AVIRAL CLASSES
CREATING SCHOLARS
FOR JEE (Main & advance), PMTs & FOUNDATION

DATE : 18.12.2016

FINAL TEST SERIES JEE -2017
TEST-02

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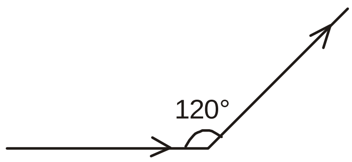
1. Answer (4)
No external force
$\therefore \quad V_{cm} = 0.$

2. Answer (1)
The force should be applied at centre of mass.

3. Answer (1)
Impulse = charge in momentum = mv_0



4. Answer (2)

$F < \mu mg$ and $F \times h > mg \times \dfrac{L}{2}$

$\Rightarrow \quad \mu mg > F > mg\dfrac{L}{2h} \qquad \Rightarrow \qquad \mu > \dfrac{L}{2h}$

5. Answer (4)

$\because \quad I \propto R^2 \qquad m = \text{vol} \times \rho$

$I \propto \dfrac{1}{\rho} \qquad = \text{area} \times \text{thickness} \times \rho$

$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{\rho_2}{\rho_1} = \dfrac{\pi_2}{\pi_1} \qquad \propto R^2.t.\rho$

$\Rightarrow R^2 \propto \dfrac{1}{\rho}$

6. Answer (4)

$mg\sin\theta(R-h) = mg\cos\theta \times \sqrt{R^2-(R-h)^2}$

$\tan^2\theta(R-h)^2 = R^2 - (R^2 + h^2 - 2Rh)$

$(R^2+h^2-2Rh)\tan^2\theta = 2Rh - h^2$

$R^2\tan^2\theta + h^2(1+\tan^2\theta) = 2Rh(1+\tan^2\theta)$

$R^2\sin^2\theta + h^2 = 2Rh$

$h^2 - 2Rh + R^2\sin^2\theta = 0$

$h = \dfrac{2R \pm \sqrt{4R^2 - 4R^2\sin^2\theta}}{2}$

$= R \pm R\sqrt{1-\sin^2\theta} \quad = R \pm R\cos\theta$

$= R[1 - \cos\theta]$

7. Answer (2)

$I_1 = \dfrac{ml^2}{12}$

$I_2 = m\left(\dfrac{l}{2\pi}\right)^2$

$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{4\pi^2}{12} = \dfrac{\pi^2}{3}$

8. Answer (1)

$p \times \dfrac{l}{4} = \dfrac{ml^2}{12} \times \omega$

$\Rightarrow \omega = \dfrac{3p}{ml}$

9. Answer (2)

$$mV_0R - mR^2 \times \frac{V_0}{2R} = mVR + mR^2\frac{v}{R}$$

$$\frac{mV_0R}{2} = 2mVR$$

$$\Rightarrow v = \frac{v_0}{4}$$

10. Answer (4)

$$K = \frac{L^2}{2I}$$

$$\frac{K_A}{K_B} = \frac{L_A^2}{L_B^2} \times \frac{I_B}{I_A} = 25 \times \frac{4}{1} = 100:1$$

11. Answer (1)

$$\frac{4}{3}\pi(R^3 - r^3) \times 8 \times g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$$8R^3 - 8r^3 = R^3$$

$7R^3 = 8r^3$

$$\frac{r}{R} = \left(\frac{7}{8}\right)^{1/3}$$

12. Answer (2)

$$2h \times \rho \times g + h \times 2\rho \times g = \frac{1}{2}(2\rho) \times v^2$$

$$4h\rho g = \rho v^2$$

$$\Rightarrow \quad v = 2\sqrt{gh}$$

13. Answer (4)

$$g = \frac{GM}{R^2} \qquad ...(1) \qquad r\omega_0 = \sqrt{\frac{GM}{r}}$$

or $GM = r^3\omega_0^2$ ...(2)

$$\therefore \; g = \frac{r^3\omega_0^2}{R^2}$$

14. Answer (1)

$$\frac{3}{5}g = g - R\omega^2$$

$$R\omega^2 = \frac{2}{5}g$$

15. Answer (1)

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\Rightarrow M = \frac{4\pi^2 r^3}{GT^2}$$

16. Answer (1)

$$-\frac{GMm}{R} + 0 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \; v = \sqrt{gR}$$

17. Answer (1)

$$K = |+E| = \frac{|U|}{2} \;\Rightarrow E = -\frac{1}{2}mv^2$$

18. Answer (3)

$$W_{\text{ext}} = \Delta K + \Delta U = \Delta ME = E_f - E_i$$

$$= -\frac{GMm}{10R} - \left(-\frac{GMm}{6R}\right) = \left(\frac{1}{6} - \frac{1}{10}\right)\frac{GMm}{R} = \frac{GMm}{15R}$$

19. Answer (3)

$v = 10\cos 2t + 2$

$v(t = 0) = 10 + 2) = 12$

$$v\left(t = \frac{\pi}{2}\right) = -8$$

$I = 5[-8 - 12] = -100$ kgms^{-1}

20. Answer (1)

$$x_{cm} = \frac{\int_0^l \lambda_0 x^5 .dx.x}{\int_0^l dx.\lambda_0 x^5} = \frac{\frac{l^7}{7}}{\frac{l^6}{6}} = \frac{6}{7}l$$

$$\therefore \quad \text{From end } B, l - \frac{6l}{7} = \frac{l}{7}.$$

21. Answer (1)

Angular velocity of rod about *IAOR* is

$$\omega = \frac{V_0}{\frac{l}{\sqrt{2}}} = \frac{\sqrt{2}V_0}{l}$$

22. Answer (3)

$$v_{cm} = \frac{v}{2}$$

$$\therefore \quad w = \frac{v}{2r}$$

$$\therefore \; a = rw^2 = \frac{v^2}{4}$$

23. Answer (4)

$$\left.\begin{aligned} 80 &= k(l_1 - l_0) \\ 100 &= k(l_2 - l_0) \end{aligned}\right\} \begin{aligned} &\text{...(i)} \\ &\text{...(ii)} \end{aligned} \Rightarrow \frac{4}{5} = \frac{l_1 - l_0}{l_2 - l_0}$$

$$4l_2 - 4l_0 - 5l_1 - 5l_0$$

$$l_0 = 5l_1 - 4l_2$$

$\therefore \quad 160 = k(l_3 - l_0)$ ...(iii)

Equation (iii) is divides by equation (i)

$$\Rightarrow 2 = \frac{l_3 - 5l_1 + 4l_2}{l_1 - 5l_1 + 4l_2}$$

$$\Rightarrow 2l_1 - 10l_1 + 8l_2 = l_3 - 5l_1 + 4l_2$$

$$l_3 = -3l_1 + 4l_2.$$

24. Answer (1)

$$\frac{1}{K} = \frac{\Delta P}{\Delta V / V}$$

$$\Rightarrow \quad \Delta P = \frac{1}{K} . \frac{\Delta V}{V} = \frac{0.2}{100} \times \frac{1}{4.4 \times 10^{-5}} = 45.4 \text{ atm}$$

25. Answer (4)

$$\tau = \frac{h\rho g}{2} \times bh \times \frac{h}{3}$$

26. Answer (4)

27. Answer (1)

28. Answer (1)

29. Answer (2)

30. Answer (4)

31. Answer (2)

$$Sb_2O_3 + IO_3^- + H^+ + H_2O \longrightarrow HSb(OH)_6 + I^-$$

Reduction

Oxidation

$$3Sb_2O_3 + 2IO_3^- + 21H_2O \longrightarrow 6H\,[Sb(OH)_6] + 2I^-$$

32. Answer (1)

$+3 + 2\,(+2) + 3x + 7\,(-2) = 0$

$7 + 3x - 14 = 0$

$x = +\dfrac{7}{3}$

33. Answer (2)

$$BaO_2 + H_2SO_4 \longrightarrow H_2O_2^{-1} + B\overset{-2}{a}SO_4$$

34. Answer (4)

$$8Al \longrightarrow 8Al^{+3} + 24e$$

$$9\,Fe^{8/3+} + 24e^- \longrightarrow 9Fe$$

35. Answer (1)

$$\frac{r_{CH_4}}{r_{gas}} = \sqrt{\frac{M_{gas}}{16}} \text{ or } 2 = \sqrt{\frac{M_{gas}}{16}}$$

36. Answer (2)

$$P_{H_2} = \frac{8}{1+8} P_{Total} = \frac{8}{9} P_{Total}$$

37. Answer (1)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$E = \frac{3}{2} RT$$

$$v_{rms} = \sqrt{\frac{2KE}{M}}$$

38. Answer (4)

39. Answer (4)

Fact

40. Answer (1)

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = \Delta U + RT$

$\Delta H > \Delta U$

41. Answer (2)

$K_p = K_c\,(RT)^{\Delta ng}$

42. Answer (3)

$\Delta ng = 1 - (2 + 1) = -2$

43. Answer (1)

$\Delta G° = -2.303 \times 2 \times 298 \log 10^{-8}$

$= +10.98$ Kcal

44. Answer (4)

Equilibrium constant depends on temperature only.

45. Answer (3)

$$\begin{array}{ccccccc|l} W & + & X & \rightleftharpoons & Y & + & Z & g = \dfrac{x^2}{(1-x)^2} \\ 1 & & 1 & & 0 & & 0 & \\ 1-x & & 1-x & & x & & x & x = 0.75 \end{array}$$

46. Answer (3)

$$\frac{1}{K_a} = \frac{K_a}{K_w} = \frac{10^{-4}}{10^{-14}} = 10^{10}$$

47. Answer (3)

$$\frac{7.9 \times 10^{-10}}{7 \times 10^{-10}} = \frac{[Sr^{+2}][F^-]^2}{[Sr^{+2}][CO_3^{-2}]}$$

$[F^-] = 3.7 \times 10^{-2}$

48. Answer (2)

$$\begin{array}{cccc|} Mg(OH)_2 & \rightleftharpoons Mg^{+2} & + & 2OH^- \\ & s & & 2s \end{array}$$

$$K_{sp} = (0.01) \times (2\,s)^2$$

$$2\,S = [OH^-] = 10^{-5}$$

49. Answer (2)

$$pOH = pk_b + \log\frac{[B^+]}{[BOH]}$$

$pk_a = 6$

$k_b = 10^{-6}$

50. Answer (1)

$$K_a = \frac{[H^+]^2[S^{-2}]}{[H_2S]}$$

$$10^{-21} = \frac{(0.1)^2 \times [S^{-2}]}{0.1}$$

$[S^{-2}] = 10^{-20}$ M

$= 10^{-20} \times 6.023 \times 10^{23} = 6.023 \times 10^{3}$

51. Answer (1)

K_a of $H_2S = 9 \times 10^{-20}$

$$[S^{-2}] = \frac{K_a[H_2S]}{[H^+]^2}$$

$[H^+] = 10^{-3}$

$$[S^{-2}] = \frac{9 \times 10^{-20} \times 0.1}{(10^{-3})^2} = 9 \times 10^{-15}$$

$[Mn^{+2}]\ (9 \times 10^{-15}) = 3 \times 10^{-22}$

$[Mn^{+2}] = 3.33 \times 10^{-8}$ M

52. Answer (1)

$S(s) \longrightarrow S(g)$ ...(1)

$\frac{1}{2}F2 \longrightarrow F(g)$ ...(2)

$SF_6(g) \longrightarrow S(g) + 6F(g)$ ...(3)

53. Answer (1)

$$Z = \frac{PV}{nRT}$$

$$\left(P + \frac{an^2}{V^2}\right)(V) = RT$$

54. Answer (2)

$m_1 = (148 - 50) = 98$ gm

$$V = \frac{m_1}{\rho} = \frac{98}{0.98} = 100 \text{ ml}$$

Mass of gases = (50.5 – 50) = 0.5

$$PV = nRT = \frac{W}{M}RT$$

M = 123 gm / mol

55. Answer (3)

$$\frac{x}{27/3} + \frac{(1.67-x)}{65.4/2} = \frac{1.69}{11.2}$$

$x = 1.239 \approx 1.24$ gm

56. Answer (3)

In very dilute solutions both source of H^+ ions from HCl and H_2O must be considered and due to common ion (H^+) supression of ionisation.

57. Answer (2)

Fact

58. Answer (2)

Graphite is more stable than diamond, hence its energy is lower than that of diamond and entropy of graphite is also lower than that of diamond.

59. Answer (1)

60. Answer (4)

Pb^{+2} is more stable than Pb^{+4} due to inert pair effect

$Pb^{+4} + 2e \longrightarrow Pb^{+2}$ (oxidising agent)

61. Answer (1)

Given, $|z_1 - z_0| = |z_2 - z_0| = a \Rightarrow \left|\dfrac{z_2 - z_0}{z_1 - z_0}\right| = 1$

$\Rightarrow \left|\dfrac{z_2 - z_0}{z_0 - z_1}\right| = 1$ and amp. $\left(\dfrac{z_2 - z_0}{z_0 - z_1}\right) = \dfrac{\pi}{2}$

$\therefore \dfrac{z_2 - z_0}{z_0 - z_1} = \left|\dfrac{z_2 - z_0}{z_0 - z_1}\right| e^{i\pi/2} = 1\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right) = i$

$\Rightarrow z_2 - z_0 = iz_0 - iz_1 \Rightarrow z_2 + iz_1 = (i + 1)z_0$

$\Rightarrow z_0 = \dfrac{z_2 + iz_1}{1+i} = \dfrac{(z_2 + iz_1)(1-i)}{1-i^2}$

$= \dfrac{1}{2} \quad \left[z_1(i - i^2) + (1-i)z_2\right]$

$= \dfrac{1}{2}\left[(1+i)z_1 + (1-i)z_2\right]$

62. Answer (1)

Let $z = a + b\omega + c\omega^2$

$\therefore |z|^2 = z\bar{z} = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \dfrac{1}{2}\ [(b - c)^2 + (c - a)^2 + (a - b)^2]$

$\therefore$ From question,

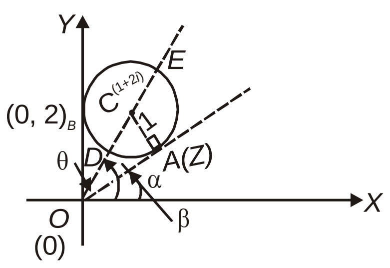
$x = |z| + |\bar{z}| = 2|z|$

$= \sqrt{2}\sqrt{[(a - b)^2 + (b - c)^2 + (c - a)^2}$

since a, b, c are distinct integers, then minimum value of $(b - c)^2 + (c - a)^2 + (a - b)^2$ is $1^2 + 1^2 + 2^2 = 6$

$\therefore$ Minimum value of $x = \sqrt{6}\,\sqrt{2} = 2\sqrt{3}$

63. Answer (4)



$\tan\beta = 2$

$\tan\theta = \dfrac{1}{2}$

$\tan\alpha = \left|\dfrac{2 - \frac{1}{2}}{1+1}\right|$

$\Rightarrow \alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$

$\therefore$ min. (arg$z$) = $\tan^{-1}\dfrac{3}{4}$

Here min.$(|z|) = OD$

$= OC - CD$

$= |1 + 2i - 0| - 1$

$= \sqrt{5} - 1$

64. Answer (1)

$\because z + \dfrac{1}{z} = 2\cos\theta$

$\Rightarrow z^2 - (2\cos\theta).z + 1 = 0$

$\Rightarrow z = \dfrac{-(-2\cos\theta) \pm \sqrt{4\cos^2\theta - 4}}{2.1}$

$= \cos\theta \pm i\sin\theta$

$\therefore$ Roots are, $z_1 = \cos\theta + i\sin\theta$

$z_2 = \cos\theta - i\sin\theta$

$\therefore z_1^n + z_2^n = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$

$= 2\cos n\theta = e^{in\theta} + e^{-in\theta}$

65. Answer (2)

Clearly locus is a circle

66. Answer (1)

Without restriction, total number of ways

$= \dfrac{7!}{2! \times 2!} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2! \times 2 \times 1}$

= 1260

Number of ways when R's come together

= A$\boxed{\text{RR}}$ANGE

$= \dfrac{6!}{2!} = \dfrac{6.5.4.3.2!}{2!} = 360$

$\therefore$ Required number of ways = 1260 – 360 = 900

Alternate Method

_A_A_N_G_E_

Required number of ways

$= {}^6C_2 \times \dfrac{5!}{2!} = 900$

67. Answer (2)

Using 2, 5 and 7 with repetition each place of n-digit number can be chosen in 3 ways. Hence, total number of n-digit numbers

$= 3 \times 3 \times 3 \times - - - - - n$ times $= 3^n$

According to question, $3^n \geq 900$

$\Rightarrow 3^{n-2} \geq 100$

$\Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$

$\Rightarrow n - 2 \geq 4 \Rightarrow n > 6$

$\Rightarrow n$ least $= 7$

68. Answer (1)

Let the six digit numbers be $x_1\, x_2\, x_3\, x_4\, x_5\, x_6$.

Then, x_1 can take values 2 or 4 in 2 ways

x_2 can take vaules 1 or 3 or 5 in 3 ways

x_3 can take values as 0 and the digit that x_1 cannot occur

x_4 can take two remaining digit that x_2 cannot occur

Lastly x_5 and x_6 each can take 1 value.

$\therefore$ Total number of possibilities

$= 2 \times 3 \times 2 \times 2 \times 1 \times 1 = 24$

69. Answer (3)

Let $b = a + d$ and $c = a + 2d$, where $d = c.d.$ of A.P.

Given equation is $a + b + c = 21$ ...(1)

$\Rightarrow a + a + d + a + 2d = 21$

$\Rightarrow a + d = 7 \quad \Rightarrow b = 7$

$\therefore$ From (1), $a + c = 14$ ...(1)

$\therefore$ No. of positive integral solutions of equation (1)

$= {}^{14-1}C_{2-1} = {}^{13}C_1 = 13$

$\therefore$ Possible number of values of a, b, c = 13

70. Answer (1)

Without restriction, number of words formed

$$= \frac{7!}{2!} = \frac{7.6.5.4.3.2!}{2!} = 2520$$

When, I, N taken together $\boxed{\text{IN}}$TEGER,

No. of words formed $= \dfrac{6!}{2!} \times 2! = \dfrac{6.5.4.3.2!}{2!} \times 2 = 720$

$\therefore$ $n_1 = 2520 - 720 = 1800$

When the words begin with I and end with R

$\boxed{\text{I}}$NTEGE$\boxed{\text{R}}$,

No. of words formed is

$$n_2 = \frac{5!}{2!} = \frac{5.4.3.2!}{2!} = 60$$

$$\therefore\ 6\left(\frac{n_1 - 2n_2}{n_2}\right) = 6\left(\frac{1800 - 120}{60}\right) = 6\left(\frac{1680}{60}\right) = 168$$

71. Answer (4)

Here $S_n < 1 + 2\left(\frac{b}{a}\right) + 3\left(\frac{b}{a}\right)^2 + \ldots$ to ∞

$$\Rightarrow\ S_n < \left(1 - \frac{b}{a}\right)^{-2} = \left(\frac{a}{a-b}\right)^2 = \left(\frac{4a^2}{4a^2 - 4ab}\right)^2$$

$$= \frac{(2a)^4}{\left[(2a)^2 - 4ab\right]^2} = \frac{(\alpha+\beta)^4}{\left[(\alpha+\beta)^2 - 4\alpha\beta\right]^2}$$

$$\Rightarrow S_n < \left(\frac{\alpha+\beta}{\alpha-\beta}\right)^4$$

72. Answer (4)

$a + c = 2b$

$\Rightarrow\ a + c - 2b = 0$

$\Rightarrow\ a^3 + c^3 - 3ac\,(-2b)$

$= (2b)^3$

$= 8b^3$

73. Answer (3)

$$t_r = \frac{4r+1}{5^r.r\,(r-1)},\ \text{where } r \geq 2$$

$$= \frac{5r - (r-1)}{5^r.r\,(r-1)} = \frac{1}{5^{r-1}(r-1)} - \frac{1}{5^r.r}$$

$$\therefore\ \sum_{r=2}^{\infty} t_r = \left[\frac{1}{5^1.1} - \frac{1}{5^2.2} + \frac{1}{5^2.2} - \frac{1}{5^3.3}\right.$$

$$\left. + \left(\frac{1}{5^3.3} - \frac{1}{5^4.4}\right) + \ldots \text{ to } \infty\right]$$

$$= \frac{1}{5}\ [\because \text{ terms tend to zero as } n \to \infty]$$

74. Answer (2)

Given, $\dfrac{a + be^y}{a - be^y} = \dfrac{b + ce^y}{b - ce^y} = \dfrac{c + de^y}{c - de^y}$

$$\Rightarrow \frac{2a - (a - be^y)}{a - be^y} = \frac{2b - (b - ce^y)}{b - ce^y} = \frac{2c - (c - de^y)}{c - de^y}$$

$$\Rightarrow \frac{2a}{a-be^y}-1=\frac{2b}{b-ce^y}-1=\frac{2c}{c-de^y}-1$$

$$\Rightarrow \frac{a-be^y}{2a}=\frac{b-ce^y}{2b}=\frac{c-de^y}{2c}$$

$$\Rightarrow \frac{1}{2}-\frac{b}{a}e^y=\frac{1}{2}-\frac{c}{b}e^y=\frac{1}{2}-\frac{d}{c}e^y$$

$$\Rightarrow \frac{b}{a}=\frac{c}{b}=\frac{d}{c} \quad \Rightarrow \quad a, b, c, d \text{ are in G.P.}$$

75. Answer (1)

A.M $\geq$ G.M.

$$\Rightarrow \frac{1+\tan x+\cot x}{3}\geq(1.\tan x\cdot\cot x)^{1/3}$$

$$\Rightarrow (1+\tan x+\tan x)\geq 3$$

$$\Rightarrow (1+\tan x+\cot x)^3\geq 27$$

76. Answer (2)

We know that $P(E) + P(E') = 1$

$$\Rightarrow 1+\lambda+\lambda^2+(1+\lambda)^2=1$$

$$\Rightarrow 2\lambda^2+3\lambda+1=0$$

$$\Rightarrow (2\lambda+1)(\lambda+1)=0$$

$$\Rightarrow \lambda=-1,-\frac{1}{2}$$

When $\lambda = -1$, $P(E) = 1 - 1 + 1 = 1$ (not possible)

when $\lambda=-\frac{1}{2}$, then $P(E) = 1+\left(-\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2$

$$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

77. Answer (1)

Let E_1 = the event of making words of 5 letters

Then $P(E_1)=\frac{n(E_1)}{n(S)}$

$$=\frac{3C_2\times 2!\times 4P_3+3C_3\times 3!\times 3C_2\times 3!}{6\times 5!}$$

$$=\frac{3\times 2!\times 4!+1\times 3!\times 3!}{6!}$$ (When 2 vowels, 3 consonants or 3vowels, 2 consonants taken)

$$=\frac{144+36\times 3}{720}=\frac{7}{20}$$

78. Answer (3)

The sum is 12 in the first three throws if they are (1, 5, 6) in any order or (2, 4 ,6) in any order or (3, 4, 5) in any order.

$\therefore$ Required probability

$$=\frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!+\frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!+\frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!$$

$$=\frac{6\times 3}{120}=\frac{3}{20}$$

79. Answer (1)

We have $x^{2n-1} + y^{2n-1} = (x + y)\,(x^{2n-2} - x^{2n-3}y + x^{2n-4}y^2 - - - - - - + y^{2n-2})$

$\therefore$ $x^{2n-1} + y^{2n-1}$ is divisible by $(x + y)$ for all $n\in N$.

80. $$\sum^{n}\sum^{r-1}{}^{n}C_r\,{}^{r}C_s\,2^s=\sum_{r=1}^{h}{}^{n}C_r\left(\sum_{s=0}^{n-1}{}^{r}C_s\,2^s\right)$$

$$=\sum_{r=1}^{n}{}^{n}C_r[3^r-2^r]$$

r = 1

$= 4^n - 3^n$

81. Answer (1)

$$t_{r+1}={}^{10}C_r.(x\sec\alpha)^{10-r}.(x^{-1}\operatorname{cosec}\alpha)^r$$

$$={}^{10}C_r.(\sec\alpha)^{10-r}\times(\operatorname{cosec}\alpha)^r.x^{10-2r}$$

$\therefore$ for the term independet of x

$10 - 2r = 0 \qquad \Rightarrow r = 5$

$\therefore$ Term independent of x

$$={}^{10}C_5.(\sec\alpha)^5\times(\operatorname{cosec}\alpha)^5$$

$$={}^{10}C_5.\frac{2^5}{(2\sin\alpha\cos\alpha)^5}$$

$$={}^{10}C_5\times 2^5\times\frac{1}{(\sin 2\alpha)^5}\geq{}^{10}C_5\times 2^5$$

$\therefore$ Least value of the term independent of x

$= {}^{10}C_5\,{}'\,2^5$

[$\because \sin 2\alpha\leq 1$

$\Rightarrow \frac{1}{\sin 2\alpha}\geq 1$

$\Rightarrow \frac{1}{(\sin 2\alpha)^5}\geq 1$]

$$=\frac{10!\times 2^5}{(5!)^2}$$

82. Answer (3)

Put $x = i$ in given identity,

$(1 + i - 1)^{48} = a_0 + a_1 i - a_2 - a_3 i + a_4 + a_5 i + \ldots + a_{96} i^{96}$

$\Rightarrow i^{48} = (a_0 - a_2 + a_4 - a_6 + \ldots + a_{96}) + i(a_1 - a_3 + a_5 - \ldots)$

$\Rightarrow 1 + i.0 = (a_0 - a_2 + a_4 \ldots + a_{96}) + i(a_1 - a_3 + a_5 - \ldots)$

Equating real parts,

$a_0 - a_2 + a_4 - \ldots + a_{96} = 1$

83. Answer (1)

Put $x - 101 = t \Rightarrow x - 100 = t + 1$

$\therefore$ From given identity,

$$\sum_{r=0}^{2n} b_r t^r = \sum_{r=0}^{2n} a_r (t+1)^r \quad \ldots(1)$$

$\therefore$ b_n = coefficient of t^n in the RHS of (1)

$$= {}^nC_n a_n + {}^{n+1}C_n a_{n+1} + \ldots + {}^{2n}C_n a_{2n}$$

$$= \sum_{k=n}^{2n} {}^kC_n a_k = \sum_{k=n}^{2n} 2^k$$

$$= 2^n \sum_{k=n}^{2n} 2^{k-n} = 2^n \cdot \frac{1\cdot\left(2^{n+1}-1\right)}{2-1}$$

$$= 2^n\left(2^{n+1}-1\right)$$

84. Answer (4)

We have, $(1 + x)^n (1 + y)^n (1 + z)^n$

$$= \left(\sum_{r=0}^{n} {}^nC_r x^r\right)\left(\sum_{s=0}^{n} {}^nC_s y^s\right)\left(\sum_{t=0}^{n} {}^nC_t z^t\right)$$

$$= \sum_{0\le r,s,t\le n} {}^nC_r . {}^nC_s . {}^nC_t x^r . y^s . z^t$$

For sum of coefficients of degree m, we must have $r + s + t = m$, where r, s, t are integers with $r, s, t \ge 0$

$\therefore$ Sum of such coefficients

$$= \sum_{\substack{r,s,t\ge 0\\ r+s+t=m}} \left({}^nC_r\right)\left({}^nC_s\right)\left({}^nC_t\right)$$

= The no. of ways of choosing a total number of m balls out of n black, n white, and n green balls

= ${}^{3n}C_m$

85. Answer (2)

$$\left(1+\sqrt{2}+3^{1/3}\right)^6 = {}^6C_0 + {}^6C_1\left(\sqrt{2}+\sqrt[3]{3}\right) + {}^6C_2\left(\sqrt{2}+\sqrt[3]{3}\right)^2 + {}^6C_3\left(\sqrt{2}+\sqrt[3]{3}\right)^3 + {}^6C_4\left(\sqrt{2}+\sqrt[3]{3}\right)^4 + {}^6C_5\left(\sqrt{2}+\sqrt[3]{3}\right)^5 + {}^6C_6\left(\sqrt{2}+\sqrt[3]{3}\right)^6$$

$\text{In}\left(\sqrt{2}+\sqrt[3]{3}\right)^6, t_{r+1} = {}^6C_r.\left(\sqrt{2}\right)^{6-r}.\left(\sqrt[3]{3}\right)^r$

which will be rational if $r = 0$ or 6

$\text{In}\left(\sqrt{2}+\sqrt[3]{3}\right)^5, t_{r+1} = {}^5C_r.\left(\sqrt{2}\right)^{5-r}.\left(\sqrt[3]{3}\right)^r$

$$= {}^5C_r.\left(\sqrt{2}\right)^{5-r}.3^{\frac{r}{3}}$$

which will be rational if $r = 3$

$\therefore$ Total no. of rational terms

$= 1 + 0 + 1 + 1 + 1 + 1 + 2 = 7$

86. Answer (4)

$$f(\theta) = \frac{|2i|}{|3 - i(\cos\theta + i\sin\theta)|} = \frac{|2|}{\sqrt{(3+\sin\theta)^2 + \cos^2\theta}}$$

$$\Rightarrow (f(\theta))^2 = \frac{4}{9 + 6\sin\theta + \sin^2\theta + \cos^2\theta}$$

$$= \frac{4}{10 + 6\sin\theta}$$

$$= \frac{2}{5 + 3\sin\theta}$$

$\therefore$ $(f(\theta))^2$ is maximum when $\sin\theta$ is minimum and its minimum value = –1

$$\therefore \max(f(\theta))^2 = \frac{2}{5-3} = 1$$

and $(f(\theta))^2$ will be minimum when $\sin\theta$ is maximum and max.($\sin\theta$) = 1

$$\therefore \min.(f(\theta))^2 = \frac{2}{5+3} = \frac{1}{4}$$

$$\therefore \frac{1}{4} \le (f(\theta))^2 \le 1$$

$$= \frac{1}{2} \le f(\theta) \le 1$$

87. Answer (1)

Since a, b, c are in H.P.

$$\therefore \quad b=\frac{2ac}{a+c} \Rightarrow \frac{b}{a}=\frac{2c}{a+c} \text{ and } \frac{b}{c}=\frac{2a}{a+c}$$

$$\therefore \quad \frac{a+b}{2a-b}=\frac{a\left(1+\frac{b}{a}\right)}{a\left(2-\frac{b}{a}\right)}=\frac{1+\frac{2c}{a+c}}{2-\frac{2c}{a+c}}=\frac{a+3c}{2a}=\frac{1}{2}+\frac{3}{2}\frac{c}{a}$$

$$\text{and } \frac{c+b}{2c-b}=\frac{c\left(1+\frac{b}{c}\right)}{c\left(2-\frac{b}{c}\right)}=\frac{1+\frac{2c}{a+c}}{2-\frac{2c}{a+c}}=\frac{3a+c}{2a}=\frac{3}{2}\frac{a}{c}+\frac{1}{2}$$

$$\therefore \quad \frac{a+b}{2a-b}+\frac{c+b}{2c-b}=\frac{1}{2}+\frac{3}{2}\frac{c}{a}+\frac{3}{2}\frac{a}{c}+\frac{1}{2}$$

$$=1+\frac{3}{2}\left(\frac{c}{a}+\frac{a}{c}\right)$$

$$\geq 1+\frac{3}{2}2\left(\frac{c}{a}.\frac{a}{c}\right)^{\frac{1}{2}}$$

$\because$ a, b, c are the positive real numbers

$$\frac{\frac{a}{c}+\frac{a}{c}}{2} \geq \left(\frac{c}{a}\cdot\frac{a}{c}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{c}{a}+\frac{a}{c} \geq 2.1$$

$$\geq 1+\frac{3}{2}.2.1$$

$$\geq 4$$

$\therefore$ Statement 1 is true

But $\dfrac{x+\frac{1}{x}}{2} \geq \left(x.\frac{1}{x}\right)^{\frac{1}{2}}$ if x > 0

$$\Rightarrow x+\frac{1}{x} \geq 2$$

$\therefore$ Statement 2 is true

88. Answer (1)

We have, 30 = 2 × 3 × 5

So, 2 can be assigned to either a or b or c i.e. 2 can be assigned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways.

$\therefore$ The no. of solutions = 3 × 3 × 3 = 27

$\therefore$ Statement-1 is true

Also statement-2 is theoretically true and it is the correct explanation of statement-1.

89. Answer (2)

The total no. of cases is $n(s)$ = 4! = 24

Let E be the event that no letter is mailed in its correct envelope.

Then $n(E)$ = no. of favourable cases

$$= 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$$= \frac{4.3.2!}{2!}-\frac{4.3!}{3!}+\frac{4!}{4!}$$

$$= 12 - 4 + 1 = 9$$

$\therefore$ Required probability $= \dfrac{n(E)}{n(s)}=\dfrac{9}{24}=\dfrac{3}{8}$

$\therefore$ Statement-1 is True.

Also the probability that all the letters are placed in the correct envelope $= \dfrac{1}{4!}=\dfrac{1}{24}$

$\therefore$ The probability that all the letters are not placed in the correct envelope $= 1-\dfrac{1}{24}=\dfrac{23}{24}$

Hence, statement-2 is True but it does not explain statement-1.

90. Answer (2)

Here, $m=\dfrac{(n+1)|x|}{|x|+1}=\dfrac{(12+1)\frac{11}{10}}{\frac{11}{10}+1}=\dfrac{\frac{13\times 11}{10}}{\frac{21}{10}}$

$$=\frac{143}{21}=6.8$$

$\therefore$ Greatest term in expansion of $(1 + x)^{12}$

$= {}^{t}[m]+1={}^{t}[6.8]+1$, where [.] is G.I.F.

$= t_7$

$\therefore$ 7^{th} term is the greatest term

$\Rightarrow$ Statement-1 is True

Also 7th term in expansion of $(1 + x)^{12}$

$$= {}^{12}C_6 x^6$$

$\therefore$ Binomial coefficient of 7th term is ${}^{12}C_6$ which is the greatest value of binomial coefficient ${}^{12}C_r$. But this is not the reason for which t_7 is the greatest term. Here, it is co-incident that the greatest term has the greatest binomial coefficient.

Hence statement-2 is also true but it is not the correct explanation for statement-1.