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## ULTIMATE TEST SERIES JEE-MAIN-2017

### TEST - 01, 01-03-2017

### [PHYSICS]

1. Answer (3)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = -\left[\frac{1}{0.90} + \frac{1}{0.45}\right] \Rightarrow \frac{1}{f} = -\frac{10}{3} \text{ m}$$

$$\Rightarrow f = -0.3 \text{ m}$$

$$-\frac{\Delta f}{f^2} = -\frac{\Delta v}{v^2} - \frac{\Delta u}{u^2} \Rightarrow \Delta f = f^2 \left[ \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} \right]$$

$$\Rightarrow \Delta f = \frac{3}{10} \times \frac{3}{10} \left[ \frac{0.01}{0.45 \times 0.45} + \frac{0.01}{0.90 \times 0.90} \right]$$

$$= \frac{3}{10} \times \frac{3}{10} \times \frac{1}{45} \times \frac{100}{45} \times \frac{5}{4} = 0.006 \text{ m}$$

$$\Rightarrow f = [0.3 \pm 0.006] \text{ m}$$

2. Answer (2)

$$h = \frac{1}{2}gt^2 \quad \dots(1)$$

$$\frac{h}{2} = \frac{1}{2}g(t-1)^2 \quad \dots(2)$$

Solving,  $t = 3.41 \text{ sec.}$

3. Answer (4)

$$200 - 120 = 12a_1 \Rightarrow a_1 = \frac{80}{12} = \frac{20}{3} \text{ ms}^{-2} \uparrow$$

$$200 - 150 = 15a_2 \Rightarrow a_2 = \frac{50}{15} = \frac{10}{3} \text{ ms}^{-2} \uparrow$$

$$a_p = \frac{\frac{20}{3} + \frac{10}{3}}{2} = 5 \text{ ms}^{-2} \uparrow$$

4. Answer (3)

For equilibrium,

$$F = -\frac{dU}{dx} = 0 = \frac{d}{dx}(x^2 - 6x) = 2x - 6 \Rightarrow x = 3 \text{ m}$$

At  $x = 3 \text{ m}$ ,  $U = 9 - 18 = -9 \text{ joule}$

$$E = K + U$$

$$11 = K + (-9) \Rightarrow K = 20 \text{ J}$$

5. Answer (2)

$$V_{\max} = \sqrt{gr \tan(\theta + \lambda)} = \sqrt{gr \frac{(\tan\theta + \tan\lambda)}{(1 - \tan\theta \tan\lambda)}}$$

$$= \sqrt{gr \frac{(\tan\theta + \mu)}{[1 - \tan\theta \times \mu]}} = \sqrt{\frac{10 \times 2 \left(\frac{3}{4} + \frac{1}{2}\right)}{\left(1 - \frac{1}{2} \times \frac{3}{4}\right)}}$$

$$= \sqrt{200 \times \frac{5 \times 8}{4 \times 5}} = \sqrt{400} = 20 \text{ ms}^{-1}$$

6. Answer (4)

$$x = 4t^2 - 3t \Rightarrow v = 8t - 3 \Rightarrow a = 8 \text{ ms}^{-2}$$

$$\Rightarrow \boxed{F = 16\text{N}} \Rightarrow dx = (8t - 3)dt$$

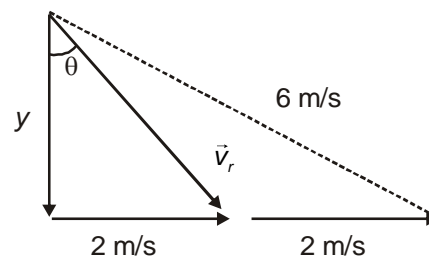
$$W = \int dW = \int Fdx = \int_0^t 16(8t - 3)dt = 16(4t^2 - 3t)$$

$$t = \frac{3}{4} \text{ s}, W = 0$$

$$\text{For } W \text{ to be minimum, } \frac{dW}{dt} = 0 \Rightarrow t = \frac{3}{8} \text{ s}$$

$$\frac{d^2W}{dt^2} = 8, \frac{d^2W}{dt^2} > 0$$

7. Answer (4)



$$\frac{2}{y} = \tan\theta$$

$$y = 2 \cot\theta$$

$$\text{Now, } \sqrt{y^2 + 16} = 36 \Rightarrow 4 \cot^2\theta + 16 = 36$$

$$\Rightarrow \cot^2\theta = 5 \Rightarrow \theta = \cot^{-1}(\sqrt{5})$$



8. Answer (2)

$$3kx_1 = 2kx_2 = kx_3 \Rightarrow 3x_1 = 2x_2 = x_3 \quad \dots(1)$$

$$x = x_1 + x_2 + x_3$$

$$\Rightarrow x = x_1 + \frac{3}{2}x_1 + 3x_1 \Rightarrow x = \frac{11x_1}{2}$$

$$\Rightarrow x_1 = \frac{2}{11}x.$$

9. Answer (1)

10. Answer (3)

$$2T - mg = ma_1 \uparrow \quad \dots(1)$$

$$Mg - T = Ma_2 \downarrow \quad \dots(2)$$

$$a_{\text{rel}} = a_1 + a_2 \quad \dots(3)$$

$$a_2 = 2a_1 \quad \dots(4)$$

$$\text{Solving, } a_1 = \frac{3g}{21}$$

$$\text{Then } a_{\text{rel}} = \frac{3g}{21} + \frac{6g}{21} = \frac{9}{21}g = \frac{3g}{7}$$

$$\text{Now, } l = \frac{1}{2}a_{\text{rel}} \cdot t^2 \Rightarrow l = \frac{1}{2} \times \frac{3g}{7} \cdot t^2 \Rightarrow t = \sqrt{\frac{14g}{3l}}$$

11. Answer (4)

$$F = -\frac{dU}{dx} \Rightarrow \frac{dU}{dx} = -F$$

$$\Rightarrow \text{For equilibrium, } F = 0 \Rightarrow \frac{dU}{dx} = 0$$

For stable equilibrium,

$$\frac{d^2U}{dx^2} > 0 \Rightarrow \frac{d}{dx} \left( \frac{dU}{dx} \right) > 0 \Rightarrow \frac{d}{dx}(-F) > 0 \Rightarrow -\frac{dF}{dx} > 0$$

$$\Rightarrow \frac{dF}{dx} < 0 \text{ for unstable equilibrium, } \frac{dF}{dx} > 1.$$

12. Answer (3)

$$\vec{v}_{\text{rel}} = (v - v \cos \theta) \hat{i} + v \sin \theta \hat{j} \Rightarrow |\vec{v}_{\text{rel}}|$$

$$= v \sqrt{(1 - \cos \theta)^2 + \sin^2 \theta} = 2v \sin \frac{\theta}{2}$$

$$|\vec{v}_{\text{rel}}| = \frac{\int_0^{2\pi} 2v \sin \theta / 2 d\theta}{2\pi \int_0^{2\pi} d\theta} = \frac{2v}{2\pi} = \frac{\left( -\cos \frac{\theta}{2} \right)_0^{2\pi}}{1} = \frac{1}{2}$$

$$= \frac{2v}{\pi} \left[ -\cos \frac{\theta}{2} \right]_0^{2\pi} = \frac{4v}{\pi}$$

13. Answer (2)

$$2.221$$

$$\times 1.2$$

$$\frac{2.6652}{2} = 2.7$$

Now,

$$238.523$$

$$\frac{2.7}{241.2}$$

Number of SF = 4

14. Answer (2)

$$t = \frac{s_{\text{rel}}}{v_{\text{rel}}} = \frac{l}{v + v \cos 60^\circ} = \frac{2l}{3v}$$

$$\text{Distance} = v \times t = v \times \frac{2l}{3v} = \frac{2l}{3} = \frac{2}{3} \times 3 = 2 \text{ m.}$$

15. Answer (4)

For no motion between blocks,

$$F_{\text{max}} = \mu_2(m_1 + m_2)g = 0.4 \times 10 \times 10 = 40 \text{ N}$$

 Thus,  $F_{\text{max}} = 40 \text{ N}$ 

 As,  $F_{\text{max}} > F$  applied.

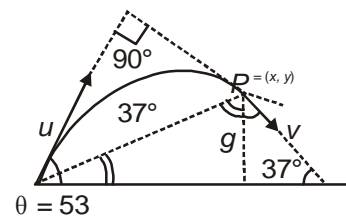
 Hence between blocks  $a_{\text{rel}} = 0$ 

$$\Rightarrow a_{\text{common}} = \frac{30}{10} = 3 \text{ ms}^{-2}$$

Now, for 4 kg

$$f = ma = 4 \times 3 = 12 \text{ N.}$$

16. Answer (3)


 When  $\vec{v} \perp \vec{u}$ 

$$t = \frac{u}{g \sin \theta} \quad \dots(1)$$

$$v = u \cot \theta \quad \dots(2)$$

$$\Rightarrow v = u \times \frac{3}{4} = 10 \times \frac{3}{4} = \frac{15}{2} \text{ ms}^{-1}$$

$$\text{At } P, \frac{v^2}{R} = g \cos 37^\circ \Rightarrow R = \frac{v^2}{g \cos 37^\circ}$$

$$\Rightarrow R = \frac{15}{2} \times \frac{15}{2} \times \frac{5}{10 \times 4}$$

$$R = \frac{225}{32} \text{ m}$$

17. Answer (4)

$$F_{\text{upon}} \text{ due to } B = (m_A + m_B)(g + a)$$

$$= (2 + 3)(10 + 2) = 60 \text{ N} \downarrow$$

$$\text{Displacement } s = ut + \frac{1}{2}at^2 = -5 \times t = \frac{1}{2} \times 2 \times t^2$$

$$= -5 \times 2 + \frac{1}{2} \times 2 \times 4 = -10 + 4 = 6 \text{ m} \downarrow$$

$$W = \vec{F} \cdot \vec{S} = 60 \times 6 = 360 \text{ J}$$

18. Answer (4)

$$\mu_s mg = m \sqrt{a_r^2 + \frac{v_m^4}{r^2}}$$

$$\Rightarrow \mu^2 g^2 = a_r^2 + \frac{v_m^4}{r^2}$$

$$\Rightarrow \frac{1}{4} \times 100 = 9 + \frac{v_m^4}{16}$$

$$\Rightarrow v_m^4 = 16 \times 16$$

$$\Rightarrow v_m = 4 \text{ ms}^{-1}$$

$$\text{Now, } a = \frac{vdv}{ds} \Rightarrow vdv = ads$$

$$\int_u^v vdv = \int_0^s ads$$

$$v^2 = u^2 + 2as$$

$$(4)^2 = (2)^2 + 2 \times 3 \times s$$

$$2 \times 6 = 6 \times s \Rightarrow \boxed{s = 2\text{m}}$$

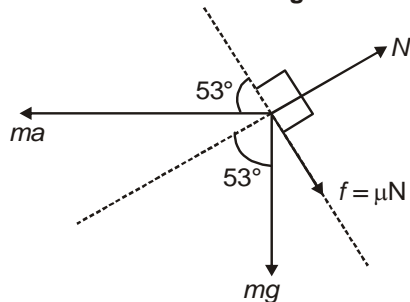
19. Answer (3)

$$N = mg \cos 53^\circ + ma \sin 53^\circ$$

$$N = mg \times \frac{3}{5} + ma \times \frac{4}{5} \quad \dots(1)$$

$$\mu N + mg \sin 53^\circ = ma \cos 53^\circ \quad \dots(2)$$

FBD of block w.r.t. wedge



$$0.1 \left[ \frac{3}{5} mg + \frac{4}{5} ma \right] + \frac{4}{5} mg = \frac{3}{5} ma$$

$$\frac{1}{10} [3mg + 4ma] + 4mg = 3ma$$

$$3mg + 4ma + 40mg = 30ma$$

$$\Rightarrow 43mg = 26ma$$

$$\Rightarrow \boxed{a = \frac{43}{26}g} \quad \dots(3)$$

$$\text{Now, } F = (M + m)a = 6 \times \frac{43}{26}g = \frac{129}{13}g \text{ N}$$

20. Answer (1)

$$T - mg = ma \quad \dots(1)$$

$$Mg - 31T = M \times \frac{a}{31} \quad \dots(2)$$

$$\text{Solving, } a = \frac{31g(M - 31m)}{M + 961m}$$

21. Answer (4)

From work-energy theorem,

$$W_F + W_{by} mg + W_{by} tr = \Delta K = 0$$

$$W_F - mgR \sin 60^\circ - \mu mg(R - R \cos 60^\circ) = 0$$

$$W_F = mgR \frac{\sqrt{3}}{2} + \frac{\mu mgR}{2} = \frac{mgR}{2} (\mu + \sqrt{3})$$

22. Answer (3)

$$F = -\frac{dU}{dx} = -\left[ \frac{d}{dx} \left( -\frac{2}{3}x^3 + \frac{3}{2}x^2 + 2x \right) \right]$$

$$= -\left[ -\frac{2}{3} \times 3x^2 + \frac{3}{2} \times 2x + 2 \right] = -[-2x^2 + 3x + 2]$$

$$F = 2x^2 - 3x - 2 \quad \dots(1)$$

$$F = 0 \Rightarrow 2x^2 - 3x - 2 = 0$$

$$\Rightarrow 2x^2 - 4x + x - 2 = 0$$

$$\Rightarrow 2x(x - 2) + 1(x - 2) = 0$$

$$x = 2, \quad x = -\frac{1}{2}$$

$$\text{Now, } \frac{dF}{dx} = 4x - 3 \quad \text{at } x = 2,$$

$$\frac{dF}{dx} = 4 \times 2 - 3 = 8 - 3 = 5$$

$$\frac{dF}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} < 0$$

Thus at  $x = 2$ , unstable equilibrium.

$$\text{At } x = -\frac{1}{2}, \frac{dF}{dx} = 4x - 3 = 4 \left( -\frac{1}{2} \right) - 3 = -5, \frac{dF}{dx} < 0$$

$$\Rightarrow \frac{d^2 y}{dx^2} > 0$$

Thus  $x = -\frac{1}{2}$ , stable equilibrium.

23. Answer (4)

Limiting friction between B and C =  $0.5 \times 4 \times 10 = 20 \text{ N}$

Maximum common acceleration =  $\frac{20}{4} = 5 \text{ ms}^{-2}$

$$\text{Now, } a_{\max} = \frac{m_{A\max} \cdot g}{m_A + 3 + 4} \Rightarrow 5 = \frac{m \times 10}{m + 7}$$

$$10m_A = 5m_A + 35 \Rightarrow m_A = 7 \text{ kg}$$

$$\text{When } m_A = 5 \text{ kg, } a = \frac{5 \times 10}{5 + 3 + 4} = \frac{50}{12} \text{ ms}^{-2}$$

$$\Rightarrow f = 4 \times \frac{50}{12} = \frac{50}{3} \text{ N}$$

24. Answer (4)

$$\text{At A, } T_A - mg = \frac{mv_A^2}{l} \Rightarrow T_A = mg + 6mg = 7mg$$

$$\text{At B, } T_B = \frac{mv_B^2}{l} = 4mg, T \text{ and C,}$$

$$T_C + mg - \frac{mv_C^2}{l} \Rightarrow T_C = m \times \frac{V_C^2}{l} - mg - 2mg - mg$$

$$\boxed{T_B = 4mg} \quad \boxed{T_A = 7mg} \quad \boxed{T_C = mg}$$

25. Answer (4)

$$\text{For UCM, } \vec{v} \perp \frac{d}{dt} \vec{v}$$

$$\text{For accelerated CM, } \vec{v} \cdot \frac{d}{dt} \vec{v} > 0$$

$$\text{For decelerated CM, } \vec{v} \cdot \frac{d}{dt} \vec{v} < 0.$$

26. Answer (4)

27. Answer (2)

28. Answer (3)

29. Answer (2)

30. Answer (1)

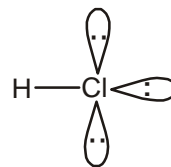
## [CHEMISTRY]

31. Answer (1)



Hence it belongs to 17<sup>th</sup> group.

32. Answer (3)



1 bond pair, 3 lone pair.

33. Answer (1)

NO (15) Bond order = 2.5

NO<sup>+</sup> (14) Bond order = 3

NO<sup>-</sup> (16) Bond order = 2

Hence, order of bond energy NO<sup>+</sup> > NO > NO<sup>-</sup>

34. Answer (2)

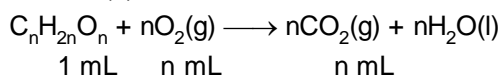
Let the uncertainty in momentum =  $x$

Uncertainty in position =  $4x$

$$\therefore 4x^2 = \frac{h}{4\pi} \Rightarrow x = \frac{1}{4} \sqrt{\frac{h}{\pi}}$$

Hence uncertainty in position =  $\sqrt{\frac{h}{\pi}}$ .

35. Answer (3)



$\therefore$  Contraction in volume =  $1 + n - n = 1 \text{ mL}$ .

36. Answer (4)

Fact.

37. Answer (2)

Electronic configuration (56) – [Xe]<sub>54</sub> 6s<sup>2</sup>

38. Answer (4)

Radius of n<sup>th</sup> shell =  $r_0 n^2$

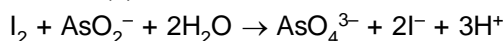
$$\Rightarrow r_0(n+1)^2 - r_0(n-1)^2 = 2r_0 n^2$$

$$\Rightarrow (n+1+n-1)(n+1-n+1) = 2n^2$$

$$\Rightarrow 2n \times 2 = 2n^2$$

$$\Rightarrow \boxed{n=2}$$

39. Answer (1)



m.eq. of HAsO<sub>2</sub> (in 50 mL) = m.eq. of I<sub>2</sub> =  $35 \times 0.05 \times 2 = 3.5$

$\therefore$  meq of HAsO<sub>2</sub> in 250 mL = 17.5

$$\text{weight of HAsO}_2 = \frac{17.5}{2} \times 108 \times 10^{-3} = 0.945 \text{ g}$$

$$\% \text{ of HAsO}_2 \text{ in the sample} = \frac{0.945}{3.78} \times 100 = 25\%$$

40. Answer (3)

Isodiaphers have same (n - p)



41. Answer (4)

 $\text{XeOF}_4$  is square pyramidal.

42. Answer (2)

 $\text{C} : \text{H} : \text{N} = 9 : 1 : 3.5$ 

Ratio by moles

$$\text{C} : \text{H} : \text{N} = \frac{9}{12} : \frac{1}{1} : \frac{3.5}{14} = \frac{3}{4} : 1 : 1 : \frac{1}{4} = 3 : 4 : 1$$

 $\text{C}_3\text{H}_4\text{N}$  is empirical formula.

$$n = \frac{108}{54} = 2$$

 Hence molecular formula  $\text{C}_6\text{H}_8\text{N}_2$ 

43. Answer (1)

Separation energy

$$= +R_H \frac{n^2}{Z^2} = 13.6 \times \frac{(2)^2}{(3)^2} = +6.04 \text{ eV}$$

 ( $\therefore$  for first excited state  $n = 2$ )

44. Answer (3)

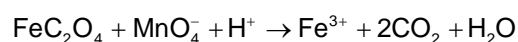
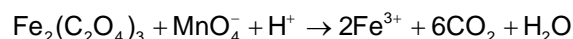
45. Answer (2)

$$2\pi r_n = n\lambda \Rightarrow 2\pi \times 0.53 \frac{n^2}{Z} = n\lambda$$

$$\Rightarrow \lambda = 2\pi \times 0.53 \times \frac{n}{Z}$$

$$E_{\text{sep}} = 3.4 = 13.6 \frac{Z^2}{n^2} \Rightarrow \frac{n}{Z} = 2$$

46. Answer (2)


 Total equivalents of  $\text{Fe}_2(\text{C}_2\text{O}_4)_3$  and  $\text{FeC}_2\text{O}_4$   
 = equivalents of  $\text{KMnO}_4$ 

 On solving,  $x = 0.9$  mole.

47. Answer (3)

 Limiting agent is  $\text{Na}_2\text{CO}_3$ .

 Now 106 g of  $\text{Na}_2\text{CO}_3$  gives 22.4 L of  $\text{CO}_2$ 

$$\therefore 5.3 \text{ g of } \text{Na}_2\text{CO}_3 \text{ gives } \frac{22.4}{106} \times 5.3 = 1.12 \text{ L}$$

48. Answer (4)

Fact.

49. Answer (3)

50. Answer (4)

51. Answer (3)

 No. of molecules in 1 gm  $\text{NH}_3$  is  $\frac{1}{17} \times N_A$ 

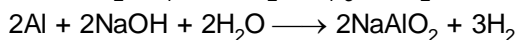
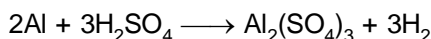
 No. of molecules in 1 gm  $\text{N}_2$  is  $\frac{1}{28} \times N_A$ 

52. Answer (2)

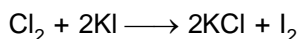
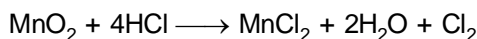
$$\frac{1}{\lambda} = 109700 \left( \frac{1}{2^2} - \frac{1}{4^2} \right) \text{ cm}^{-1}$$

$$\lambda = 486 \text{ nm}$$

53. Answer (1)

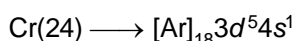


54. Answer (3)


 The number of moles of  $\text{MnO}_2$  equals that of  $\text{I}_2$ .

 Thus 25.4 of  $\text{I}_2$  is liberated.

55. Answer (1)


 $19^{\text{th}}$  electron is in 4s orbital hence 4, 0, 0,  $+\frac{1}{2}$ 

56. Answer (4)

57. Answer (1)

58. Answer (3)

59. Answer (1)

60. Answer (2)

## [MATHEMATICS]

61. Answer (2)

 $\sec A + \sec B$  is minimum when

$$A = B = \frac{\pi}{6}$$

 Hence minimum value =  $\frac{4}{\sqrt{3}}$ 

62. Answer (2)

 $\triangle ABC$  is isosceles.

 Let  $A$  and  $C$  are equal.

$$\Rightarrow 2 \tan A + \tan B = 100$$

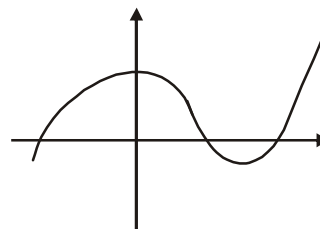
$$\Rightarrow 2x + \left( -\frac{2x}{1-x^2} \right) = 100$$

$$\Rightarrow 2x - 2x^3 - 2x = 100 - 100x^2$$

$$\Rightarrow 2x^3 - 100x^2 + 100 = 0$$

$$\Rightarrow x^3 - 50x^2 + 50 = 0$$

Hence there are 2 possible positive roots.



63. Answer (3)

$$S_n = \cos^n \theta + \sin^n \theta$$

$$S_4 = \cos^4 \theta + \sin^4 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta$$

$$= 1 - 2\sin^2 \theta \cos^2 \theta$$

$$S_6 = \cos^6 \theta + \sin^6 \theta$$

$$= (\cos^2 \theta)^3 + (\sin^2 \theta)^3$$

$$= (\cos^2 \theta + \sin^2 \theta)(\cos^4 \theta + \sin^4 \theta - \cos^2 \theta \sin^2 \theta)$$

$$S_6 = S_4 - \left(\frac{1 - S_4}{2}\right)$$

$$\Rightarrow S_6 = \frac{3S_4 - 1}{2}$$

$$\Rightarrow 3S_4 - 2S_6 = 1$$

64. Answer (1)

$$\sec \theta = m \text{ and } \tan \theta = n, \text{ then } \frac{1}{m} \left[ (m+n) + \frac{1}{(m+n)} \right]$$

$$\frac{1}{\sec \theta} \left[ (\sec \theta + \tan \theta) + \frac{1}{(\sec \theta + \tan \theta)} \right]$$

$$\cos \theta \left[ \frac{1}{(\sec \theta - \tan \theta)} + \frac{1}{\sec \theta + \tan \theta} \right]$$

$$\cos \theta \left[ \frac{\sec \theta + \tan \theta + \sec \theta - \tan \theta}{\sec^2 \theta - \tan^2 \theta} \right] = 2$$

65. Answer (4)

$$a^2 - 4a + 6$$

Its minimum value is 2.

 Hence,  $\sin x + \cos x = 1$ 

$$\text{General solution} \rightarrow n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$$

66. Answer (4)

$$2a^2 + 4b^2 + c^2 = 4ab + 2ac$$

$$(a - 2b)^2 + (a - c)^2 = 0$$

$$a = 2b, a = c$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{4b^2 + 4b^2 - b^2}{2(4b^2)}$$

$$= \frac{7b^2}{8b^2} = \frac{7}{8}$$

67. Answer (3)

$$\tan A = \frac{4}{3}$$

$$\cos A = \frac{3}{5}$$

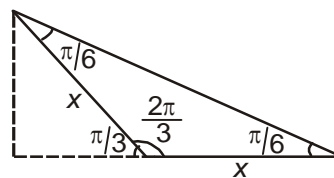
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(b+c)^2 - 2bc - a^2}{2bc}$$

$$= \frac{(61)^2 - 2(820) - a^2}{1640} \Rightarrow \frac{3}{5} = \frac{3721 - 1640 - a^2}{1640}$$

$$2081 - a^2 = 984 \Rightarrow a^2 = 2081 - 984 = 1097$$

68. Answer (3)



$$\frac{\sin \frac{\pi}{6}}{x} = \frac{\sin \frac{2\pi}{3}}{y} = \frac{1}{2x} = \frac{\sqrt{3}}{2y}$$

$$r = \frac{\Delta}{s} \quad y = \sqrt{3}x$$

$$\Rightarrow (2 - \sqrt{3}) = \frac{\frac{1}{2} \times x \times x \sin \frac{\pi}{3}}{\frac{2x + \sqrt{3}x}{2}}$$

$$\Rightarrow (2 - \sqrt{3})(2 + \sqrt{3})x = \frac{1}{2} \times x^2 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{2}{\sqrt{3}}$$

$$\text{Area} = \frac{1}{2} \times \frac{4}{3} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{\sqrt{3}}$$

69. Answer (4)

No. of one to one functions

$$= 6 \times 5 \times 4 \times 3$$

$$= 360$$

70. Answer (4)

$$f\left(\frac{3x+x^3}{1+3x^2}\right) = \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$$

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+x}{1-x}\right)^2 = 2f(x)$$

$$\Rightarrow f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right) = f(x)$$

71. Answer (2)

$$f(f(x)) = g(f(x))$$

$$\Rightarrow a(ax+b) + b = c(ax+b) + d$$

$$\Rightarrow a^2x + ab + b = cax + bc + d$$

$$a^2 = ac, ab + b = bc + d$$

$$a = 0 \quad \left| \quad a = c \right.$$

$$b = bc + d \quad \left| \quad b = d \right.$$

$$\Rightarrow f(b) = g(b) \quad \left| \quad \text{Not possible} \right.$$

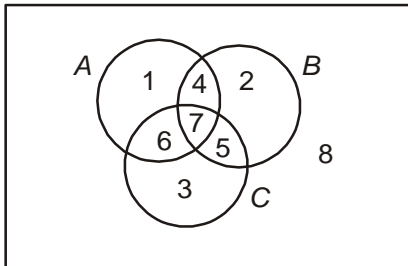
72. Answer (3)

$$f(x) = \{x\} - \frac{1}{2}$$

$$f(x) = \frac{1}{2}, \text{ when } \{x\} = 1$$

Not possible

73. Answer (4)



$$A - B = 1 + 6$$

$$B - C = 2 + 4$$

$$C - A = 3 + 5$$

$$(A - B) \cup (B - C) \cup (C - A) = 1 + 2 + 3 + 4 + 5 + 6$$

$$= (A \cup B \cup C) - (A \cap B \cap C)$$

74. Answer (1)

$$y = 41 - 2x \quad x, y \in \mathbb{N}$$

$$\text{Domain} \Rightarrow \{1, 2, 3, \dots, 20\}$$

Hence correct answer is (1)

75. Answer (4)

76. Answer (4)

$$[x]^2 - 5[x] + 6 = 0$$

$$([x] - 2)([x] - 3) = 0$$

$$[x] = 2, 3$$

$$x \in [2, 4)$$

77. Answer (2)

Case I

$$x \geq 1$$

$$x > 2\{x\}$$

$$\Rightarrow x \geq 1$$

Case II

$$x < 1$$

$$x < 2\{x\}$$

$$\Rightarrow x \in (-\infty, 0) \cup (0, 1)$$

$$\text{Taking union} \Rightarrow (-\infty, 0) \cup (0, \infty)$$

78. Answer (3)

$$\sin^3 x \sin 3x = C_1 \cos x + C_2 \cos 2x + \dots + C_n \cos nx$$

$$= \frac{(3 \sin x - \sin 3x) \sin 3x}{4}$$

$$= \frac{(3 \times 2)}{8} \sin x \sin 3x - \frac{1}{4} \sin^2 3x$$

$$= \frac{3}{8} (2 \sin x \sin 3x) - \frac{1}{4} \sin^2 3x$$

$$= \frac{3}{8} [\cos 2x - \cos 4x] - \frac{1}{4} \frac{(1 - \cos 6)}{2}$$

$$= \frac{3}{8} [\cos 2x] - \frac{3}{8} \cos 4x - \frac{1}{8} + \frac{1}{8} \cos 6x$$

 Hence  $n = 6$ 

79. Answer (1)

 $p$ : Both are decreasing functions

 Hence larger of the two is  $\cos(\ln \theta)$ 

$$\text{as } \cos\left(\ln e^{-\pi/2^+}\right) > \ln\left(\cos\left(e^{-\pi/2^+}\right)\right)$$

$$\text{and } \cos\left(\frac{\pi}{2}^+\right) > \ln\left(\cos\frac{\pi}{2}^+\right)$$

$$q: \cos x + \cos\left(\frac{2\pi}{3} - x\right) = \frac{3}{2}$$

$$\text{Hence } \cos x + \left(\left(-\frac{1}{2} \cos x\right) + \frac{\sqrt{3}}{2} \sin x\right) = \frac{3}{2}$$

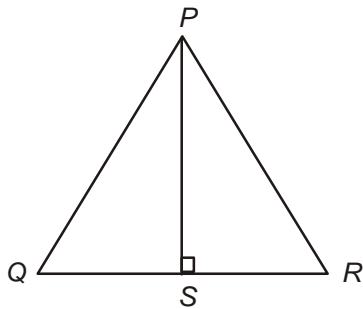
$$\Rightarrow \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = \frac{3}{2}$$

Never possible.

 Hence  $p$  and  $q$ , both are correct.

80. Answer (1)

81. Answer (2)



$\sin P, \sin Q, \sin R$  are in A.P.

$$2 \sin Q = \sin P + \sin R$$

$$2q = p + r$$

$$2 \times \frac{2\Delta}{A_q} = \frac{2\Delta}{A_p} + \frac{2\Delta}{A_r}$$

$$\frac{2}{A_q} = \frac{1}{A_p} + \frac{1}{A_r}$$

Altitudes are in H.P.

82. Answer (1)

$$f(x) = ax^7 + bx^3 + cx - 5$$

$$f(-7) = a(-7)^7 + b(-7)^3 + c(-7) - 5$$

$$\Rightarrow 7 = -a \cdot 7^7 - b \cdot 7^3 - c \cdot 7 - 5$$

$$\Rightarrow 7 = -(a \cdot 7^7 + b \cdot 7^3 + c \cdot 7) - 5$$

$$\Rightarrow 7 = -(f(7) + 5) - 5$$

$$\Rightarrow 7 = -f(7) - 10$$

$$\Rightarrow f(7) = -17$$

Range of  $f(7) + 17 \cos x$  is  $[-34, 0]$

83. Answer (3)

$$f(x) = \frac{e^x - e^{|x|}}{e^x + e^{|x|}}$$

$$x > 0$$

$$\frac{e^x - e^x}{e^x + e^x} = 0$$

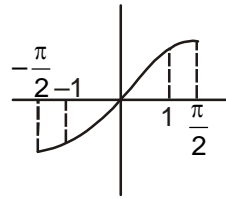
$$x < 0$$

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = 1 - \frac{2}{e^{2x} + 1}$$

$$\Rightarrow \text{Range} \Rightarrow (-1, 0]$$

84. Answer (2)

$$-1 \leq \cos x \leq 1$$



$\sin(1)$  maximum.

85. Answer (3)

$$\sqrt{2 + \sqrt{4 \cos^2 2\theta}} = \sqrt{2 + |2 \cos 2\theta|}$$

$$\pi < 2\theta < \frac{3\pi}{2} \Rightarrow \sqrt{2 - 2 \cos 2\theta} = \sqrt{4 \sin^2 \theta} = |2 \sin \theta|$$

$$\frac{\pi}{2} < \theta < \frac{3\pi}{4} \Rightarrow = 2 \sin \theta$$

86. Answer (3)

Statement-2 is false.

$$R = \frac{abc}{4\Delta} \quad r = \frac{\Delta}{S}$$

$$\frac{R}{r} = \frac{Sabc}{4\Delta^2} = \frac{4 \times 5 \times 6}{4 \left(\frac{15}{2} - 4\right) \left(\frac{15}{2} - 5\right) \left(\frac{15}{2} - 6\right)} = \frac{16}{7}$$

87. Answer (4)

$$f\left(\frac{2 \tan x}{1 + \tan^2 x}\right) = \frac{2}{2 \cos^2 x (1 + \tan^2 x + 2 \tan x)}$$

$$= \frac{1 + \tan^2 x}{1 + \tan^2 x + 2 \tan x}$$

$$= \frac{1}{1 + \frac{2 \tan x}{1 + \tan^2 x}}$$

$$f(\sin 2x) = \frac{1}{1 + \sin 2x}$$

$$f(x) = \frac{1}{1+x}, \quad f(x) \neq 0$$

88. Answer (4)

89. Answer (4)

90. Answer (1)

$$f(x) = a \sin^2 x + b \operatorname{cosec}^2 x$$

$$\frac{a \sin^2 x + \frac{b}{\sin^2 x}}{2} \geq \sqrt{ab}$$

$$a \sin^2 x + \frac{b}{\sin^2 x} \geq 2\sqrt{ab}$$

$$f_m(4, 9) = 2\sqrt{4 \times 9} = 12$$

$$f_m(9, 4) = 2\sqrt{4 \times 9} = 12$$