - Q_1}{R}$$

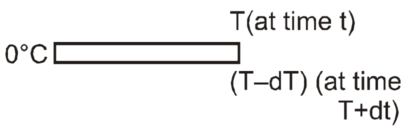
24. 

$$\frac{\Delta Q_{AB}}{\Delta t} = \frac{dQ_A}{dt} - \frac{dQ_B}{dt}$$

$$\Rightarrow \frac{\Delta Q}{\Delta t} = i_A - i_B$$

$$\Rightarrow (ms)\frac{dQ_{AB}}{dt} = KA\left(\frac{dT}{dx}\right)_A - KA\left(\frac{dT}{dx}\right)_B$$

$$\Rightarrow 0.40\ \frac{dQ_{AB}}{dt} = 200 \times (1 \times 10^{-4})\ \left[(5 - 2.6) \times 10^2\right]$$

25. 

$$i = \frac{dQ}{dt} = \frac{KA(T-0)}{\ell}$$

$$Q_T = \int dQ = \int_0^{(10\times60)Sec} \frac{KAT}{\ell}dt \qquad \text{....... (1)}$$

$$\text{Now, } T = \frac{t}{10} \qquad \text{....... (2)}$$

So,

$$Q_T = \int dQ = \int_0^{600} \frac{KAt}{\ell 10}dt = \frac{KA\left[t^2\right]_0^{600}}{20\ell} = 1800\ J$$

26. Here

$$\frac{50.1 - 49.9}{5} = K\ (50 - 30°)$$

$$\frac{0.2}{5} = K \times 20 \qquad \Rightarrow \qquad K = \frac{1}{500}$$

Now,

$$\frac{40.1 - 39.9}{t} = K\ (40 - 30)$$

$$\frac{0.2}{t} = \frac{0.2}{5 \times 20}\ (10) \qquad \Rightarrow t = 10sec.$$

27. $M_{cube} = m_{sphere}$

$$\Rightarrow \rho \times a^3 = \rho \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{a}{r} = \left(\frac{4}{3}\pi\right)^{\frac{1}{3}}$$

Rate of cooling $\frac{-dT}{dt} = eA\sigma(T^4 - T_0^4)$

Ratio : $\frac{Cube}{Sphere} = \frac{A_{Cu}}{A_{Sp}} = \frac{6a^2}{4\pi r^2}$

$$= \frac{3}{2\pi} \times \left(\frac{4}{3}\pi\right)^{\frac{2}{3}} = \left(\frac{6}{\pi}\right)^{\frac{1}{3}}$$

28. $\frac{dQ}{dt}$ (loss)

$$= ms\left(\frac{-dT}{dt}\right) = \left(\frac{4}{3}\pi R^3\right)\rho s\left(\frac{-dT}{dt}\right)$$

$$\frac{dQ_1}{dt} \Big/ \frac{dQ_2}{dt} = \frac{\rho_1}{\rho_2} \times \frac{s_1}{s_2}$$

$$= \frac{8}{1} \times \frac{1}{4} = 2$$

29. $\lambda_{max} = \frac{b}{T}$

$$\Rightarrow \frac{\lambda_s}{\lambda_{st}} = \frac{T_{star}}{T_{sum}}$$

$$\Rightarrow T_{Star} = \frac{4753}{9506} \times 6050\ \ K$$

$= 3025$ K

30. $\sigma(300)^4 = 5$ ....(1)

$\sigma(600)^4 = x$ ....(2)

Now, eq (2)/(1)

$$\frac{x}{5} = \left(\frac{600}{300}\right)^4$$

$\Rightarrow x = 80\ watt/cm^2$

31. $100 = e\ \sigma\ A_s T^4$

$$\Rightarrow T^4 = \frac{100}{e\sigma A_s}$$

$\because A_s = 2\pi R\ell$

$T = 1696.7$ K

≈ 1700 K

32. $i = e\sigma A\ (T^4 - T_0^4) = 210$

$= e\sigma A\ ((500)^4 - (300)^4)$

$= 210$ .... (1)

Now,

$i = 700 = \sigma A\ [(500)^4 - (300)^4]$ ....(2)

(1)/(2)

$$\ell = \frac{210}{700} = 0.3$$

33. $\frac{P_s}{4\pi d_{ES}^2}\left(\frac{\pi d_E^2}{4}\right) = \sigma A_E T_E^4$

and $P_s = \sigma A_s T_s^4$

$$\Rightarrow \frac{T_E^4}{4} = \frac{R_s^2}{d_{ES}^2}.\ T_s^4$$

$$T_E = \frac{1}{\sqrt{2}}\left(\frac{6.9\times10^8}{1.49\times10^{11}}\right)^{1/2} 6000\text{ K}$$
= 288.7 K

$$\boxed{T_E = 15.7°C}$$

34. $T = \dfrac{b}{\lambda_{max}}$

$T_B = \dfrac{0.3}{5000\times10^{-8}} = 6 \times 10^3$ K

$T_R = \dfrac{0.3}{7500\times10^{-8}} = 4 \times 10^3$ K

35. $T = \dfrac{b}{\lambda_{max}} = \dfrac{0.3}{5000\times10^{-8}} = 6000$ K

$E = \sigma A T^4$

$= (5.6 \times 10^{-8})(1 \times 10^{-4})(6000)^4$

$= 7.2576 \times 10^3$ watt

$= 7.2576 \times 10^{10} \dfrac{\text{erg}}{\text{cm}^2\text{ sec}}$

36. $E\lambda_{max} = 16 \longrightarrow$

$\lambda'_{max} = 6$ at 500 K

$\lambda'_m T$ = Constant

$6 \times 500 = \lambda_m 1000$

$$\boxed{\lambda_m = 3\mu m}$$

37. $\dfrac{70-60}{5} = K\left(\dfrac{70+60}{2} - 30\right)$

2 = K (65 – 30) .... (1)

Now,

$\dfrac{60-50}{t} = K\left(\dfrac{60+50}{2} - 30\right)$

(1) ÷ (2)

$\dfrac{2t}{10} = \dfrac{35}{55-30}$

$t = \dfrac{35\times5}{25} = 7$ min

38. $\dfrac{50-45}{5} = K\left(\dfrac{50+45}{2} - \theta_0\right)$

Or $2 = K(95 - 2\theta_0)$ .... (1)

$\dfrac{45-40}{8} = K\left(\dfrac{45+40}{2} - \theta_0\right)$

cos $\dfrac{5\times2}{8} = K(85 - 2\theta_0)$ ...... (2)

(1) ÷ (2)

$\dfrac{8}{5} = \dfrac{95-2\theta_0}{85-2\theta_0}$

$\Rightarrow 205 = 6\theta_0$

$$\boxed{\theta_0 = 34.16°C}$$

Exercise-3 Level-II

1. Steam = 330 – 200 – 100 = 30 g

Let ice = xg & water = (200 – x) g

loss (steam) = gain (ice + water + calorimeter)

$30 \times 2.25 \times 10^5 = x \times 3.36 \times 10^5$

$+ 200 \times 4.2 \times 10^3 \times 50$

$+ 100 \times 0.42 \times 10^3 \times 50$

$\Rightarrow$ x = 70g & water = 200 – x = 130 g

Ratios = $\dfrac{7}{13}$

2. –10ºC → – 2ºC

$Q = ms\Delta T$

$64 = 10 \times S_L \times 8 \Rightarrow S_L = 0.8$

1ºC to 3ºC $\Rightarrow$ Q = 900 – 880 = 20 cal

$Q = ms\Delta T \Rightarrow 20 = 10 \times S_s \times 2 \Rightarrow S_s = 1$

Now, –2ºC to + 1ºC

$880 = 10 \times 0.8 \times (t_m + 2) + 10\,L + 10 \times 1 \times (1 - t_m)$

$\Rightarrow L = 85.4 + 0.2\,t_m$ (in cal/gm)

for cal/kg $\Rightarrow L = 85400 + 200\,t_m$

3. (a) $Q = ms\Delta T$

In one second

(a) = 180 × 0.1 × 0.5 = 9 cal/s

= 37.8 water

(b) $P = \tau w \Rightarrow \tau = \dfrac{p}{w} = \dfrac{37.8}{6T_1}$

4. $Q = ms\Delta T$

in one second

$250 = \dfrac{0.2\times10}{1000}\times5\times25 \Rightarrow S = 5000$

5. If y length of ice melts then

y – 0.5 length of water forms

$\Rightarrow$ Ay × 0.9 = A(y – 0.5) × 1

$\Rightarrow$ y = 5 cm ice melts.

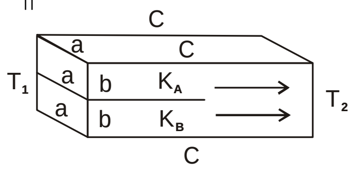
loss (water) = goin (ice)

A × 10 × 1 × 1 × T

= A × 10 × 0.9 × 0.5 × 20

+ A × 5 × 0.9 × 80 $\Rightarrow$ T = 45ºC

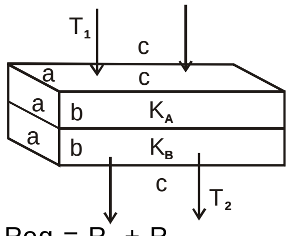
6. $K_{||}$



$\dfrac{1}{R} = \dfrac{kA}{l}$

$\Rightarrow \dfrac{K_{||}(2ab)}{C} = \dfrac{K_A(ab)}{C} + \dfrac{K_B(cb)}{C}$

$\Rightarrow K_{||} = \dfrac{Ka + Kb}{2}$

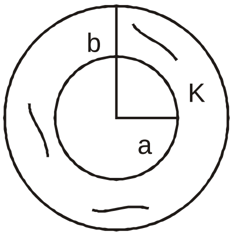


$R_{eq} = R_1 + R_2$

$\dfrac{2b}{keq(ac)} = \dfrac{b}{K_A(ac)} + \dfrac{b}{K_B(ac)} \Rightarrow \boxed{Keq = \dfrac{2K_AK_B}{K_A + K_B}}$

7. for cylinder (a to b)

Req = $\frac{\ln(b/a)}{2\pi kl}$



for conductor (R = 0 to R = a)

dQ = msdT = s × πa^2 l × dT

$\frac{dQ}{dt} = \pi a^2 sl \times \frac{dT}{dt} = \frac{T_0 - T}{Req}$

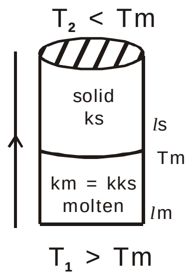
$\Rightarrow \pi a^2 sl \times \frac{dT}{dt} = \frac{T_0 - T}{\ln(b/a)} \times 2\pi k l$

$\Rightarrow \frac{a^2 s}{2k}\int_{T_1}^{T_2}\frac{dT}{T_0 - T} = \frac{1}{\ln(b/a)}\int_0^T dt$

$\Rightarrow t = \frac{a^2 s}{2k} \ln\left(\frac{b}{a}\right) \ln\left(\frac{T_0 - T_1}{T_0 - T_2}\right)$

8. $\left(\frac{dQ}{dt}\right)_{molten} = \left(\frac{dQ}{dt}\right)_{solid}$

$\frac{K_m A(T_1 - T_m)}{l_m} = \frac{K_s A(T_m - T_2)}{l_s}$



$\frac{K(T_1 - T_m)}{T_m - T_2} = \frac{l_m}{l_s}$

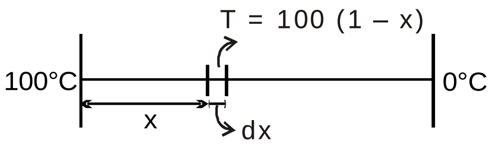
fraction of moltan metar

$= \frac{l_m}{l_m + l_s} = \frac{1}{1 + \frac{l_s}{l_m}}$

$\frac{1}{1 + \frac{T_m - T_2}{k(T_1 - T_m)}} = \frac{k(T_1 - T_m)}{K(T_1 - T_m) + (T_m - T_2)}$

9. **(a)** $\frac{dT}{dx} = \frac{0 - 100}{1}$ = –100º C/m

(b)

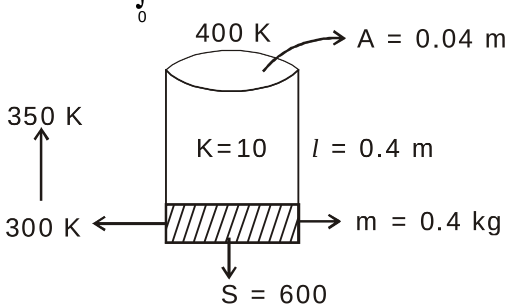


dQ = dm (T – 0) × s

= 2dx×10×100(1–x)

$Q = 2000\int_0^1 (1 - x)\,dx$ = 1000 J

10.



$\frac{dQ}{dt} = ms\frac{dT}{dt}$

$\frac{10(0.04)(400 - T)}{0.4} = 0.4 \times 600 \times \frac{dT}{dt}$

t = 240 ln 2 = 166.3 s

11. $-\frac{dT}{dt} = \frac{dQ/dt}{ms}\frac{eA\sigma(T^4 - T_0^4)}{ms}$

$-\frac{dT}{dt} = \frac{K(T^4 - T_0^4)}{R}$ (R = Radius)

$\Rightarrow 2.8 = \frac{K}{R}$ $(400^4 - 300^4)$

& $-\frac{dT}{dt} = \frac{k}{2R}$ $(600^4 - 300^4)$

dividing $-\frac{dT}{dt}$ = –9.72º C/s

12. $\lambda T = b \Rightarrow T = \frac{b}{\lambda} = \frac{3\times10^{-3}}{7.5\times10^{-6}} \Rightarrow T = 400$ K

$\frac{KA(T_A - T_B)}{l} = A\sigma\, T_B^4 \Rightarrow \frac{17\times(T_A - 400)}{0.5}$

$= 5.67\times10^{-8}\times400^4 \Rightarrow T_A$=423ºKalvin

13. The shell of a space station is a blackened

Rate of loss initially

P = A σ T^4 = A σ $(500)^4$

later half of radiation emmited are emmited back by shell but not loss must be same. So, it radiated double P' = 2p

A σ T^4 = 2 A σ $(500)^4$ $\boxed{T = 200 \times 2^{\frac{1}{4}}}$

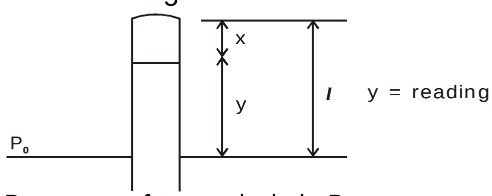
14. $-\frac{dT}{dt} = k(T - T_0) \Rightarrow \ln\left(\frac{T_1 - T_0}{T_2 - T_0}\right) = kt$

$\ln\left(\frac{80 - 20}{50 - 20}\right) = k \times 5 \Rightarrow k = \frac{1}{5}\ln 2$

and $\ln\left(\frac{60 - 20}{30 - 20}\right) = kt$

$\Rightarrow \ln 4 = \frac{1}{5}\ln 2 \times t$ t = 10 min

15.



Pressure of traped air is P_0 – y

and for T = cons ; PV = cons or Pl = cons

and Pl = $(P_0 - y)$ (l – y) = 1 cm

(a) Pl(73–69)(l–69) = (75–70) (l–70) ⇒ l=74cm

Pl = (75 – 70) (74 – 70) = 20

(b) Pl = $(P_0 - y)$ (l – y) = 20

⇒ $(P_0 - 69.5)$(74–69.5) = 20 ⇒ P_0 = 73.94 cm

(c) Pl = $(P_0 - y)$ (l – y) = 20 ⇒ (74–y)(74–y) = 20

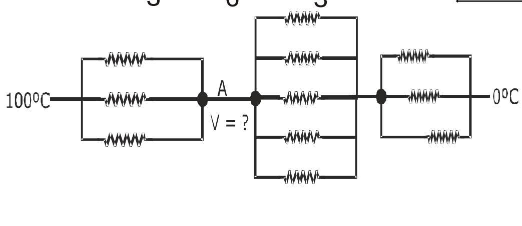
⇒ 74 – y = $\sqrt{20}$ ⇒ y = 69.52 cm

16. (i) $p_1 = P_{H_2}$ = 1.25 × 10^6 Pa ;

$p_2 = p_{H_2} + p_{O_2} + p_{N_2}$ = 2.8125 × 10^6 Pa;

$p_3 = p_{H_2} + p_{N_2}$ = 1.5625 × 10^6 Pa

17. 100ºC — $\frac{R}{3}$ A $\frac{R}{6}$ — $\frac{R}{3}$ —• 0ºC $\boxed{T_A = 60^\circ C}$



Exercise-4 Level-I

1. A

2. B

Spectrometer is an instrument which is used to obtain a pure spectrum of white light. It is used to determine the wavelength of different colours of white light (or wavelength of monochromatic light source), the refractive index of the material of the prism and the dispersive power of the material of the prism. Pyrometer is infrared sensitive device, so it is used to detect infrared radiations.

Nanometer is the small unit of distance and is not a device.

Photometer is used to measure luminuous intensity, illuminance and other photometric quantities.

3. B

The thermal capacity of a substance is defined as the amount of heat required to raise its temperature by 1^0C

4. C

Temperature of a gas is determined by the total translational kinetic energy measured with respect to the centre of mass of of the gas. Therefore, the motion of centre of mass of the gas does not affect the temperature. Hence, the temperature of gas will remain same.

5. A

Energy radiated per second by a body which has surface area A at temperature T is given by stefen's law,

$E = \sigma AT^4$

Therefore, $\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{1}{4}\right)^2\left(\frac{4000}{2000}\right)^4$

[Since bodies are of same material, so $e_1 = e_2$]

$\Rightarrow \frac{E_1}{E_2} = \frac{16}{16} = \frac{1}{1} = 1:1$

6. A

According to the mass -energy equivalence, mass and energy remain conserved. So, when water is cooled to form ice, water loses its energy, so change in energy increases the mass of water.

7. D

According to Newton's law of cooling

$\frac{dQ}{dt} \propto \Delta\theta$

But $\frac{dQ}{dt} \propto (\Delta\theta)^n$ (given)

$\therefore \quad n = 1$

8. D

From stefan law, the energy radiated by sun in given by $P = \sigma eAT^4$.

In 1st case, $P_1 = \sigma e \times 4\pi R^2 \times T^4$

In 2nd case, $P_2 = \sigma e \times 4\pi(2R^2) \times (2T)^4$

$= \sigma e \times 4\pi R^2 \times T^4 \times 64 = 64P_1$

The rate at which energy received at earth is

$E = \frac{P}{4\pi R_{SE}^2} \times A_E$

where A_E = Area of earth

R_{SE} = Distance between sun and earth

So, In 1st case,

$E_1 = \frac{P_1}{4\pi R_{SE}^2} \times A_E$

$E_2 = \frac{P_2}{4\pi R_{SE}^2} \times A_E$

$= 64E_1$

9. D

Let the temperature of common interface be T^0C. Rate of heat flow

$H = \frac{Q}{t} = \frac{KA\Delta T}{l}$

$\therefore \quad H_1 = \left(\frac{Q}{t}\right)_1 = \frac{2KA(T - T_1)}{4x}$

and $H_2 = \left(\frac{Q}{t}\right)_2 = \frac{2KA(T_2 - T)}{x}$

In steady state, the rate of heat flow should be same in whole system ie,

$H_1 = H_2$

$\Rightarrow \quad \frac{2KA(T - T_1)}{4x} = \frac{KA(T_2 - T)}{x}$

or $\quad \frac{(T - T_1)}{2} = (T_2 - T)$

or $\quad T - T_1 = 2T_2 - 2T$

or $\quad T = \frac{2T_2 + 2T_1}{3}$ ...(i)

Hence, heat flow from composite slab is

$H = \frac{KA(T_2 - T)}{x}$

$= \frac{KA}{x}\left(T_2 - \frac{2T_2 + T_1}{3}\right)$

$= \frac{KA}{3x}(T_2 - T_1)$ ...(ii)

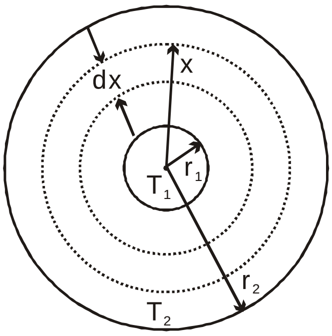
Accordingly, $H = \left[\frac{A(T_2 - T_1)K}{x}\right]f$ ..(iii)

By comparing Eqs. (ii) and (iii), we get

$f = \frac{1}{3}$

10. C

To measure the radial rate of heat flow, we have to go for integration technique as here the area of the surface through which heat will flow is not constant.



Let us consider an element (spherical shell) of thickness dx and radius x as shown in figure. Let us first find the equivalent thermal resistance between inner and outer sphere.

Rsistance of shell $= dR = \frac{dx}{K \times 4\pi x^2}$

$$\left[\begin{array}{l} R = \frac{1}{KA} \\ \text{where } K \to \text{thermal conductivity} \end{array}\right]$$

$$\Rightarrow \int dR = R = \int_{r_1}^{r_2} \frac{dx}{4\pi k x^2}$$

$$= \frac{1}{4\pi K}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{r_2 - r_1}{4\pi K(r_1 r_2)}$$

Rate of heat flow = H

$$= \frac{T_1 - T_2}{R}$$

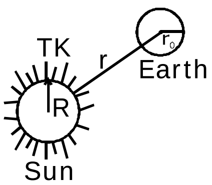
$$= \frac{T_1 - T_2}{r_2 - r_1} \times 4\pi K(r_1 r_2) \quad \propto \frac{r_1 r_2}{r_2 - r_1}$$

11. B

From Stefan's law, the rate at which energy is radiated by sun at its surface is

$P = \sigma \times 4\pi R^2 \times T^4$

[Sun is a perfectly black body as it emits radiations of all wavelengths and so for it e =1.]



The intensity of this power at earth's surface [under the assumption r >> r_0] is

$$I = \frac{P}{4\pi r^2}$$

$$= \frac{\sigma \times 4\pi R^2 r^4}{4\pi r^2} = \frac{\sigma R^2 T^4}{r^2}$$

The area of earth which receives this energy is only one half of total surface area of earth, whose projection would be πr_0^2

∴ Total radiant power as received by earth

$$= \pi r_0^2 \times I$$

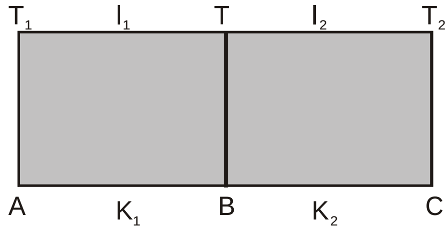
$$= \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2}$$

$$= \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$$

12. C

Let temperature at the interface is T

For part AB,



$$\frac{Q_1}{t} \propto \frac{(T_1 - T)K_1}{l_1}$$

For part BC, $\frac{Q_2}{t} \propto \frac{(T - T_2)K_2}{l_2}$

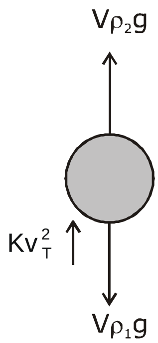
At equilibrium, $\frac{Q_1}{t} = \frac{Q_2}{t}$

$$\Rightarrow \frac{(T_1 - T)K_1}{l_1} = \frac{(T - T_2)K_2}{l_2}$$

or $$T = \frac{T_1 K_1 l_2 + T_2 K_2 l_1}{K_1 l_2 + K_2 l_1}$$

13. A

The forces acting on the ball are gravity force, buoyancy force and viscous force. When ball acquires terminal speed, it is in dynamic equilibrium, let terminal speed of ball is v_T. So,



$$V\rho_2 g + k v_T^2 = V\rho_1 g$$

or $$V_T = \sqrt{\frac{V(\rho_1 - \rho_2)g}{k}}$$

14. D

According to Newton's law of cooling, rate of fall in temperature is proportional to the difference in temperature of the body with surrounding i.e.

$$-\frac{d\theta}{dt} = k(\theta - \theta_0)$$

$$\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -k\,dt$$

In $(\theta - \theta_0)$ = kt + C

Which is a straight line with negative slope.

Exercise-4 Level-II

1. $m_v[L_v + S_w\Delta\theta_1] = 100\,S_w\Delta\theta_2$

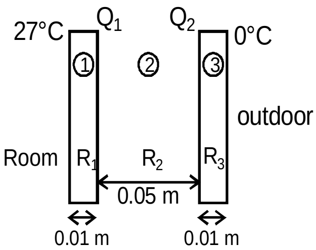
$m_v[540 + 1(100-90)] = 100(1)(90-24)$

$m_v = 12$ gm

2. $\frac{R_A}{R_B} = \frac{K_B}{K_A} = \frac{200}{300} = \frac{2}{3}$

$\frac{\Delta T_A}{\Delta T_B} = \frac{R_A}{R_B} = \frac{2}{3} \Rightarrow \frac{100-T}{T-0} = \frac{2}{3} \Rightarrow T = 60°C$

3. $R_1 = R_3 = \frac{0.01}{(0.8)(1)}$



$i = i_1 = i_2 = i_3$

$\Rightarrow \frac{27-Q_1}{0.0125} = \frac{Q_1-Q_2}{0.625} = \frac{Q_2-0}{0.0125}$

$\Rightarrow Q_1 = 26.48°C$

$Q_2 = 0.52°C$ hence i = 41.6 watt.

4. **D**

$P \propto A\,T^4$

$\frac{P'}{P} = \frac{(A/4)(2T)^4}{A\,T^4}$

$P' = 450\left(\frac{1}{4}\right)(16) = 1800$ W

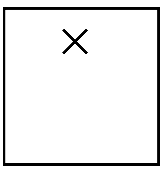
5. $\Delta Q_{ice} = (0.28)\,(3.3 \times 10^5 \text{ J/kg}) = 9.24 \times 10^4$

Heat received/second = (1400) (0.2) = 280 J

Time taken = $\frac{9.24\times10^4}{280\times60} = 5.5$ min

6. For $t \le t_1$

Newton's law of cooling

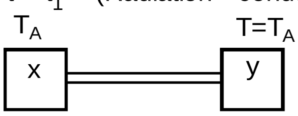


$-\frac{dT}{dt} = k(T-T_A)$

$\int_{T_0}^{T_1}\frac{dT}{T-T_A} = -k\int_0^{t_1} dt$

$kt_1 = -\ell n\left(\frac{350-300}{400-300}\right) = \ln(2)$

For $t > t_1$ (Radiation + conduction)



Rate of cooling $-\frac{dT}{dt} = k(T-T_A) + \frac{kA}{CL}(T-T_A)$

Let at $t = 3t_1$, temp. of x becomes T_2 then

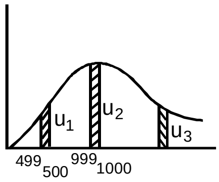
$\int_{T_1=350}^{T_2}\frac{dT}{T-T_A} = -\left(k+\frac{kA}{LC}\right)\int_{t_1}^{3t_1} dt$

$\Rightarrow \ell n\left(\frac{T_2-300}{350-300}\right) = -\left(2kt_1 + \frac{2kA}{LC}t_1\right)$ $\{kt_1 = \ln 2\}$

$T_2 = 300 + 50\exp\left[-\left(\frac{kA}{L}+\frac{\ln 2}{t_1}\right)2t_1\right]$

7. **D**

Wien's Displacement law



$\lambda_m T = b \Rightarrow \lambda_m = \frac{b}{T}$

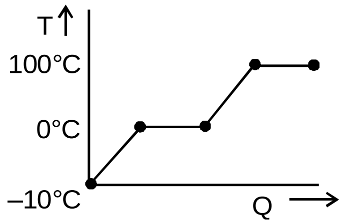
$\Rightarrow \lambda_m = 1000$ nm

from graph

$U_2 > U_1$

8. **A**

Initially the heat absorbed goes into increasing the temperature of ice. At 0ºC the phase changes. So temperature is constant. Till 100º again this process and at 100ºC agian phase change occues.



9. **B**

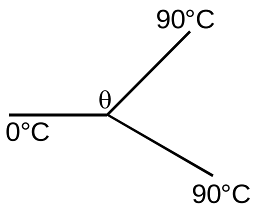
$\lambda_m \propto \frac{1}{T}$

from fig. $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

$T_1 > T_3 > T_2$

10. **B**

Using junction law and assuming the temperature of the junction to be θ.



$\frac{90-\theta}{R} + \frac{90-\theta}{R} = \frac{\theta-0}{R}$

$\Rightarrow \theta = 60°C$

11. **D**

Initially it willl absorb all the radiant energy so it will be the darkest one.

Then radiate maximum energy and it will be the brightest of all.

12. Heat gained by ice = heat lost by container

$(0.1)\,(8 \times 10^4) + (0.1)\,(10^3)\,(27)$

$= m\int_{500}^{300}(A+BT)\,dT$

$10700 = -m\left[AT + \frac{BT^2}{2}\right]_{500}^{300} \Rightarrow m = 0.495$ kg

13. **A**

$Q_{w_{20\to0}} = 5(10^3)(20-0) = 10^5$ cal

$Q_{ice_{-20\to0}} = 2(500)(20) = 0.2\times10^5$ cal

$\Delta Q = 0.8 \times 10^5$ cal will melt a mass m of ice

$m = \frac{Q}{L} = \frac{0.8\times10^5}{80\times10^3} = 1$kg

$m_{water} = 5 + 1 = 6$ kg

14. A

$-\frac{dT}{dt} \propto e \Rightarrow \left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$ from graph

$\therefore\ e_x > e_y \Rightarrow e_x > a_y$

15. (a) Rate of heat loss per unit area

$I = e\,\sigma\,(T^4 - T_0^4)$

$= 0.6 \times \frac{17}{3} \times 10^{-8}((400)^4 - (300)^4) = 595\ w/m^2$

(b) $\frac{\theta - 127}{(\ell/kA)} = 595A$

$\ell = 0.005 \quad k = 0.149$

$\theta = 146.96\ °C = 419.96\ k$

16. B

$Q \propto AT^4$ and $\lambda_m T$ = const.

hence $Q \propto \frac{A}{\lambda_m^4} \propto \frac{r^2}{\lambda_m^4} \Rightarrow Q_A : Q_B : Q_C = \frac{2^2}{3^4} : \frac{4^2}{4^4} : \frac{6^2}{5^4}$

17. D

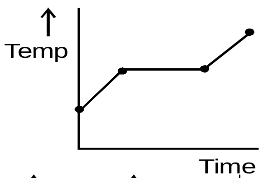
$\frac{dQ}{dt} \propto L\left(\frac{dm}{dt}\right)$ and $\frac{\Delta T}{R} = L\frac{dm}{dt}$

$\Rightarrow \frac{dQ}{dt} \propto \frac{1}{R} \Rightarrow q \propto \frac{1}{R}$

$I \rightarrow$ Parallel $R_{eq.} = R/2 \quad \Rightarrow \quad II \rightarrow$ Series $R_{eq} = 2R$

$\frac{q_2}{q_1} = \left[\frac{2R}{(R/2)}\right]^{-1} = 1/4$

18. C



19. $T\uparrow \quad V_{cube}\uparrow \quad \rho_\ell \downarrow$

Depth of container submerged in liquid remains same

Upthrust = Weight

$v_i \rho_L g = v'_i \rho'_L g$

$(Ah_i)\rho_L g = A(1 + 2\alpha_s \Delta T)h_i\left(\frac{\rho_L}{1 + \gamma_\ell \Delta T}\right)g \Rightarrow \gamma_\ell = 2\alpha_s$

20. Rate of heat conduction = Rate of heat lost from right end

$\frac{kA(T_1 - T_2)}{L} = eA\sigma(T_2^4 - T_5^4)$

As $T_2^4 = (T_5 + \Delta T)^4 = T_5^4\left(1 + \frac{4\Delta T}{T_s}\right)$

hence $\frac{k(T_1 - T_s - \Delta T)}{L} = 4e\sigma T_5^3 \Delta T$

$\Rightarrow \Delta T = \frac{k(T_1 - T_5)}{(4e\sigma L T_5^3 + K)}$

on comparing proportional const. $= \frac{k}{4e\sigma L T_5^3 + K}$

21. A

$\lambda_m T$ = const.

from graph $T_3 > T_2 > T_1$

22. C

Glass of bulb heats due to filament by radiation

23. A

Energy gained by water (in 1 s)

= 1000 – 160 = 8405

Time required t

$= \frac{ms\Delta\theta}{\text{Rate by which energy}} = \frac{2(4.2 \times 10^3)(50)}{840}$ = 500 Sec.

24. A $\frac{dQ}{dt} = e\sigma AT^4 = 0.6\ s\ A\ T^4$

25. B

We know that 14.5°C to 15.5°C at 760 mm of Hg.

26. A,D

Using theory.

27. 0.05 kg steam at (373 k) $\xrightarrow{Q_1 = 50 \times 540 = 27 kcal}$

0.05 kg water at 373 k

0.05 kg water $\xrightarrow{Q_2 = 50 \times 100 = 5 kcal}$

0.05 kg water at 273 k

0.45 kg ice at $\xrightarrow{Q_3 = 450 \times 0.5 \times 20 = 4.5 kcal}$

0.45 kg water at 273 k

$Q_1 + Q > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$

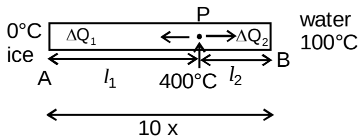
whole ice will not melt

T = 273 k

28. (A) → S, Q ; (B) → Q ; (C) → P, Q ; (D) → D, R or

(A) → S (B) → Q ; (C) → P ; (D) → R (Using theory.)

29.



Let the mass of water that melts or evaporates is m

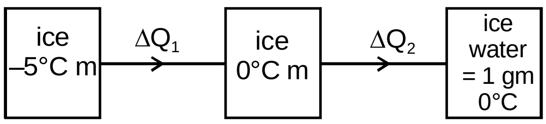
$\Rightarrow \Delta Q_1 = \frac{kA}{\ell_1}(400 - 0) = m \times 80 \times 42$

$\Delta Q_2 = \frac{kA}{\ell_2}(400 - 100) = m \times 540 \times 42$

$\Rightarrow \frac{\Delta Q_1}{\Delta Q_2} = \frac{\ell_2}{\ell_1} \times \frac{400}{300} = \frac{80}{540} \Rightarrow \frac{\ell_2}{\ell_1} = \frac{1}{9}$

$\Rightarrow \frac{\ell_2 + \ell_1}{\ell_1} = \frac{10}{9} \Rightarrow \ell_1 = \frac{9}{10} \times 10 = 9$

30. Total Heat given ΔQ = 420 J



$\Delta Q_1 = m \times 2100\ (0 - (-5)) = m \times 2100 \times 5J$

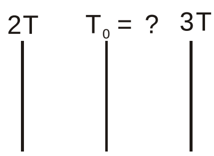
$\Delta Q_2 = 10^{-3} \times 3.36 \times 10^5 = 336\ J$

$\Delta Q = \Delta Q_1 + \Delta Q_2$

$420 = m \times 2100 \times 5 + 336$

$\Rightarrow m = \frac{84}{2100 \times 5}$ kg = 8 gm

31. C



For middle plate(in unit time)

Heat absorbed/Area

= Heat emitted/Area

$\sigma\,(2T)^4 + \sigma(3T)^4 = \sigma(T_0)^4 \times 2$

$T_0 = \left(\frac{97}{2}\right)^{1/4} T$