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FINAL TEST SERIES JEE -2017 TEST-02

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1. Answer (4)

No external force

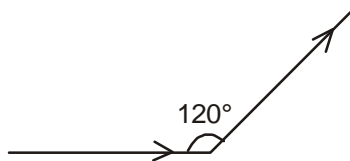
$$\therefore V_{cm} = 0.$$

2. Answer (1)

The force should be applied at centre of mass.

3. Answer (1)

$$\text{Impulse} = \text{change in momentum} = mv_0$$



4. Answer (2)

$$F < \mu mg \text{ and } F \times h > mg \times \frac{L}{2}$$

$$\Rightarrow \mu mg > F > mg \frac{L}{2h} \Rightarrow \mu > \frac{L}{2h}$$

5. Answer (4)

$$\therefore I \propto R^2 \quad m = \text{vol} \times \rho$$

$$I \propto \frac{1}{\rho} \quad = \text{area} \times \text{thickness} \times \rho$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{\rho_2}{\rho_1} = \frac{\pi_2}{\pi_1} \propto R^2 \cdot t \cdot \rho$$

$$\Rightarrow R^2 \propto \frac{1}{\rho}$$

6. Answer (4)

$$mg \sin \theta (R - h) = mg \cos \theta \times \sqrt{R^2 - (R - h)^2}$$

$$\tan^2 \theta (R - h)^2 = R^2 - (R^2 + h^2 - 2Rh)$$

$$(R^2 + h^2 - 2Rh) \tan^2 \theta = 2Rh - h^2$$

$$R^2 \tan^2 \theta + h^2 (1 + \tan^2 \theta) = 2Rh (1 + \tan^2 \theta)$$

$$R^2 \sin^2 \theta + h^2 = 2Rh$$

$$h^2 - 2Rh + R^2 \sin^2 \theta = 0$$

$$h = \frac{2R \pm \sqrt{4R^2 - 4R^2 \sin^2 \theta}}{2}$$

$$= R \pm R \sqrt{1 - \sin^2 \theta} = R \pm R \cos \theta$$

$$= R[1 - \cos \theta]$$

7. Answer (2)

$$I_1 = \frac{ml^2}{12}$$

$$I_2 = m \left(\frac{l}{2\pi} \right)^2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{4\pi^2}{12} = \frac{\pi^2}{3}$$

8. Answer (1)

$$\rho \times \frac{l}{4} = \frac{ml^2}{12} \times \omega$$

$$\Rightarrow \omega = \frac{3\rho}{ml}$$

9. Answer (2)

$$mV_0R - mR^2 \times \frac{V_0}{2R} = mVR + mR^2 \frac{v}{R}$$

$$\frac{mV_0R}{2} = 2mVR$$

$$\Rightarrow v = \frac{V_0}{4}$$

10. Answer (4)

$$K = \frac{L^2}{2I}$$

$$\frac{K_A}{K_B} = \frac{L_A^2}{L_B^2} \times \frac{I_B}{I_A} = 25 \times \frac{4}{1} = 100 : 1$$

11. Answer (1)

$$\frac{4}{3}\pi(R^3 - r^3) \times 8 \times g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$$8R^3 - 8r^3 = R^3$$

$$7R^3 = 8r^3$$

$$\frac{r}{R} = \left(\frac{7}{8}\right)^{1/3}$$

12. Answer (2)

$$2h \times \rho \times g + h \times 2\rho \times g = \frac{1}{2}(2\rho) \times v^2$$

$$4h\rho g = \rho v^2$$

$$\Rightarrow v = 2\sqrt{gh}$$

13. Answer (4)

$$g = \frac{GM}{R^2} \quad \dots(1) \quad r\omega_0 = \sqrt{\frac{GM}{r}}$$

$$\text{or } GM = r^3\omega_0^2 \quad \dots(2)$$

$$\therefore g = \frac{r^3\omega_0^2}{R^2}$$

14. Answer (1)

$$\frac{3}{5}g = g - R\omega^2$$

$$R\omega^2 = \frac{2}{5}g$$

15. Answer (1)

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\Rightarrow M = \frac{4\pi^2 r^3}{GT^2}$$

16. Answer (1)

$$-\frac{GMm}{R} + 0 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{gR}$$

17. Answer (1)

$$K = | +E | = \frac{|U|}{2} \Rightarrow E = -\frac{1}{2}mv^2$$

18. Answer (3)

$$W_{\text{ext}} = \Delta K + \Delta U = \Delta ME = E_f - E_i$$

$$= -\frac{GMm}{10R} - \left(-\frac{GMm}{6R}\right) = \left(\frac{1}{6} - \frac{1}{10}\right)\frac{GMm}{R} = \frac{GMm}{15R}$$

19. Answer (3)

$$v = 10\cos 2t + 2$$

$$v(t=0) = 10 + 2 = 12$$

$$v\left(t = \frac{\pi}{2}\right) = -8$$

$$I = 5[-8 - 12] = -100 \text{ kgms}^{-1}$$

20. Answer (1)

$$x_{\text{cm}} = \frac{\int_0^l \lambda_0 x^5 \cdot dx \cdot x}{\int_0^l dx \cdot \lambda_0 x^5} = \frac{l^7}{l^6} = \frac{6}{7}l$$

$$\therefore \text{From end B, } l - \frac{6l}{7} = \frac{l}{7}$$

21. Answer (1)

Angular velocity of rod about IAOR is

$$\omega = \frac{V_0}{\frac{l}{\sqrt{2}}} = \frac{\sqrt{2}V_0}{l}$$

22. Answer (3)

$$v_{cm} = \frac{v}{2}$$

$$\therefore w = \frac{v}{2r}$$

$$\therefore a = rw^2 = \frac{v^2}{4}$$

23. Answer (4)

$$\left. \begin{aligned} 80 &= k(l_1 - l_0) \dots(i) \\ 100 &= k(l_2 - l_0) \dots(ii) \end{aligned} \right\} \Rightarrow \frac{4}{5} = \frac{l_1 - l_0}{l_2 - l_0}$$

$$4l_2 - 4l_0 - 5l_1 - 5l_0$$

$$l_0 = 5l_1 - 4l_2$$

$$\therefore 160 = k(l_3 - l_0) \dots(iii)$$

Equation (iii) is divided by equation (i)

$$\Rightarrow 2 = \frac{l_3 - 5l_1 + 4l_2}{l_1 - 5l_1 + 4l_2}$$

$$\Rightarrow 2l_1 - 10l_1 + 8l_2 = l_3 - 5l_1 + 4l_2$$

$$l_3 = -3l_1 + 4l_2$$

24. Answer (1)

$$\frac{1}{K} = \frac{\Delta P}{\Delta V/V}$$

$$\Rightarrow \Delta P = \frac{1}{K} \cdot \frac{\Delta V}{V} = \frac{0.2}{100} \times \frac{1}{4.4 \times 10^{-5}} = 45.4 \text{ atm}$$

25. Answer (4)

$$\tau = \frac{h\rho g}{2} \times bh \times \frac{h}{3}$$

26. Answer (4)

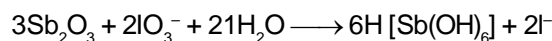
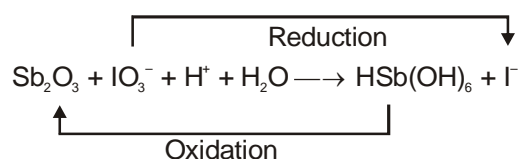
27. Answer (1)

28. Answer (1)

29. Answer (2)

30. Answer (4)

31. Answer (2)



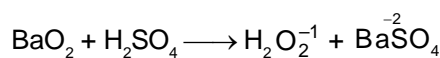
32. Answer (1)

$$+3 + 2(+2) + 3x + 7(-2) = 0$$

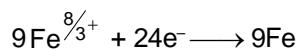
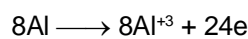
$$7 + 3x - 14 = 0$$

$$x = +\frac{7}{3}$$

33. Answer (2)



34. Answer (4)



35. Answer (1)

$$\frac{r_{\text{CH}_4}}{r_{\text{gas}}} = \sqrt{\frac{M_{\text{gas}}}{16}} \text{ or } 2 = \sqrt{\frac{M_{\text{gas}}}{16}}$$

36. Answer (2)

$$P_{\text{H}_2} = \frac{8}{1+8} P_{\text{Total}} = \frac{8}{9} P_{\text{Total}}$$

37. Answer (1)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$E = \frac{3}{2} RT$$

$$v_{\text{rms}} = \sqrt{\frac{2KE}{M}}$$

38. Answer (4)

39. Answer (4)

Fact

40. Answer (1)

$$\Delta H = \Delta U + \Delta n_g RT$$

$$\Delta H = \Delta U + RT$$

$$\Delta H > \Delta U$$

41. Answer (2)

$$K_p = K_c (RT)^{\Delta n_g}$$

42. Answer (3)

$$\Delta n_g = 1 - (2 + 1) = -2$$

43. Answer (1)

$$\Delta G^\circ = -2.303 \times 2 \times 298 \log 10^{-8}$$

$$= + 10.98 \text{ Kcal}$$

44. Answer (4)

Equilibrium constant depends on temperature only.

45. Answer (3)

$$\begin{array}{cccc|c} W & + & X & \rightleftharpoons & Y & + & Z & \\ 1 & & 1 & & 0 & & 0 & \\ 1-x & & 1-x & & x & & x & \end{array} \quad \begin{array}{l} g = \frac{x^2}{(1-x)^2} \\ x = 0.75 \end{array}$$

46. Answer (3)

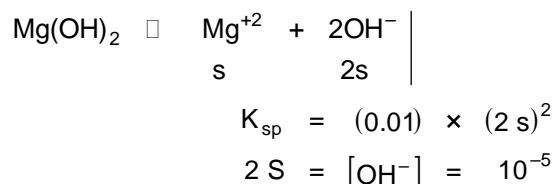
$$\frac{1}{K_a} = \frac{K_a}{K_w} = \frac{10^{-4}}{10^{-14}} = 10^{10}$$

47. Answer (3)

$$\frac{7.9 \times 10^{-10}}{7 \times 10^{-10}} = \frac{[Sr^{+2}][F^-]^2}{[Sr^{+2}][CO_3^{2-}]}$$

$$[F^-] = 3.7 \times 10^{-2}$$

48. Answer (2)



49. Answer (2)

$$pOH = pk_b + \log \frac{[B^+]}{[BOH]}$$

$$pk_a = 6$$

$$k_b = 10^{-6}$$

50. Answer (1)

$$K_a = \frac{[H^+]^2[S^{-2}]}{[H_2S]}$$

$$10^{-21} = \frac{(0.1)^2 \times [S^{-2}]}{0.1}$$

$$[S^{-2}] = 10^{-20} \text{ M}$$

$$= 10^{-20} \times 6.023 \times 10^{23} = 6.023 \times 10^3$$

51. Answer (1)

$$K_a \text{ of } H_2S = 9 \times 10^{-20}$$

$$[S^{-2}] = \frac{K_a[H_2S]}{[H^+]^2}$$

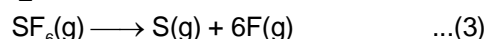
$$[H^+] = 10^{-3}$$

$$[S^{-2}] = \frac{9 \times 10^{-20} \times 0.1}{(10^{-3})^2} = 9 \times 10^{-15}$$

$$[Mn^{+2}] (9 \times 10^{-15}) = 3 \times 10^{-22}$$

$$[Mn^{+2}] = 3.33 \times 10^{-8} \text{ M}$$

52. Answer (1)



53. Answer (1)

$$Z = \frac{PV}{nRT}$$

$$\left(P + \frac{an^2}{V^2} \right) (V) = nRT$$

54. Answer (2)

$$m_1 = (148 - 50) = 98 \text{ gm}$$

$$V = \frac{m_1}{\rho} = \frac{98}{0.98} = 100 \text{ ml}$$

$$\text{Mass of gases} = (50.5 - 50) = 0.5$$

$$PV = nRT = \frac{W}{M} RT$$

$$M = 123 \text{ gm / mol}$$

55. Answer (3)

$$\frac{x}{27/3} + \frac{(1.67-x)}{65.4/2} = \frac{1.69}{11.2}$$

$$x = 1.239 \approx 1.24 \text{ gm}$$

56. Answer (3)

In very dilute solutions both source of H^+ ions from HCl and H_2O must be considered and due to common ion (H^+) suppression of ionisation.

57. Answer (2)

Fact

58. Answer (2)

Graphite is more stable than diamond, hence its energy is lower than that of diamond and entropy of graphite is also lower than that of diamond.

59. Answer (1)

60. Answer (4)

Pb^{+2} is more stable than Pb^{+4} due to inert pair effect
 $Pb^{+4} + 2e \longrightarrow Pb^{+2}$ (oxidising agent)

61. Answer (1)

$$\begin{aligned} \text{Given, } |z_1 - z_0| &= |z_2 - z_0| = a \Rightarrow \left| \frac{z_2 - z_0}{z_1 - z_0} \right| = 1 \\ \Rightarrow \left| \frac{z_2 - z_0}{z_0 - z_1} \right| &= 1 \text{ and amp. } \left(\frac{z_2 - z_0}{z_0 - z_1} \right) = \frac{\pi}{2} \\ \therefore \frac{z_2 - z_0}{z_0 - z_1} &= \left| \frac{z_2 - z_0}{z_0 - z_1} \right| e^{i\pi/2} = 1 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = i \\ \Rightarrow z_2 - z_0 &= iz_0 - iz_1 \Rightarrow z_2 + iz_1 = (i + 1)z_0 \\ \Rightarrow z_0 &= \frac{z_2 + iz_1}{1+i} = \frac{(z_2 + iz_1)(1-i)}{1-i^2} \\ &= \frac{1}{2} [z_1(j-i^2) + (1-i)z_2] \\ &= \frac{1}{2} [(1+i)z_1 + (1-i)z_2] \end{aligned}$$

62. Answer (1)

$$\begin{aligned} \text{Let } z &= a + b\omega + c\omega^2 \\ \therefore |z|^2 &= z\bar{z} = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega) \\ &= a^2 + b^2 + c^2 - ab - bc - ca \\ &= \frac{1}{2} [(b-c)^2 + (c-a)^2 + (a-b)^2] \end{aligned}$$

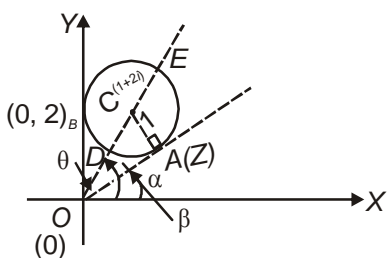
∴ From question,

$$\begin{aligned} x &= |z| + |\bar{z}| = 2|z| \\ &= \sqrt{2} \sqrt{[(a-b)^2 + (b-c)^2 + (c-a)^2]} \end{aligned}$$

since a, b, c are distinct integers, then minimum value of $(b-c)^2 + (c-a)^2 + (a-b)^2$ is $1^2 + 1^2 + 2^2 = 6$

$$\therefore \text{Minimum value of } x = \sqrt{6} \sqrt{2} = 2\sqrt{3}$$

63. Answer (4)



$$\tan \beta = 2$$

$$\tan \theta = \frac{1}{2}$$

$$\tan \alpha = \left| \frac{2 - \frac{1}{2}}{1 + 1} \right|$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{3}{4} \right)$$

$$\therefore \min. (\arg z) = \tan^{-1} \frac{3}{4}$$

Here $\min. (|z|) = OD$

$$\begin{aligned} &= OC - CD \\ &= |1 + 2i - 0| - 1 \\ &= \sqrt{5} - 1 \end{aligned}$$

64. Answer (1)

$$\therefore z + \frac{1}{z} = 2 \cos \theta$$

$$\Rightarrow z^2 - (2 \cos \theta)z + 1 = 0$$

$$\Rightarrow z = \frac{-(-2 \cos \theta) \pm \sqrt{4 \cos^2 \theta - 4}}{2 \cdot 1}$$

$$= \cos \theta \pm i \sin \theta$$

$$\therefore \text{Roots are, } z_1 = \cos \theta + i \sin \theta$$

$$z_2 = \cos \theta - i \sin \theta$$

$$\begin{aligned} \therefore z_1^n + z_2^n &= \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta \\ &= 2 \cos n\theta = e^{in\theta} + e^{-in\theta} \end{aligned}$$

65. Answer (2)

Clearly locus is a circle

66. Answer (1)

Without restriction, total number of ways

$$\begin{aligned} &= \frac{7!}{2! \times 2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2! \times 2 \times 1} \\ &= 1260 \end{aligned}$$

Number of ways when R's come together

$$\begin{aligned} &= A \overline{RR} ANGE \\ &= \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 360 \end{aligned}$$

$$\therefore \text{Required number of ways} = 1260 - 360 = 900$$

Alternate Method

A A N G E

Required number of ways

$$= {}^6C_2 \times \frac{5!}{2!} = 900$$

67. Answer (2)

Using 2, 5 and 7 with repetition each place of n -digit number can be chosen in 3 ways. Hence, total number of n -digit numbers

$$= 3 \times 3 \times 3 \times \dots \times n \text{ times} = 3^n$$

According to question, $3^n \geq 900$

$$\Rightarrow 3^{n-2} \geq 100$$

$$\Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$$

$$\Rightarrow n - 2 \geq 4 \Rightarrow n > 6$$

$$\Rightarrow n \text{ least} = 7$$

68. Answer (1)

Let the six digit numbers be $x_1 x_2 x_3 x_4 x_5 x_6$.

Then, x_1 can take values 2 or 4 in 2 ways

x_2 can take values 1 or 3 or 5 in 3 ways

x_3 can take values as 0 and the digit that x_1 cannot occur

x_4 can take two remaining digit that x_2 cannot occur

Lastly x_5 and x_6 each can take 1 value.

\therefore Total number of possibilities

$$= 2 \times 3 \times 2 \times 2 \times 1 \times 1 = 24$$

69. Answer (3)

Let $b = a + d$ and $c = a + 2d$, where $d = c.d.$ of A.P.

Given equation is $a + b + c = 21$... (1)

$$\Rightarrow a + a + d + a + 2d = 21$$

$$\Rightarrow a + d = 7 \Rightarrow b = 7$$

\therefore From (1), $a + c = 14$... (1)

\therefore No. of positive integral solutions of equation (1)

$$= {}^{14-1}C_{2-1} = {}^{13}C_1 = 13$$

\therefore Possible number of values of $a, b, c = 13$

70. Answer (1)

Without restriction, number of words formed

$$= \frac{7!}{2!} = \frac{7.6.5.4.3.2!}{2!} = 2520$$

When, I, N taken together **INTEGER**,

$$\text{No. of words formed} = \frac{6!}{2!} \times 2! = \frac{6.5.4.3.2!}{2!} \times 2 = 720$$

$$\therefore n_1 = 2520 - 720 = 1800$$

When the words begin with I and end with R

INTEGER,

No. of words formed is

$$n_2 = \frac{5!}{2!} = \frac{5.4.3.2!}{2!} = 60$$

$$\therefore 6 \left(\frac{n_1 - 2n_2}{n_2} \right) = 6 \left(\frac{1800 - 120}{60} \right) = 6 \left(\frac{1680}{60} \right) = 168$$

71. Answer (4)

Here $S_n < 1 + 2 \left(\frac{b}{a} \right) + 3 \left(\frac{b}{a} \right)^2 + \dots$ to ∞

$$\begin{aligned} \Rightarrow S_n &< \left(1 - \frac{b}{a} \right)^{-2} = \left(\frac{a}{a-b} \right)^2 = \left(\frac{4a^2}{4a^2 - 4ab} \right)^2 \\ &= \frac{(2a)^4}{[(2a)^2 - 4ab]^2} = \frac{(\alpha + \beta)^4}{[(\alpha + \beta)^2 - 4\alpha\beta]^2} \\ \Rightarrow S_n &< \left(\frac{\alpha + \beta}{\alpha - \beta} \right)^4 \end{aligned}$$

72. Answer (4)

$$a + c = 2b$$

$$\Rightarrow a + c - 2b = 0$$

$$\Rightarrow a^3 + c^3 - 3ac(-2b)$$

$$= (2b)^3$$

$$= 8b^3$$

73. Answer (3)

$$t_r = \frac{4r+1}{5^r \cdot r(r-1)}, \text{ where } r \geq 2$$

$$= \frac{5r - (r-1)}{5^r \cdot r(r-1)} = \frac{1}{5^{r-1}(r-1)} - \frac{1}{5^r \cdot r}$$

$$\therefore \sum_{r=2}^{\infty} t_r = \left[\frac{1}{5^1 \cdot 1} - \frac{1}{5^2 \cdot 2} + \frac{1}{5^2 \cdot 2} - \frac{1}{5^3 \cdot 3} \right.$$

$$\left. + \left(\frac{1}{5^3 \cdot 3} - \frac{1}{5^4 \cdot 4} \right) + \dots \text{ to } \infty \right]$$

$$= \frac{1}{5} [\because \text{ terms tend to zero as } n \rightarrow \infty]$$

74. Answer (2)

$$\text{Given, } \frac{a + be^y}{a - be^y} = \frac{b + ce^y}{b - ce^y} = \frac{c + de^y}{c - de^y}$$

$$\Rightarrow \frac{2a - (a - be^y)}{a - be^y} = \frac{2b - (b - ce^y)}{b - ce^y} = \frac{2c - (c - de^y)}{c - de^y}$$

$$\Rightarrow \frac{2a}{a-be^y} - 1 = \frac{2b}{b-ce^y} - 1 = \frac{2c}{c-de^y} - 1$$

$$\Rightarrow \frac{a-be^y}{2a} = \frac{b-ce^y}{2b} = \frac{c-de^y}{2c}$$

$$\Rightarrow \frac{1}{2} - \frac{b}{a}e^y = \frac{1}{2} - \frac{c}{b}e^y = \frac{1}{2} - \frac{d}{c}e^y$$

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \Rightarrow a, b, c, d \text{ are in G.P.}$$

75. Answer (1)

A.M \geq G.M.

$$\Rightarrow \frac{1 + \tan x + \cot x}{3} \geq (1 \cdot \tan x \cdot \cot x)^{1/3}$$

$$\Rightarrow (1 + \tan x + \cot x) \geq 3$$

$$\Rightarrow (1 + \tan x + \cot x)^3 \geq 27$$

76. Answer (2)

We know that $P(E) + P(E') = 1$

$$\Rightarrow 1 + \lambda + \lambda^2 + (1 + \lambda)^2 = 1$$

$$\Rightarrow 2\lambda^2 + 3\lambda + 1 = 0$$

$$\Rightarrow (2\lambda + 1)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = -1, -\frac{1}{2}$$

When $\lambda = -1$, $P(E) = 1 - 1 + 1 = 1$ (not possible)

when $\lambda = -\frac{1}{2}$, then $P(E) = 1 + \left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

77. Answer (1)

Let E_1 = the event of making words of 5 letters

Then $P(E_1) = \frac{n(E_1)}{n(S)}$

$$= \frac{3C_2 \times 2! \times 4P_3 + 3C_3 \times 3! \times 3C_2 \times 3!}{6 \times 5!}$$

$$= \frac{3 \times 2! \times 4! + 1 \times 3! \times 3!}{6!} \text{ (When 2 vowels, 3 consonants or 3 vowels, 2 consonants taken)}$$

$$= \frac{144 + 36 \times 3}{720} = \frac{7}{20}$$

78. Answer (3)

The sum is 12 in the first three throws if they are (1, 5, 6) in any order or (2, 4, 6) in any order or (3, 4, 5) in any order.

\therefore Required probability

$$= \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! + \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! + \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3!$$

$$= \frac{6 \times 3}{120} = \frac{3}{20}$$

79. Answer (1)

We have $x^{2n-1} + y^{2n-1} = (x + y) (x^{2n-2} - x^{2n-3}y + x^{2n-4}y^2 - \dots + y^{2n-2})$

$\therefore x^{2n-1} + y^{2n-1}$ is divisible by $(x + y)$ for all $n \in N$.

80. $\sum_{r=1}^n \sum_{s=0}^{r-1} {}^n C_r {}^r C_s 2^s = \sum_{r=1}^n {}^n C_r \left(\sum_{s=0}^{r-1} {}^r C_s 2^s \right)$

$$= \sum_{r=1}^n {}^n C_r [3^r - 2^r]$$

$$r = 1$$

$$= 4^n - 3^n$$

81. Answer (1)

$$t_{r+1} = {}^{10}C_r \cdot (x \sec \alpha)^{10-r} \cdot (x^{-1} \operatorname{cosec} \alpha)^r$$

$$= {}^{10}C_r \cdot (\sec \alpha)^{10-r} \times (\operatorname{cosec} \alpha)^r \cdot x^{10-2r}$$

\therefore for the term independent of x

$$10 - 2r = 0 \Rightarrow r = 5$$

\therefore Term independent of x

$$= {}^{10}C_5 \cdot (\sec \alpha)^5 \times (\operatorname{cosec} \alpha)^5$$

$$= {}^{10}C_5 \cdot \frac{2^5}{(2 \sin \alpha \cos \alpha)^5}$$

$$= {}^{10}C_5 \times 2^5 \times \frac{1}{(\sin 2\alpha)^5} \geq {}^{10}C_5 \times 2^5$$

\therefore Least value of the

term independent of x

$$= {}^{10}C_5 \cdot 2^5$$

$$= \frac{10! \times 2^5}{(5!)^2}$$

$$\left[\begin{array}{l} \therefore \sin 2\alpha \leq 1 \\ \Rightarrow \frac{1}{\sin 2\alpha} \geq 1 \\ \Rightarrow \frac{1}{(\sin 2\alpha)^5} \geq 1 \end{array} \right]$$

82. Answer (3)

Put $x = i$ in given identity,

$$(1 + i - 1)^{48} = a_0 + a_1 i - a_2 - a_3 i + a_4 + a_5 i + \dots + a_{96} i^{96}$$

$$\Rightarrow i^{48} = (a_0 - a_2 + a_4 - a_6 + \dots + a_{96}) + i(a_1 - a_3 + a_5 - \dots)$$

$$\Rightarrow 1 + i \cdot 0 = (a_0 - a_2 + a_4 - \dots + a_{96}) + i(a_1 - a_3 + a_5 - \dots)$$

Equating real parts,

$$a_0 - a_2 + a_4 - \dots + a_{96} = 1$$

83. Answer (1)

Put $x - 101 = t \Rightarrow x - 100 = t + 1$

\therefore From given identity,

$$\sum_{r=0}^{2n} b_r t^r = \sum_{r=0}^{2n} a_r (t+1)^r \dots (1)$$

$$\therefore b_n = \text{coefficient of } t^n \text{ in the RHS of (1)}$$

$$= {}^n C_n a_n + {}^{n+1} C_n a_{n+1} + \dots + {}^{2n} C_n a_{2n}$$

$$= \sum_{k=n}^{2n} {}^k C_n a_k = \sum_{k=n}^{2n} 2^k$$

$$= 2^n \sum_{k=n}^{2n} 2^{k-n} = 2^n \cdot \frac{1 \cdot (2^{n+1} - 1)}{2 - 1}$$

$$= 2^n (2^{n+1} - 1)$$

84. Answer (4)

We have, $(1 + x)^n (1 + y)^n (1 + z)^n$

$$= \left(\sum_{r=0}^n {}^n C_r x^r \right) \left(\sum_{s=0}^n {}^n C_s y^s \right) \left(\sum_{t=0}^n {}^n C_t z^t \right)$$

$$= \sum_{0 \leq r, s, t \leq n} {}^n C_r \cdot {}^n C_s \cdot {}^n C_t x^r \cdot y^s \cdot z^t$$

For sum of coefficients of degree m , we must have

$$r + s + t = m, \text{ where } r, s, t \text{ are integers with } r, s, t \geq 0$$

\therefore Sum of such coefficients

$$\sum_{\substack{r, s, t \geq 0 \\ r+s+t=m}} ({}^n C_r) ({}^n C_s) ({}^n C_t)$$

= The no. of ways of choosing a total number of m balls out of n black, n white, and n green balls

$$= {}^{3n} C_m$$

85. Answer (2)

$$\left(1 + \sqrt{2} + 3^{1/3}\right)^6 = {}^6 C_0 + {}^6 C_1 (\sqrt{2} + \sqrt[3]{3}) + {}^6 C_2 (\sqrt{2} + \sqrt[3]{3})^2 + {}^6 C_3 (\sqrt{2} + \sqrt[3]{3})^3 + {}^6 C_4 (\sqrt{2} + \sqrt[3]{3})^4 + {}^6 C_5 (\sqrt{2} + \sqrt[3]{3})^5 + {}^6 C_6 (\sqrt{2} + \sqrt[3]{3})^6$$

$$\ln(\sqrt{2} + \sqrt[3]{3})^6, t_{r+1} = {}^6 C_r (\sqrt{2})^{6-r} \cdot (\sqrt[3]{3})^r$$

which will be rational if $r = 0$ or 6

$$\ln(\sqrt{2} + \sqrt[3]{3})^5, t_{r+1} = {}^5 C_r (\sqrt{2})^{5-r} \cdot (\sqrt[3]{3})^r$$

$$= {}^5 C_r (\sqrt{2})^{5-r} \cdot 3^{r/3}$$

which will be rational if $r = 3$

\therefore Total no. of rational terms

$$= 1 + 0 + 1 + 1 + 1 + 1 + 2 = 7$$

86. Answer (4)

$$f(\theta) = \frac{|2i|}{|3 - i(\cos\theta + i\sin\theta)|} = \frac{|2|}{\sqrt{(3 + \sin\theta)^2 + \cos^2\theta}}$$

$$\Rightarrow (f(\theta))^2 = \frac{4}{9 + 6\sin\theta + \sin^2\theta + \cos^2\theta}$$

$$= \frac{4}{10 + 6\sin\theta}$$

$$= \frac{2}{5 + 3\sin\theta}$$

$\therefore (f(\theta))^2$ is maximum when $\sin\theta$ is minimum and its minimum value = -1

$$\therefore \max(f(\theta))^2 = \frac{2}{5-3} = 1$$

and $(f(\theta))^2$ will be minimum when $\sin\theta$ is maximum and $\max(\sin\theta) = 1$

$$\therefore \min(f(\theta))^2 = \frac{2}{5+3} = \frac{1}{4}$$

$$\therefore \frac{1}{4} \leq (f(\theta))^2 \leq 1$$

$$= \frac{1}{2} \leq f(\theta) \leq 1$$

87. Answer (1)

Since a, b, c are in H.P.

$$\therefore b = \frac{2ac}{a+c} \Rightarrow \frac{b}{a} = \frac{2c}{a+c} \text{ and } \frac{b}{c} = \frac{2a}{a+c}$$

$$\therefore \frac{a+b}{2a-b} = \frac{a\left(1+\frac{b}{a}\right)}{a\left(2-\frac{b}{a}\right)} = \frac{1+\frac{2c}{a+c}}{2-\frac{2c}{a+c}} = \frac{a+3c}{2a} = \frac{1}{2} + \frac{3c}{2a}$$

$$\text{and } \frac{c+b}{2c-b} = \frac{c\left(1+\frac{b}{c}\right)}{c\left(2-\frac{b}{c}\right)} = \frac{1+\frac{2c}{a+c}}{2-\frac{2c}{a+c}} = \frac{3a+c}{2a} = \frac{3a}{2c} + \frac{1}{2}$$

$$\therefore \frac{a+b}{2a-b} + \frac{c+b}{2c-b} = \frac{1}{2} + \frac{3c}{2a} + \frac{3a}{2c} + \frac{1}{2}$$

$$= 1 + \frac{3}{2} \left(\frac{c}{a} + \frac{a}{c} \right)$$

$$\geq 1 + \frac{3}{2} \cdot 2 \left(\frac{c}{a} \cdot \frac{a}{c} \right)^{\frac{1}{2}} \left[\begin{array}{l} \therefore a, b, c \text{ are the positive} \\ \text{real numbers} \\ \frac{a}{c} + \frac{a}{c} \geq \left(\frac{c}{a} \cdot \frac{a}{c} \right)^{\frac{1}{2}} \\ \Rightarrow \frac{c}{a} + \frac{a}{c} \geq 2.1 \end{array} \right]$$

$$\geq 1 + \frac{3}{2} \cdot 2.1$$

$$\geq 4$$

\therefore Statement 1 is true

$$\text{But } \frac{x+\frac{1}{x}}{2} \geq \left(x \cdot \frac{1}{x}\right)^{\frac{1}{2}} \text{ if } x > 0$$

$$\Rightarrow x + \frac{1}{x} \geq 2$$

\therefore Statement 2 is true

88. Answer (1)

We have, $30 = 2 \times 3 \times 5$

So, 2 can be assigned to either a or b or c i.e. 2 can be assigned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways.

\therefore The no. of solutions = $3 \times 3 \times 3 = 27$

\therefore Statement-1 is true

89. Answer (2)

The total no. of cases is $n(s) = 4! = 24$

Let E be the event that no letter is mailed in its correct envelope.

Then $n(E) =$ no. of favourable cases

$$\begin{aligned} &= 4! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right) \\ &= \frac{4 \cdot 3 \cdot 2!}{2!} - \frac{4 \cdot 3!}{3!} + \frac{4!}{4!} \\ &= 12 - 4 + 1 = 9 \end{aligned}$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(s)} = \frac{9}{24} = \frac{3}{8}$$

\therefore Statement-1 is True.

Also the probability that all the letters are placed in

$$\text{the correct envelope} = \frac{1}{4!} = \frac{1}{24}$$

\therefore The probability that all the letters are not placed

$$\text{in the correct envelope} = 1 - \frac{1}{24} = \frac{23}{24}$$

Hence, statement-2 is True but it does not explain statement-1.

90. Answer (2)

$$\text{Here, } m = \frac{(n+1)|x|}{|x|+1} = \frac{(12+1)\frac{11}{10}}{\frac{11}{10}+1} = \frac{13 \times 11}{\frac{21}{10}}$$

$$= \frac{143}{21} = 6.8$$

\therefore Greatest term in expansion of $(1+x)^{12}$

$$\begin{aligned} &= {}^t[m]+1 = {}^t[6.8]+1, \text{ where } [.] \text{ is G.I.F.} \\ &= t_7 \end{aligned}$$

\therefore 7th term is the greatest term

\Rightarrow Statement-1 is True

Also 7th term in expansion of $(1+x)^{12}$

$$= {}^{12}C_6 x^6$$

\therefore Binomial coefficient of 7th term is ${}^{12}C_6$ which is the greatest value of binomial coefficient ${}^{12}C_r$. But this is not the reason for which t_7 is the greatest term. Here, it is co-incident that the greatest term has the greatest binomial coefficient.

Also statement-2 is theoretically true and it is the correct explanation of statement-1.

Hence statement-2 is also true but it is not the correct explanation for statement-1.