AVIRAL CLASSES
CREATING SCHOLARS
FOR JEE (Main & advance), PMTs & FOUNDATION

DATE : 15.01.2017

FINAL TEST SERIES JEE -2017
TEST-04

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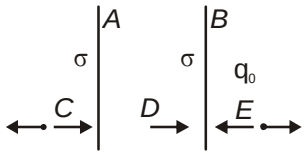
[PHYSICS]

1. Answer (2)



$$E_D = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} = \frac{2}{2\times\pi}\frac{\times 10^{-9}}{8.85\times10^{-12}} + \frac{1}{2\pi}\frac{\times10^{-9}}{8.85\times10^{-12}}$$

$$= \frac{10^{-9}\times10^{12}}{\pi\times8.85}\times\frac{3}{2} = \frac{10^3\times1.5}{8.85\times\pi} = 54$$

2. Answer (4)

torque = $pE\sin\theta$

$\tau_{max} = pE = 2\times10^{-3}$ N-m

$$W = \int_{\theta_0}^{\theta} pE\sin\theta d\theta = pE\left(\cos\theta_0 - \cos\theta\right)$$

$$= pE\left[1-(-1)\right] = 2pE = 4\times10^{-3}\ \text{J}$$

3. Answer (3)

$$C = \frac{\varepsilon_0 A}{d} = 1.77\times10^{-11}\ \text{F}$$

after dielectric is inserted, $C' = kC = 4.6\times10^{-11}$F

$Q_{initial} = CV = 5.31\times10^{-9}$C

When battery is disconnected , Q = constant,

$$\therefore\ V = \frac{Q}{kC} = 115\ \text{V}$$

Surface charge density remains constant in both the cases

$$\sigma = \frac{q}{A} = 5.31\times10^{-7}\ \text{c/m}^2$$

4. Answer (4)

The given system can be taken as combination of two spherical capacitors, both being at same potential difference and they are in parallel

$$C = 4\pi\varepsilon_0 b + 4\pi k\varepsilon_0\left(\frac{ab}{b-a}\right) = \frac{32}{15}\times10^{-10}\ \text{F}$$

5. Answer (1)

$$q_1 + q_2 = Q \qquad ...(1)$$

$$\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \qquad ...(2)$$

From (1) and (2)

$$q_1 = \frac{Qr^2}{R^2+r^2},\ q_2 = \frac{QR^2}{r^2+R^2}$$

$$V_{centre} = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{R^2+r^2}$$

6. Answer (4)

Let q' be the charge on inner shell when it is earthed. $V_{inner} = 0$

$$\therefore\ \frac{1}{4\pi\varepsilon_0}\left(\frac{q'}{r} + \frac{q}{3r}\right) = 0 \Rightarrow q' = -\frac{q}{3}$$

i.e. $+\frac{q}{3}$ charge will flow from inner shell to earth.

7. Answer (4)

A, B, C are at same potential, $W = q\Delta V = 0$

8. Answer (1)

9. Answer (4)

$$W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 10 \times 2 \times 10^{-12} \times 9 \times 10^9 \left(\frac{1}{0.6} - \frac{1}{0.8}\right)$$

$= 0.075$ J

10. Answer (2)

When key is open

$$Q = \frac{CV}{2}$$

When key is closed $Q' = CV$

Charge flow $\Delta Q = Q' - Q$

$$= \frac{CV}{2}$$

11. Answer (3)

$\because$ $\vec{E}$ between capacitor plates given is uniform,

$\therefore$ Neglecting the edge effect, effective area s A_1

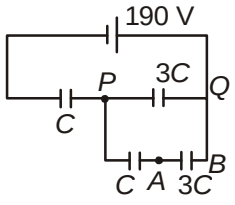
$$C = \frac{\varepsilon_0 A_1}{d}$$

12. Answer (3)

Both capacitors are in series, so charge on both is same.

$$\therefore \quad Q = \frac{C \times 2C}{C + 2C} \times 10 = 40\ \mu\text{C}$$

13. Answer (3)



$$V_P - V_Q = 190\left[\frac{C}{C + \frac{15C}{4}}\right] = 40 \text{ V}$$

It will be divided between C and $3C$ which are in series

$$V_C = \left(\frac{C}{C + 3C}\right) 40 = 10 \text{ V}.$$

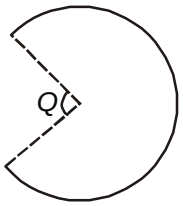
14. Answer (3)

In absence of dielectric, $Q = CV = 10\ \mu$C

with dielectric $Q' = kCV = 30\ \mu$C

$$k = \frac{30}{10} = 3$$

15. Answer (2)



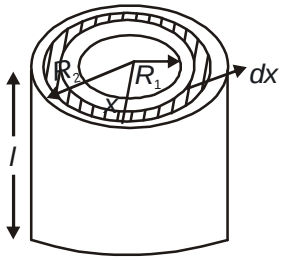
$\vec{E}$ due to an arc at its centre = $\frac{K\lambda}{R} 2\sin\frac{\theta}{2}$

Let $E = \vec{E}$ due to remaining portion

$$(E_{net})_{centre} = 0 \qquad \Rightarrow \qquad \frac{K\lambda}{R} 2\sin\frac{\theta}{2} + \vec{E} = 0$$

$$\Rightarrow |\vec{E}| = \frac{Q}{4\pi^2 \varepsilon_0 R^2} \sin\frac{\theta}{2}$$

16. Answer (1)

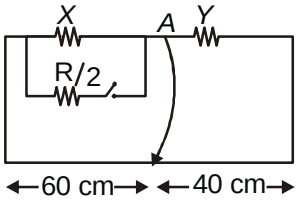


$$R = \rho \int_{R_1}^{R_2} \frac{dx}{2\pi x l}$$

$$= \frac{\rho \ln \frac{R_2}{R_1}}{2\pi l}$$

$$i = \frac{\varepsilon}{R} = \frac{2\pi l \varepsilon}{\rho \ln \frac{R_2}{R_1}}$$

17. Answer (1)



$$\frac{X}{Y}=\frac{60}{40}=\frac{3}{2} \qquad ...(1)$$

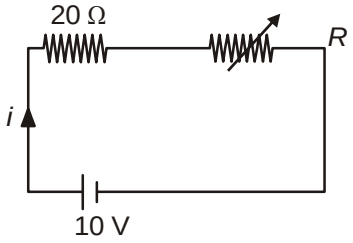
When X is shunked

$$X'=\frac{X.\frac{X}{2}}{X+\frac{X}{2}}=\frac{X}{3} \quad ...(2)$$

Now, $\frac{\left(\frac{X}{3}\right)}{Y}=\frac{l}{100-l} \Rightarrow l = 33.3$ cm

Shift = 60 – 33.3 = 26.7 cm

18. Answer (2)



$$R=\frac{5}{60}t\,\Omega/\text{s}$$

$$i=\frac{dq}{dt}=\frac{10}{20+R}$$

$$dq=\frac{10}{20+\frac{5}{60}t}dt$$

$$q=120\int_0^{240}\frac{\frac{1}{12}dt}{20+\frac{t}{12}}=120\ln 2$$

19. Answer (1)

$$f=\frac{V}{2\pi r}=6.6\times10^{15} \text{ rPs}$$

$i = ef$ = 1.06 mA

20. Answer (1)

$n = 8.85 \times 10^{28}$ m^{-3}, $\rho = 1.72 \times 10^{-8}\ \Omega$

$$\tau=\frac{m}{\eta e^2\rho}=2.4\times10^{-14}\text{s}$$

21. Answer (2)

Since $V \alpha I$

$\Rightarrow$ R is constant $\Rightarrow P = i^2R \Rightarrow P \propto i^2$

22. Answer (4)

$$V=iR=\frac{\in R}{R+r}$$

23. Answer (1)

$$R' = R + (R+1) + ...(R+n)$$

$$=(n+1)R+\frac{n(n+1)}{2}=(n+1)\left(R+\frac{n}{2}\right)$$

24. Answer (4)

$V = IR \Rightarrow \log V = \log I + \log R$

$\Rightarrow$ Straight line but not passing through origin.

25. Answer (1)

$$R_0=\frac{R_t}{1+\alpha t}=\frac{133}{1+(0.0045)\times150}=79\ \Omega$$

$$R_{500°C}=R_0\left(1+\alpha t_{500}\right)$$

$$= 79\ [1 + 0.0045 \times 500] = 257\ \Omega$$

26. Answer (3)

27. Answer (3)

28. Answer (4)

29. Answer (3)

30. Answer (1)

[CHEMISTRY]

31. Answer (2)
In S_8, sulphur is sp^3 hybridized.
32. Answer (1)
33. Answer (4)
F_2 and Cl_2 exist in gaseous state, Br_2 in liquid state and I_2 exist in solid state.
34. Answer (2)

$$M = \frac{5.6}{11.2} = \frac{1}{2}M$$

35. Answer (1)
Bicarbonates of Mg^{2+} and Ca^{2+} cause temporary hardness.
36. Answer (1)
Ag_2O on heating form Ag and O_2. So, Ag_2O is reduced on heating.

$$Ag_2O \xrightarrow{\Delta} 2Ag + \frac{1}{2}O_2$$

37. Answer (4) 38. Answer (1)
39. Answer (2)

$$MgCl_2.6H_2O \xrightarrow{\Delta} MgO + 2HCl$$

40. Answer (1)

$$MgH_2 + 2H_2O \rightarrow Mg(OH)_2 + 2H_2$$

41. Answer (4)
N_2, O_2 and PH_3 do not react with H_2O have can be collected over water.
42. Answer (4)
Hydrated P_4O_{10} form H_3PO_4 which react with basic material like NH_4OH, CaO and MgO.
43. Answer (1)

$$CO + H_2O \xrightarrow{\Delta} CO_2 + H_2$$

44. Answer (3)
45. Answer (1)
$(NH_4)_2SO_4.FeSO_4.6H_2O$ is not an alum because Fe is in +2 oxidation state.
46. Answer (2)
47. Answer (3)
48. Answer (1)
Zn on reaction with dilute HNO_3 form N_2O in gaseous form.
49. Answer (1)
Solubility order is LiF < NaF < KF < CsF.
50. Answer (4)
In order to complete octet Be form four banana bonds around it.
51. Answer (4) 52. Answer (3)
53. Answer (3) 54. Answer (4)
55. Answer (1)
$HCl + D_2O \rightarrow DCl + HOD$
56. Answer (3)
Due to cage like structure $H_2O(s)$ has higher volume than $H_2O(l)$ hence ρ of $H_2O(s)$ is lower than $H_2O(l)$.
57. Answer (2) 58. Answer (2)
59. Answer (4) 60. Answer (4)

[MATHEMATICS]

61. Answer (3)

$$f(x,y) = \begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -y & 1+y \end{vmatrix} = xy$$

62. Answer (1)

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow x + y + z = xyz$

63. Answer (2)

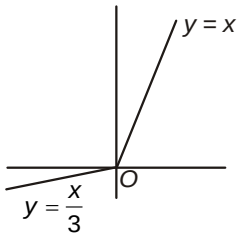
$$f(x) = \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$\Rightarrow f(x)$ is many one as well as into.

64. Answer (1)

$$\lim_{x\to 0} \frac{x(1+x) - \left(x - \frac{x^2}{2}\right)}{x^2} = \frac{3}{2}$$

65. Answer (1)



$x + |y| = 2y$

$y > 0 \Rightarrow x = y$

$y < 0 \Rightarrow x = 3y$

66. Answer (3)

$A^2 - A + I = 0$

$\Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$

$\Rightarrow A - I + A^{-1} = 0 \Rightarrow A^{-1} = I - A.$

67. Answer (4)

$$\cot^{-1}\left(\frac{(1-\sin x)-(1+\sin x)}{1-\sin x+1+\sin x-2\sqrt{1-\sin^2 x}}\right)$$

$$= \cot^{-1}\left(\frac{-2\sin x}{2-2\cos x}\right) = \cot^{-1}\left(-\frac{\sin x}{1-\cos x}\right)$$

$$= \cot^{-1}\left(-\cot\frac{x}{2}\right)$$

$$= \pi - \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$= \pi - \frac{x}{2}.$$

68. Answer (1)

$A^T = -A$

$\Rightarrow$ $(A^n)^T = (A^T)^n = (-A)^n = (-1)^n A^n = A^n$.

$\Rightarrow$ Symmetric matrix.

69. Answer (1)

$f(x) = 0 \ \forall x \in R$

70. Answer (2)

$$\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}$$

$$= \lim_{x\to 0}\frac{2f''(x)-12f''(2x)+16f''(4x)}{2}$$

$$= \frac{2f''(0)-12f''(0)+16f''(0)}{2} = 3f''(0) = 12$$

71. Answer (1)

$$\theta = \frac{\pi}{2} + \tan^{-1}x$$

$$\Rightarrow \quad \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{4}\right] \text{ as } x \in [0,1].$$

72. Answer (1)

$C_1 \to C_1 - C_2, \qquad C_2 \to C_2 - C_3,$

$$(a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}$$

$R_3 \to R_3 - R_2 - R_1$

$$(a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 2(a-b) & -2a & 2ab \end{vmatrix}$$

$= 2abc(a + b + c)^3$

73. Answer (4)

$\lim\limits_{x\to\infty}\left(\sin\sqrt{x+1} - \sin\sqrt{x}\right) = 0$ but the actual value can be positive or negative.

So, $\lim\limits_{x\to 0}\left[\sin\sqrt{x+1} - \sin\sqrt{x}\right]$ is undefined.

74. Answer (1)

$\sin^{-1}\sin 12 = \sin^{-1}\sin(12 - 4\pi) + \cos^{-1}(\cos(4\pi - 12)$

$= 12 - 4\pi + 4\pi - 12 = 0$

75. Answer (4)

$$\lim_{n\to\infty}\frac{s_1\left(1+\frac{s_3}{8}\right)}{s_2^2} = \lim_{n\to\infty}\frac{\frac{n(n+1)}{2}\left(1+\frac{n^2(n+1)^2}{32}\right)}{\frac{n^2(n+1)^2(2n+1)^2}{36}}$$

$$\frac{36}{2\times 32}\lim_{n\to\infty}\frac{32+n^2(n+1)^2}{n(n+1)(2n+1)^2}$$

$$= \frac{9}{16}\times\frac{1}{4} = \frac{9}{64}$$

76. Answer (2)

$(x, x) \Rightarrow x^2 + x^2 = 1$ (Not true for $\forall x \in R$)

Hence not reflexive so not identity relation

$(y, x) \Rightarrow y^2 + x^2 = 1$ (True if $x^2 + y^2 = 1$)

Hence symmetric

Also $(x, y) \Rightarrow x^2 + y^2 = 1$

and $(y, z) \Rightarrow y^2 + z^2 = 1$

$x^2 + z^2 = 1$

Hence not transitive.

77. Answer (4)

R_4 is identify relation, Hence it is equivalence relation.

78. Answer (2)

$x = y\ln xy$

$$1 = y'\ln(xy) + \frac{y(y+xy')}{xy}$$

$$\Rightarrow \quad 1 = y'\cdot\frac{x}{y} + \frac{y}{x} + y'$$

$$\Rightarrow \quad (x-y)\frac{y}{x} = y'(x+y)$$

$$\Rightarrow \quad y' = \left(\frac{x-y}{x+y}\right)\frac{y}{x}.$$

79. Answer (3)

$$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}}{\frac{1}{x}}$$

$= \dfrac{xe^{\sin^{-1}x}}{\sqrt{1-x^2}}$

80. Answer (4)

Fundamental period $= \dfrac{2\pi}{\sqrt{[a]}} = \pi$

$\Rightarrow \sqrt{[a]} = 2$

$\Rightarrow [a] = 4$

$\Rightarrow a \in [4, 5)$

81. Answer (4)

$$\lim_{x\to 0^+} e^{\frac{\ln \operatorname{cosec} x}{\ln x}}$$

$$= \lim_{x\to 0^+} e^{\frac{\frac{-1}{\operatorname{cosec} x}\cdot \operatorname{cosec} x \cdot \cot x}{\frac{1}{x}}}$$

$$= \lim_{x\to 0^+} e^{\frac{-x}{\tan x}} = \frac{1}{e}$$

82. Answer (1)

$$\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a^3 & (a+1)^3 - a^3 & (a+2)^3 - a^3 \\ a & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow 2[(a + 1)^3 - a^3] - [(a + 2)^3 - a^3] = 0$

$2(3a^2 + 3a + 1) - (3a^2 \times 2 + 3a \times 4 + 8) = 0$

$-6a - 6 = 0$

$\Rightarrow a = -1.$

83. Answer (2)

$(\text{adj}A)\cdot(\text{adj}(\text{adj}A)) = |\text{adj}A| I_n$

$\Rightarrow A(\text{adj}A)\cdot(\text{adj}(\text{adj}A)) = |A|^{n-1} I_n \cdot A$

$\Rightarrow |A|\text{adj}(\text{adj}A) = |A|^{n-1}A$

$\Rightarrow \text{adj}(\text{adj}A) = |A|^{n-2}A.$

84. Answer (2)

$$y = \cos\tan^{-1}\sin\frac{\pi}{6} = \cos\tan^{-1}\frac{1}{2}$$

$$= \cos\cos^{-1}\frac{2}{\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}.$$

85. Answer (4)

$f(1^-) = f(1) = f(1^+)$

$\Rightarrow a - b = 2$

86. Answer (3)

S-1 : $f'(0^+) = \lim\limits_{h\to 0^+} \dfrac{|h|\cosh}{h} = 1$

$$f'(0^-) = \lim_{h\to 0^+} \frac{|-h|\cos(-h)}{-h} = -1$$

Hence not differentiable at $x = 0$

S-2 : $y = |x^2|$ is differentiable everywhere, hence '*R*' is false.

87. Answer (4)

S-1 : $\{1, 2\} \subset \{1, 2, 3\}$ but $\{1, 2, 3\} \not\subset \{1, 2\}$
Hence relation is not symmetric.

S-2 : True.

88. Answer (3)

S-1 : $|A| = 14 - (1 + 4) = 9$

S-2 : Let $A = \begin{bmatrix} 1+i & 1 \\ 1 & 1+i \end{bmatrix}$

$\Rightarrow |A| = (1 + i)^2 - 1 = 2i - 1$ (not real)
Hence Statement-2 is false.

89. Answer (3)

We have,

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x\sin\left(\dfrac{1}{x}\right) - \cos\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is discontinuous at $x = 0$, because $\cos\left(\dfrac{1}{x}\right)$ has no limit as $x \to 0$, Statement-1 is true, Statement-2 is false.

90. Answer (4)

S-1 : consider $f(x) = \dfrac{1}{x}$

$\Rightarrow f^{-1}(x) = \dfrac{1}{x}$

So, the curves are equal *i.e.* they meet on all points in the domain.

S-2 : $y = f(x)$ and $y = f^{-1}(x)$ are image of each other in $y = x$ and image of (α, β) in $y = x$ is (β, α).