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ULTIMATE TEST SERIES JEE-MAIN-2017
TEST - 02, 03-03-2017

[PHYSICS]

1. Answer (1)
2. Answer (4)

$$\left.\begin{aligned} 36 &= \frac{\omega^2 - \frac{\omega^2}{4}}{2\alpha} \\ \theta &= \frac{\frac{\omega^2}{4} - 0}{2\alpha} \end{aligned}\right\}$$

$$\Rightarrow \frac{\theta}{36} = \frac{1}{3} \Rightarrow \theta = 12$$

3. Answer (2)

$$a = \frac{g\sin 90}{1+1} = \frac{g}{2}$$

4. Answer (4)

$$\frac{1}{2}\frac{ml^2}{3}\omega^2 = mgh$$

$$\Rightarrow \quad h = \frac{l^2\omega^2}{6g}$$

5. Answer (4)

$$m_1 \times d = m_2 \times x$$

6. Answer (3)

$$-\frac{2GM_1m}{d} - \frac{2GM_2m}{d} + \frac{1}{2}mV_e^2 = 0$$

$$\Rightarrow V_e = \sqrt{\frac{4G(M_1+M_2)}{d}}$$

7. Answer (2)
8. Answer (1)

$$\left.\begin{aligned} U &= -2E \\ KE &= E \end{aligned}\right\} \therefore \text{ Total energy} = -E$$

$\therefore$ Binding energy = E.

9. Answer (4)

$$\frac{GM}{x^2} = G \times \frac{4m}{(r-x)^2}$$

$$\Rightarrow x = \frac{r-x}{2} \Rightarrow x = \frac{r}{3}$$

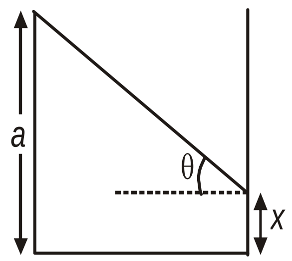
$$\therefore V = -\frac{3GM}{r} - \frac{3G\times 4m}{2r} = -\frac{9GM}{r}$$

10. Answer (2)

$$\sqrt{2gx} \times \sqrt{\frac{2h}{g}} = R$$

$4hx = R^2$.

11. Answer (2)



$$\frac{1}{2} \times a^2 \times (a-x) = \frac{a^3}{3}$$

$$3a - 3x = 2a$$

$$x = \frac{a}{3}$$

$$\tan\theta = \frac{\frac{2a}{3}}{a} = \frac{a_0}{g}$$

$$\Rightarrow a_0 = \frac{2g}{3}$$

12. Answer (2)
13. Answer (4)

$$a = \frac{g}{3}$$

$$\therefore\ a_{cm} = \frac{m\left(\frac{g}{3}\right) - 2m \times \frac{g}{3}}{3m} = -\frac{g}{9}$$

14. Answer (4)

$$0 = m(v_0 - v) - 2mV$$

$$v_0 - v = 2v \Rightarrow v = \frac{v_0}{3}$$

$$\therefore \text{ Velocity of girl } = v_0 - \frac{v_0}{3} = \frac{2v_0}{3}$$

15. Answer (3)

$$I = \frac{MR^2}{2} - \left[\frac{3}{2} \times \frac{M}{4}\left(\frac{R}{2}\right)^2\right] = \left(\frac{1}{2} - \frac{3}{32}\right)MR^2 = \frac{13}{32}MR^2$$

16. Answer (2)

$$u_1 = \frac{P}{m}, u_2 = 0,\ I = p - mv_1,\ I = mv_2$$

$$\Rightarrow v_1 = \frac{p - I}{m} \quad \Rightarrow v_2 = \frac{I}{m}$$

$$\therefore e = \frac{\frac{I}{m} - \frac{p}{m} + \frac{I}{m}}{u_1} = \frac{2I - p}{p}$$

17. Answer (3)
18. Answer (4)

$$T = \left(\frac{M}{2}\right)\left(\frac{3l}{4}\right)\omega^2 = \frac{3}{8}ml\omega^2$$

19. Answer (3)
20. Answer (2)
21. Answer (1)
22. Answer (2)

$$F_2 = \frac{a_2}{a_1} \times F_1 = \frac{(15)^2}{(1.25)^2} \times 400 = 57600 \text{ N}$$

23. Answer (4)

$B = 20 - 16 = 4$ N

$\therefore$ So weight read = 36 + 4 = 40 N

24. Answer (2)

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R+h} = 0$$

25. Answer (2)
26. Answer (4)
27. Answer (2)
28. Answer (2)
29. Answer (3)
30. Answer (4)

[CHEMISTRY]

31. Answer (3)

$H_2O_2 \longrightarrow O_2 + 2e^-$

$5e^- + ClO_2 \longrightarrow Cl^-$

1 mol of ClO_2 = $\frac{5}{2}$ mole of H_2O_2

32. Answer (3)

$$3Cl_2 + 6OH^- \longrightarrow ClO_3^- + 5Cl^- + 3H_2O$$

33. Answer (3)

$^{+4}CO_2$

34. Answer (2)

$Mn^{+2} \longrightarrow Mn^{+4} + 2e$

n-factor = 2

35. Answer (2)

$$\rho = \frac{PM}{RT}$$

36. Answer (1)

$$\frac{v_2}{v_1} = \sqrt{\frac{1200}{300}} \times 0.3 = 0.6\ ms^{-1}$$

37. Answer (2)

Fact

38. Answer (3)

$$n = \frac{PV}{RT} = \frac{1.56 \times 10}{0.082 \times 317} = 0.6 \text{ mol}$$

Let $^{n}C_xH_8 = a$

$^{n}C_xH_{12} = (0.6 - a)$

Total mass of C in mixture

= 12 ax + 12 (.6 – a)x = 7.2x

% of C in mixture = $\frac{7.2x}{41.4} \times 100$ = 87%

or $\frac{720x}{41.4}$ = 87 or x = 5

39. Answer (3)

Slope of the adiabatic curve $\propto \gamma$

He (γ = 1.66), O_2 (γ = 1.44)

40. Answer (1)

$$W = -2.303 \text{ nRT} \log \frac{P_1}{P_2}$$

$$= -2.303 \times 1 \times 8.314 \times 273 \log \frac{1}{0.1} = -5.227 \text{ kJ}$$

q = –W = +5.227 kJ

41. Answer (1)

N_2	+	$3H_2$	$\rightleftharpoons$	$2NH_3$
0.2		0.6		0

0.2 – x   0.6 – 3x   2x

$0.4 = \frac{x}{0.2} \Rightarrow x = 0.08$

$\frac{V_f}{V_i} = \frac{0.8 - 2x}{0.8} = \frac{4}{5}$

42. Answer (2)

$\alpha = \frac{46 - 38.3}{38.3} = 0.2$

$2\alpha = 2 \times .2 = 0.4$

43. Answer (2)

$A + B \rightleftharpoons C + D$

4   4   0   0

4 – x   4 – x   x   x

x = 2

$K = \frac{2 \times 2}{2 \times 2} = 1$

44. Answer (4)

$2HI \rightleftharpoons H_2 + I_2$ | $K = \frac{0.5 \times 0.5}{(2 - 2 \times 0.5)^2} = 0.25$

$(2–2\alpha)$ α α

45. Answer (1)

Fact

46. Answer (1)

$[NH_4^+] = [NH_2^-] = \sqrt{KNH_3} = 10^{-15}$ M

1 mm^3 solution contains = $6.02 \times 10^{23} \times 10^{-15} \times 10^{-6}$

= 602

47. Answer (1)

$[H^+]_{mix} = \sqrt{1.8 \times 10^{-4} \times 0.1 + 3.1 \times 10^{-4} \times 0.1}$

$= \sqrt{4.9 \times 10^{-5}} = 7 \times 10^{-3}$

48. Answer (3)

Buffer capacity = $\frac{2}{3.4 - 2.9} = 4$

49. Answer (4)

pH = $-\log_{10}(2 \times 0.05) = 1$

50. Answer (2)

Fact

51. Answer (4)

$\Delta H = -20.6 - 8.8 = -29.4$ J

52. Answer (2)

$2KCN + AgNO_3 \longrightarrow [Ag(CN)_2]^- + KNO_3 + K^+$

(.1 – 0.06)   0   0.03   0.03   0.03

$[Ag(CN)_2]^- \rightleftharpoons Ag^+ + 2CN–$

(0.03 – a)   a   (0.04 + a)

$K_C = \frac{a \times (0.04)^2}{0.03} = 4 \times 10^{-19} [0.03 - a \approx a \text{ as } K_C = 4 \times 10^{-19}]$

53. Answer (1)

$2AB_2(g) \rightleftharpoons 2AB(g) + B_2(g)$

2 – 2x   2x   x

$$K_P = \frac{\left(\frac{2x}{2+x}\right)P^2 \times \left(\frac{x}{2+x}\right)P}{\frac{(2-2x)^2}{2+x}P^2} = \frac{x^3P}{(2+x)(1-x)^2}$$

54. Answer (1)

Number of meq of $H^+ = 10 \times 1 + 20 \times 2 = 50$

Number of meq of $OH^- = 30 \times 1 = 30$

Number of meq of H^+ left = 50 – 30 = 20

55. Answer (3)

56. Answer (3)

Fact

57. Answer (2)

58. Answer (3)

59. Answer (2)

$V_{rms} \propto \sqrt{T}$

Collision frequency $\propto \sqrt{T}$

60. Answer (1)

$XeO^{-4} + 2F^- + 6H^+ \longrightarrow XeO_3^{+6} + F_2^{\,0} + 3H_2O$

[MATHEMATICS]

61. Answer (3)

Here product of roots $= \frac{(m^2+1)^2}{m^2+1} = m^2 + 1 \geq 1$

$\therefore$ b = least value of product of roots = 1

and sum of roots $= -\frac{(-3)}{m^2+1} = \frac{3}{m^2+1}$

$\therefore$ Greatest value of sum of roots is possible if $m^2 + 1$ is minimum and its minimum value = 1.

$\therefore$ Greatest value of sum of roots $= \frac{3}{1} = 3 = c$ (given)

$\therefore$ 1st term, $a = b + 2 = 1 + 2 = 3$

and $c.r$, $r = 2c^{-1} = \frac{2}{c} = \frac{2}{3} < 1$

$\therefore$ $S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{2}{3}} = 9.$

62. Answer (1)

A.M. $\geq$ G.M.

$$\frac{\frac{a}{b}+\frac{b}{c}+\frac{c}{a}}{3} \geq (1)^{1/3}$$

$\Rightarrow \frac{a}{b}+\frac{b}{c}+\frac{c}{a} \geq 3$

63. Answer (2)

$$\frac{S_n}{S_n^1} = \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]}$$

$$= \frac{7n+1}{4n+17}$$

$$\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+17}$$

Put $n = 19$

Required ratio = $\frac{134}{93}$

$\therefore$ They are in G.P.

64. Answer (1)

Here $\frac{a^2+b^2}{2} \ge (a^2b^2)^{\frac{1}{2}}$

$\Rightarrow a^2 + b^2 \ge 2ab$ ...(1)

$b^2 + c^2 \ge 2bc$ ...(2)

and $c^2 + a^2 \ge 2ca$ ...(3)

Adding (1), (2) and (3), we get

$2(a^2+b^2+c^2) \ge 2(ab+bc+ca)$

$\Rightarrow 1 \ge \frac{ab+bc+ca}{a^2+b^2+c^2} \quad \Rightarrow \frac{ab+bc+ca}{a^2+b^2+c^2} \le 1$ ...(4)

Also, $a < b + c$ (for any Δ)

$\Rightarrow a\cdot a < a(b+c)$ $[\because \ a > 0]$

$\Rightarrow a^2 < ab + ca$ ...(5)

Similarly, $b^2 < b(c + a) \Rightarrow b^2 < bc + ab$ ...(6)

and $c^2 < c(a + b) \Rightarrow c^2 < ca + bc$ ...(7)

By [(5), (6) & (7)]

$a^2 + b^2 + c^2 < 2(ab + bc + ca)$

$\Rightarrow \frac{1}{2} < \frac{ab+bc+ca}{a^2+b^2+c^2}$ ...(8)

$\therefore$ From (4) and (8) $\frac{1}{2} < \frac{ab+bc+ca}{a^2+b^2+c^2} \le 1.$

65. Answer (1)

From given expression,

$(15x)^2+(5y)^2+(3z)^2-15x.5y-5y.3z-15x.3z=0$

$\Rightarrow (15x - 5y)^2 + (5y - 3z)^2 + (3z - 15x)^2 = 0$

$\therefore 15x = 5y = 3z$

$\Rightarrow \frac{x}{\frac{1}{15}} = \frac{y}{\frac{1}{5}} = \frac{z}{\frac{1}{3}} = k$, say

$\therefore x = \frac{k}{15}, y = \frac{k}{5}, z = \frac{k}{3}$

$\therefore x+z = \frac{k}{15}+\frac{k}{3} = \frac{k+5k}{15} = \frac{6k}{15} = \frac{2k}{5} = 2y$

$\Rightarrow$ x, y, z are in A.P.

66. Answer (1)

Order of letter are ELMOUV 601th word will be VELMOU

67. Answer (1)

Let a, b, c, d represent 4 persons and x_1, x_2, x_3, x_4, x_5 denote the number of vacant seats on either side of the persons as shown below

$x_1(a)x_2\ (b)x_3\ (c)\ x_4\ (d)x_5$

We have $x_1 + x_2 + x_3 + x_4 + x_5 = 12 - 4 = 8$

where $x_1, x_5 \ge 0$ and $x_2, x_3, x_4 \ge 1$

$x_2 - 1 \ge 0, x_3 - 1 \ge 0, x_4 - 1 \ge 0$

Let $x_2 - 1 = p, x_3 - 1 = q, x_4 - 1 = r$

$\Rightarrow x_2 = p+1, x_3 = q+1, x_4 = r+1$

so that $p \ge 0, q \ge 0, r \ge 0$

$\therefore x_1 + p + 1 + q + 1 + r + 1 + x_5 = 8$

$\Rightarrow x_1 + p + q + r + x_5 = 5$ ...(i)

where each of $x_1, p, q, r, x_5 \ge 0$

$\therefore$ Number of non-negative integral solutions of equation (1)

$= {}^{5+5-1}C_{5-1} = {}^9C_4.$

Also 4 persons can be arranged amongst themselves in 4! ways.

$\therefore$ Required number of seating arrangements

$$= 4!\times {}^9C_4 = 4!\times\frac{9!}{4!\times 5!} = \frac{9.8.7.6.5!}{5!} = 3024.$$

68. Answer (1)

Let the 4 persons be given a, b, c, d things respectively.

Then $a + b + c + d = 16$,

where $a, b, c, d \ge 3$ ...(1)

$\Rightarrow a - 3 \ge 0, b - 3 \ge 0, c - 3 \ge 0, d - 3 \ge 0$

Put $a - 3 = x, b - 3 = y, c - 3 = z, d - 3 = t$

$\Rightarrow a = 3 + x, b = 3 + y, c = 3 + z, d = 3 + t$

$\therefore$ from (1) $3 + x + 3 + y + 3 + z + 3 + t = 16$

$\Rightarrow x + y + z + d = 4, x, y, z, t \ge 0$

$\therefore$ Required number = number of ways of distributing 4 identical things among 4 persons when each person can get zero or more things

$$= {}^{4+4-1}C_{4-1} = {}^7C_3 = \frac{7!}{3!\times 4!} = \frac{7.6.5.4!}{6.4!} = 35$$

69. Answer (2)

$4n + 2 = 2(2n + 1)$

Number of divisors of the form $4n + 2$ is

$1 \times 4 \times 4 \times 6 = 96$

but $n \in N$, So required number = 96 – 1 = 95

70. Answer (2)

Twenty pearls ≡ 10 pearls of one colour and 10 pearls of another colour.

Step-1: First arrange pearls of same colour in necklace in $\frac{1}{2}(10-1)! = \frac{1}{2}\times 9!$.

Step-2: Now arrange pearls of another colour in between the arranged 10 pearls in 10! ways.

∴ Required number of ways $= \frac{1}{2}\times 9! \times 10!$

$$= \frac{1}{2}\times 9! \times 10 \times 9! = 5\times(9!)^2$$

71. Answer (1)

Let one real root be α and other root be purely imaginary as $i\beta$.

Now α and $i\beta$ are the roots of equation

$x^2 + ax + b = 0$

∴ $\alpha + i\beta = -a$ ...(1)

and $\alpha i\beta = b$ ...(2)

Now from (1) $\overline{\alpha + i\beta} = -\bar{a}$

⇒ $\alpha - i\beta = -\bar{a}$ ...(3)

Adding (1) and (3)

$2\alpha = -a - \bar{a} = -(a+\bar{a})$ ...(4)

Subtracting (3) from (1)

$2i\beta = -a + \bar{a} = -(a-\bar{a})$ ...(5)

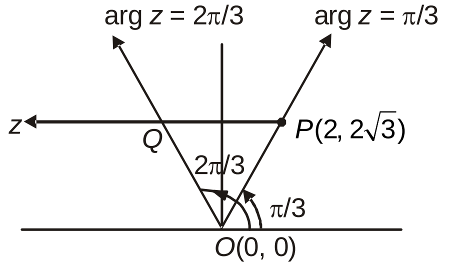
Multiplying (4) and (5)

$4i\alpha\beta = (a^2 - \bar{a}^2)$

$\Rightarrow 4b = a^2 - \bar{a}^2$ [from (3)]

72. Answer (1)

The rays represented by these equations are drawn in the following figure:



In the above figure, ΔOPQ is equilateral

Whose one side $OP = \sqrt{(2-0)^2 + (2\sqrt{3}-0)^2} = 4$

∴ Area of bounded region as ΔOPQ

$$= \frac{\sqrt{3}}{4}\times(\text{side})^2 = \frac{\sqrt{3}}{4}\times 16 = 4\sqrt{3}\text{ sq. units}$$

73. Answer (1)

Let $z = r\cos\theta + ir\sin\theta$

So that $|z| = r \neq 0$ and arg $z = \theta$

$$\therefore \quad \frac{z}{|z|} = \frac{r(\cos\theta + i\sin\theta)}{r} = \cos\theta + i\sin\theta$$

$$\Rightarrow \frac{z}{|z|} - 1 = (\cos\theta - 1) + i\sin\theta$$

$$\Rightarrow \left|\frac{z}{|z|} - 1\right| = \sqrt{(\cos\theta-1)^2 + \sin^2\theta} = \sqrt{2-2\cos\theta}$$

$$= \sqrt{4\sin^2\frac{\theta}{2}} = 2\left|\sin\frac{\theta}{2}\right|$$

$$\Rightarrow \left|\frac{z}{|z|} - 1\right| = 2\left|\sin\frac{\theta}{2}\right| \le 2\left|\frac{\theta}{2}\right| = |\theta|$$

$$[\because \sin\theta \le \theta \Rightarrow |\sin\theta| \le |\theta|]$$

$$\Rightarrow \left|\frac{z}{|z|} - 1\right| \le |\arg z|$$

74. Answer (1)

Since $\cos\alpha + i\sin\alpha$ is the root of the given equation,

we have $\sum_{k=0}^{n} p_k(\cos\alpha + i\sin\alpha)^{n-k} = 0$ [where $p_0 = 1$]

$$\Rightarrow (\cos\alpha + i\sin\alpha)^n \sum_{k=0}^{n} p_k(\cos\alpha + i\sin\alpha)^{-k} = 0$$

$$\Rightarrow \sum_{k=0}^{n} p_k(\cos\alpha k - i\sin\alpha k) = 0$$

$$\Rightarrow \sum_{k=0}^{n} p_k \sin k\alpha = 0$$

⇒ $0 + p_1\sin\alpha + p_2\sin2\alpha + ... + p_n\sin n\alpha = 0$.

75. Answer (1)

By $c_1 \to c_1 + c_2 + c_3$

$$= \begin{vmatrix} x+(1+\omega+\omega^2) & \omega & \omega^2 \\ \omega + x + \omega^2 + 1 & x+\omega^2 & 1 \\ \omega^2 + 1 + x + \omega & 1 & x+\omega \end{vmatrix}$$

$$= \begin{vmatrix} x & \omega & \omega^2 \\ x & x+\omega^2 & 1 \\ x & 1 & x+\omega \end{vmatrix} [\because 1+\omega+\omega^2 = 0]$$

$$= x(x^2 + x\omega + \omega^2 x + 1 - 1) - x(\omega x + \omega^2 - \omega^2)$$

$$+ x(\omega - \omega^2 x - \omega)$$

$= x(x^2 - x) - x^2(\omega + \omega^2) = x^3 - x^2 + x^2 = x^3.$

76. Answer (2)

$(1 + x^3 + 3x^2 + 3x)^{10} = (1 + x)^{30}$

Largest binomial coeff. = ${}^{30}C_{15}$

77. Answer (3)

$$3^{51} = 3.3^{50} = 3.(3^2)^{25} = 3(1+8)^{25}$$

$$= 3[1 + {}^{25}C_1.8 + {}^{25}C_2.8^2 + ... + {}^{25}C_{25}.8^{25}]$$

$$= 3 + 3.8[{}^{25}C_1 + {}^{25}C_2.8 + ... + {}^{25}C_{25}.8^{24}]$$

$$= 3 + 24K, \text{ where } K = {}^{25}C_1 + {}^{25}C_2.8 + ... + {}^{25}C_{25}.8^{24}$$

= an integer.

$\therefore$ 3^{51} when divided by 24 leaves the remainder 3.

78. Answer (3)

Given, $(1 + ax)^n = 1 + 8x + 24x^2 + ...$

$$\Rightarrow 1 + n(ax) + \frac{n(n-1)}{2!}a^2x^2 + ... = 1 + 8x + 24x^2 + ...$$

Comparing, we get

$$na = 8 \quad \Rightarrow a = \frac{8}{n}$$

and $\dfrac{n(n-1)}{2!}a^2 = 24 \Rightarrow \dfrac{n(n-1)}{2} \times \dfrac{64}{n^2} = 24$

$$\Rightarrow \frac{(n-1)}{n} = \frac{24}{32} = \frac{3}{4} \quad \Rightarrow 1 - \frac{1}{n} = \frac{3}{4}$$

$$\Rightarrow 1 - \frac{3}{4} = \frac{1}{n}$$

$\therefore$ $n = 4$

$$\therefore \quad a = \frac{8}{n} = \frac{8}{4} = 2$$

$$\therefore \quad \frac{a-n}{a+n} = \frac{2-4}{2+4} = -\frac{2}{6} = -\frac{1}{3}$$

79. Answer (3)

Here $t_{r+1} = {}^{15}C_r.3^{\frac{15-r}{5}}.2^{\frac{r}{3}}$

which will be rational if $3 - \dfrac{r}{5}$ and $\dfrac{r}{3}$ are integers

Also, $0 \le r \le 15$

$\therefore$ r must be a multiple of (l.c.m. of 3 and 5)

$\Rightarrow$ r must be multiple of 15

$\therefore$ $r = 0, 15$

$\therefore$ Sum of rational terms

$$= {}^{15}C_0.3^3.2^0 + {}^{15}C_{15}.3^0.2^5 = 1.27.1 + 1.1.32 = 59$$

80. Answer (3)

Given expression $= (1+x)^{2m}.\left(\dfrac{x}{1-x}\right)^{-2m}$

$$= \frac{1}{x^{2m}}\left[(1+x)(1-x)\right]^{2m} = \frac{1}{x^{2m}}.(1-x^2)^{2m}$$

Here, $t_{r+1} = \dfrac{1}{x^{2m}}.{}^{2m}C_r.(1)^{2m-r}.(-x^2)^r$

$$= \frac{1}{x^{2m}}.{}^{2m}C_r.1.(-1)^r.x^{2r}$$

$$= {}^{2m}C_r.(-1)^r.x^{2r-2m}$$

For the term independent of x, $2r - 2m = 0$

$\Rightarrow \boxed{r = m}$

$\therefore$ Term independent of $x = (-1)^m.{}^{2m}C_m$.

81. Answer (1)

Here determinant of order 2 will be of the form given below.

$$\begin{vmatrix} \square & \square \\ \square & \square \end{vmatrix}$$

and each of the four places can be filled up by 0 or 1 in 2 ways.

so that $n(S) = 2 \times 2 \times 2 \times 2 \times 2 = 16$

Let E = the event of getting non-negative determinants.

Then E^1 = the event of getting negative determinates.

But negative determinants of the second order that can be made with 0, 1 are

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\therefore \; P(E^1) = \frac{n(E')}{n(S)} = \frac{3}{16}$$

$$\therefore \; P(E) = 1 - P(E') = 1 - \frac{3}{16} = \frac{13}{16}.$$

82. Answer (3)

$P(E_n)$ = probability of the die showing $n = kn$

$$\therefore \quad P(E_1) + P(E_2) + ... + P(E_n) = 1$$

$$\Rightarrow \quad k(1+2+3+4+5+6) = 1$$

$$\Rightarrow \quad k = \frac{1}{\left(\frac{6.7}{2}\right)} = \frac{1}{21}$$

$$\therefore P(E_3) = k.3 = \frac{1}{21} \times 3 = \frac{1}{7}$$

83. Answer (1)

$$n(E) = {}^3C_2 \times 2 \times \frac{10!}{3!3!4!}$$

$$P(E) = \frac{8}{11}$$

84. Answer (2)

$Z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

$= \frac{3-4\sin^2\theta}{1+2\sin^2\theta} + i\frac{8\sin\theta}{1+4\sin^2\theta}$, Z is paralell

$\Rightarrow$ $\sin\theta = 0$

$\theta = n\pi, n \in Z$

85. Answer (3)

As there are three letters alike out of six letters and the three others are different.

Selections can be made in the following ways:

(i) All different ${}^4C_4 = 1$

(ii) 2 alike, 2 different ${}^1C_1 \times {}^3C_2 = 3$

(iii) 3 alike and 1 different ${}^1C_1 \cdot {}^3C_1 = 3$

$\therefore$ Total number of ways of selections

$= 1 + 3 + 3 = 7.$

86. Answer (1)

We have $n(n + 1) = n^2 + n < n^2 + n + n + 1 \ \forall\ n \geq 2$

$\Rightarrow n(n+1) < (n+1)^2$

$\Rightarrow \sqrt{n(n+1)} < (n+1)\ \forall n \geq 2$ ...(1)

$\Rightarrow$ Statement-2 is true.

Also from (1)

$\sqrt{n} < \sqrt{n+1}$

$\Rightarrow \frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}, n \geq 2$

$\therefore \frac{1}{\sqrt{1}} > \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}, ... \forall n \geq 2$

Adding all of them, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} > \frac{n}{\sqrt{n}}$$

$\therefore$ Statement-1 is also true and statement-2 is the correct explanation for statment-1.

87. Answer (4)

Variance of first n even natural numbers

$$= \left[\frac{1}{n}\Sigma x_i^2\right] - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \frac{1}{n}[2^2 + 4^2 + 6^2 + ... + \text{to } n \text{ terms}]$$

$$- \left(\frac{2+4+6+...+\text{to } n \text{ terms}}{n}\right)^2$$

$$= \frac{1}{n} \times 2^2(1^2 + 2^2 + 3^2 + ... \text{ to } n \text{ terms})$$

$$- \left[\frac{2(1+2+3+...+n)}{n}\right]^2$$

$$= \frac{1}{n} \times 4 \times \frac{n(n+1)(2n+1)}{6} - \left[\frac{2n(n+1)}{n2}\right]^2$$

$$= \frac{2}{3}(n+1)(2n+1) - (n+1)^2$$

$$= \frac{(n+1)}{3}[2(2n+1) - 3(n+1)]$$

$$= \frac{(n+1)}{3}(n-1) = \frac{n^2-1}{3}$$

$\therefore$ Statement-1 is false.

But statement-2 is true.

88. Answer (2)

For unique solution

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$$

where $a, b, c, d \in \{0, 1\}$

$\therefore$ Total number of cases = 2 × 2 × 2 × 2 = 16

Since each of a, b, c or d can take any one of two values 0 or 1.

Favourable number of cases = 6 (either ad = 1, bc = 0, or ad = 0, bc = 1)

$\therefore$ Probability that system of equations has unique solution $= \frac{6}{16} = \frac{3}{8} \Rightarrow$ statment-1 is true.

Since x = 0, y = 0 satisfy both the equations.

$\Rightarrow$ The system has atleast one solution.

$\therefore$ Statement-2 is also correct, but it is not the correct explanation for statement-1.

89. Answer (4)

Sum to n terms of AP is

$$S_n = \frac{n}{2}[2a + (n-1)d] = na + \frac{n}{2}(n-1)d$$

$$= na + n^2.\frac{d}{2} - n.\frac{d}{2} = n\left(a - \frac{d}{2}\right) + \left(\frac{d}{2}\right)n^2$$

which is of the form $pn^2 + qn$

Hence statment-2 is true.

$\therefore$ Statment-1 is false

90. Answer (1)

Since coefficient of x^r in $(1 - x)^{-n} = {}^{n+r-1}C_r$

$\therefore$ Coefficient of x^n in $(1 - x)^{-2}$

$$= {}^{2+n-1}C_n = {}^{n+1}C_n = \frac{(n+1)!}{n!1!} = n + 1$$

$\therefore$ Statment-2 is correct and statement-1 is also correct and statement-2 is the correct explanation for statement-1.