



AVIRAL CLASSES

CREATING SCHOLARS

FOR JEE (Main & advance), PMTs & FOUNDATION

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FINAL TEST SERIES JEE -2017

BRANCH OFFICE-1 : HO : World Trade Tower, 99- Rajpur Road. Dehradun (U.K.)

BRANCH OFFICE-2 : GMS Road, Ballupur Chowk, Dehradun (U.K.)

PH: 0135-6557701 / 02 / 03, **MOB.:** 9045062312 / 14 / 15 / 16 / 17 / 18 / 20. **web:** www.aviral.co.in

1. Answer (3)

$$g = (\tan\theta)x - \left(\frac{g}{2u\cos^2\theta}\right)x^2$$

$$= \tan\theta \left[x - \frac{g}{2u\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \cdot x^2 \right]$$

$$= \tan\theta \left[x - \frac{g}{2u^2 \cdot \sin\theta \cos\theta} \cdot x^2 \right]$$

$$= \tan\theta \left[x - \frac{x^2}{R} \right] = x \tan\theta \left[1 - \frac{x}{R} \right]$$

Comparing with $y = x - x^2 \Rightarrow R = 1 \text{ m}$

2. Answer (4)

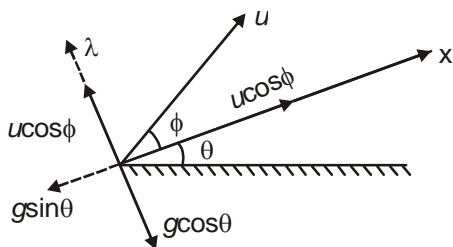
$$\vec{v} = \hat{i} + 2\hat{j} \Rightarrow v = \sqrt{5} \text{ ms}^{-1}, \vec{a} = 2\hat{i} - 3\hat{j} \Rightarrow a = \sqrt{13} \text{ ms}^{-2}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{v}}{av} = \frac{2 - 6}{\sqrt{5}\sqrt{13}} = \frac{-4}{\sqrt{5}\sqrt{13}}$$

$$\Rightarrow \sin\theta = \sqrt{1 - \frac{16}{5 \times 13}} = \frac{7}{\sqrt{65}}$$

Now $a \sin\theta = \frac{v^2}{R} \Rightarrow R = \frac{5\sqrt{5}}{7} \text{ m}$

3. Answer (2)



Time of flight $T = \frac{2u \sin\phi}{g \cos\theta} \dots(1)$

Along the plane

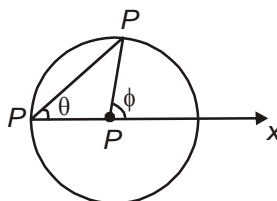
$$v_x = u_x + ax.t$$

$$0 = u \cos\phi - g \sin\theta \times \frac{2u \sin\phi}{g \cos\theta}$$

$$\Rightarrow \frac{1}{2} = \tan\theta \times \tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \theta + \phi = \frac{\pi}{6} + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

4. Answer (4)



Given: $\frac{d\theta}{dt} = \omega = 1 \text{ rad/s}$

Now $\phi = 2\theta \Rightarrow \frac{d\phi}{dt} = \omega' = 2\omega = 2 \text{ rad/s}$

Then, $a = \omega'^2 \cdot R = 4 \times 1 \text{ cm/s}^2 = 4 \text{ cm/s}^2$

5. Answer (4)

$$\left(\frac{\alpha E}{mv}\right) = [M^0 L^0 T^0] \Rightarrow \alpha = \left[\frac{MLT^{-1}}{ML^2T^{-2}}\right] = [L^{-1}T]$$

Now $[MLT^{-2}] = \frac{\beta}{\alpha} \Rightarrow \beta = [MLT^{-2}][L^{-1}T] = [ML^0T^{-1}]$

$$\Rightarrow [\sqrt{\beta}] = [M^{\frac{1}{2}}L^0T^{-\frac{1}{2}}]$$

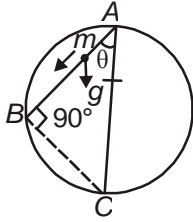
6. Answer (4)

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = 4\pi^2 \frac{l}{T^2} \Rightarrow$$

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2 \frac{\Delta T}{T} \times 100$$

$$\begin{aligned}
 &= \frac{\Delta l}{l} \times 100 + 2 \frac{\Delta t}{t} \times 100 \\
 &= \frac{0.1}{100.3} \times 100 + 2 \frac{0.5}{10.5} \times 100 \\
 &= \frac{100}{1003} + 2 \times \frac{500}{105} = 9.6\%
 \end{aligned}$$

7. Answer (3)



$$AB = l = \frac{1}{2} g \cos \theta \cdot t^2 \Rightarrow t = \sqrt{\frac{2l}{g \cos \theta}}$$

Now $\triangle ABC$, $\frac{l}{2R} = \cos \theta$

$$\Rightarrow t = \sqrt{\frac{2 \times 2R \cos \theta}{g \cos \theta}} = 2\sqrt{\frac{R}{g}}$$

8. Answer (4)

For A, $W = \int dW = \int x dx + y dy = \int_{x_1}^{x_2} x dx + \int_{y_1}^{y_2} y dy$

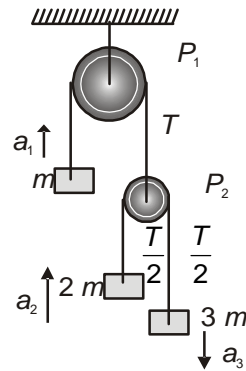
$$= \frac{1}{2} [(x_2^2 - x_1^2)] [(y_2^2 - y_1^2)]$$

For B, $W = \int x dy + y dx = \int_{x_1, y_1}^{x_2, y_2} d(xy) = [xy]_{x_1, y_1}^{x_2, y_2} = (x_2 y_2 - x_1 y_1)$

For C, $W = \int xy^2 dx + yx^2 dy = \frac{1}{2} \int d(x^2 y^2) = \left[\frac{1}{2} x^2 y^2 \right]_{x_1, y_1}^{x_2, y_2}$

For D, $W = \int y^2 dx + \int x^2 dy$
= Work done is path independent

9. Answer (3)



For A, $T - mg = ma_1 \dots(1)$

For B, $\frac{T}{2} - 2mg = 2ma_2 \dots(2)$

For C, $3mg - \frac{T}{2} = 3ma_3 \dots(3)$

For P₂, $a_1 = \frac{a_3 - a_2}{2} \dots(4)$

From (4) $2a_1 = a_3 - a_2 \Rightarrow a_3 = 2a_1 + a_2 \dots(5)$

From (1) and (2) $\frac{T}{2} - \frac{mg}{2} = \frac{ma_1}{2}$

$$\frac{T}{2} - 2mg = 2ma_2$$

$$\frac{3}{2} mg = m \left[\frac{a_1}{2} - 2a_2 \right] \Rightarrow 3g = a_1 - 4a_2 \dots(6)$$

From (2)&(3) $3mg - \frac{T}{2} = 3ma_3$

$$\frac{T}{2} - 2mg = 2ma_2 \Rightarrow g = 3a_3 + 2a_2 \dots(7)$$

$$mg = m[3a_3 + 2a_2]$$

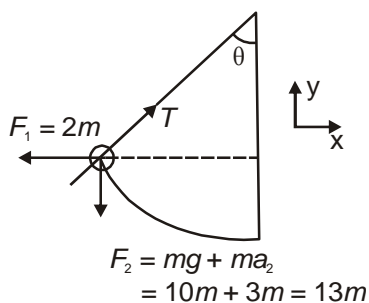
From (5) & (7)

$$g = 3(2a_1 + a_2) + 2a_2 = 6a_1 + 5a_2 \dots(8)$$

From (6) & (8)

$$a_2 = -\frac{17}{29}g$$

10. Answer (1)



FBD of pendulum bob. w.r.t. box.

Work energy theorem

$$2 ml \sin\theta - 13 ml (1 - \cos\theta) = 0$$

$$2 l \sin\theta = 13 l (1 - \cos\theta)$$

$$2 \times 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 13 \times 2 \sin^2 \frac{\theta}{2}$$

$$\Rightarrow \cot \frac{\theta}{2} = \frac{13}{2} \Rightarrow \theta = 2 \cot^{-1} \left(\frac{13}{2} \right)$$

11. Answer (2)

For A, Work energy theorem

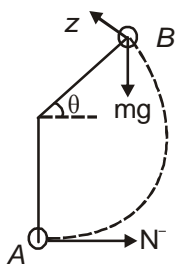
$$Fx = \frac{1}{2} kx^2 - \mu mgx = \Delta k = 0$$

$$F = \mu mgx = \frac{kx}{2} \dots(1)$$

For B to slide $kx = (2\mu)(2mg) = 4\mu mg \dots(2)$

Thus $F_{\min} = \mu mg + 2\mu mg = 3\mu mg$

12. Answer (3)



Energy conservation of A and B

$$v'^2 = v^2 - 2gl(1 + \sin\theta) \dots(1)$$

For circular motion at B

$$mg \sin\theta = \frac{mv'^2}{l} \dots(2)$$

$$\Rightarrow gl \sin\theta = v^2 - 2gl(1 + \sin\theta)$$

$$gl \sin\theta = 4gl - 2gl(1 + \sin\theta)$$

$$\sin\theta = 4 - 2(1 + \sin\theta) \Rightarrow \sin\theta = 4 - 2 - 2\sin\theta$$

$$3\sin\theta = 2 \Rightarrow \sin\theta = \frac{2}{3}$$

$$\text{Then } \cos\theta = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}}$$

$$\text{Tangential acceleration} = g \cos\theta = \frac{g\sqrt{5}}{3}$$

13. Answer (3)

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3} \text{ K}\Omega$$

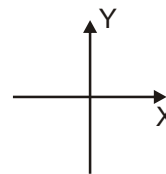
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\frac{\Delta R}{R} = \frac{10}{3} \left(\frac{0.2}{25} + \frac{0.1}{100} \right)$$

$$\Rightarrow \frac{\Delta R}{R} \times 100 = \frac{10}{3} \left(\frac{2}{250} + \frac{1}{1000} \right) = 3\%$$

$$\Rightarrow r = 3.3 \text{ k}\Omega \pm 3\%$$

14. Answer (4)



Given

$$\vec{v}_A = 5\hat{i} \dots(1)$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 6\cos\theta\hat{i} + 6\sin\theta\hat{j}$$

$$= 6 \times \frac{1}{2} \hat{i} + 6 \frac{\sqrt{3}}{2} \hat{j}$$

$$\vec{v}_B - \vec{v}_A = 3\hat{i} + 3\sqrt{3}\hat{j} \dots(2)$$

$$\vec{v}_C - \vec{v}_B = -10\hat{i} \dots(3)$$

From (2) & (3)

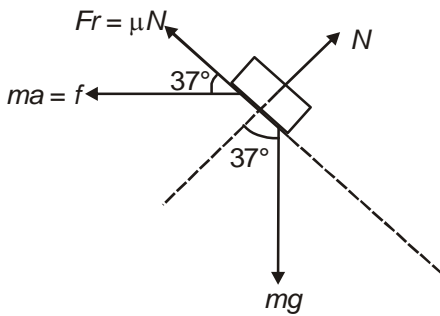
$$\vec{v}_B - 5\hat{j} = 3\hat{i} + 3\sqrt{3}\hat{j} \Rightarrow \vec{v}_B = 8\hat{i} + 3\sqrt{3}\hat{j} \dots(4)$$

From (3) & (4)

$$\vec{v}_C = \vec{v}_B - 10\hat{i} = 8\hat{i} + 3\sqrt{3}\hat{j} - 10\hat{i} = -2\hat{i} + 3\sqrt{3}\hat{j}$$

$$v_C = \sqrt{4 + 27} = \sqrt{31} \text{ ms}^{-1}$$

15. Answer (3)



$$a = \frac{F}{M+m} \dots(1)$$

$$N = ma \sin 37^\circ + mg \cos 37^\circ \dots(2)$$

$$mg \sin 37^\circ = \mu N + ma \cos 37^\circ \dots(3)$$

$$\text{From (2)} \quad N = ma \times \frac{3}{5} + mg \times \frac{4}{5} = \frac{1}{5}[3ma + 4mg]$$

$$\Rightarrow mg \times \frac{3}{5} = \mu \cdot \frac{1}{5}[3ma + 4mg] + \frac{4}{5} ma$$

$$\frac{3}{5} mg = \frac{1}{10}[3ma + 4mg] + \frac{4}{5} ma$$

$$= \frac{3}{10} ma + \frac{4}{10} mg + \frac{4}{5} ma$$

$$\frac{6}{10} mg = \frac{11}{10} ma + \frac{4}{10} mg$$

$$\Rightarrow \frac{2}{10} mg \cdot \frac{11}{10} ma \Rightarrow a = \frac{2}{11} g$$

$$\text{Then } F_{mi} = (M+m)a = 8 \times \frac{2}{11} g = \frac{6}{11} g$$

16. Answer (3)

$$\vec{v} = \hat{i} - \hat{j} \text{ ms}^{-1} \Rightarrow v = \sqrt{2} \text{ ms}^{-1}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} \text{ ms}^{-2} \Rightarrow a = \sqrt{9+4} = \sqrt{13} \text{ ms}^{-2}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{v}}{av} = \frac{3-2}{\sqrt{13}\sqrt{2}} = \frac{1}{\sqrt{26}}$$

$$a_T = a \cos \theta = \sqrt{13} \times \frac{1}{\sqrt{26}} = \frac{1}{\sqrt{2}} \text{ ms}^{-2}$$

$$F_T = ma_T = \frac{5}{\sqrt{2}} \text{ N} \Rightarrow P = F_T v = \frac{5}{\sqrt{2}} \times \sqrt{2} = 5 \text{ watt}$$

17. Answer (3)

$$P = \vec{F} \cdot \vec{v} = Fv = mav = mv \frac{d\theta}{dt}$$

$$v \frac{dv}{dt} = \frac{P}{m} \Rightarrow \frac{v^2}{2} = \frac{P}{m} \cdot t \Rightarrow v = \sqrt{\frac{2P}{m}} \cdot t^{1/2} \Rightarrow v \propto \sqrt{t}$$

$$\Rightarrow v \propto \frac{1}{\sqrt{P}}$$

$$\frac{ds}{dt} = \sqrt{\frac{2P}{m}} \cdot t^{1/2} \Rightarrow s \propto t^{3/2}, v \propto \sqrt{P}, v \propto \sqrt{t}$$

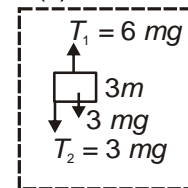
18. Answer (2)

In equilibrium

$$T_1 = 6mg \dots(1)$$

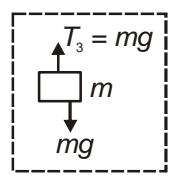
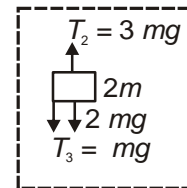
$$T_2 = 3mg \dots(2)$$

$$T_3 = mg \dots(3)$$



When string (1) is cut.

$T_1 = 0$; T_2 & T_3 unchanged



Thus,

Acceleration of $m_1 = g \downarrow$

$$\text{Acceleration of } 2m_1 = \frac{3mg - 2mg}{2m} = \frac{g}{2} \uparrow$$

Acceleration of $3m_1 = 0$

19. Answer (3)

$$KE_{\text{blocks}} = 2 \times \frac{1}{2} mv^2 = mv^2 \dots(1)$$

$$KE_{\text{spring}} = 2 \int dk = 2 \int \frac{1}{2} (dm)(v_x)^2$$

$$= \int \left(\frac{m}{l} dx \right) \left(\frac{v}{l/2} \cdot x \right)^2 = \int_{x=0}^{x=l/2} \frac{m}{l} \cdot dx \times \left(\frac{2v}{l} \cdot x \right)^2$$

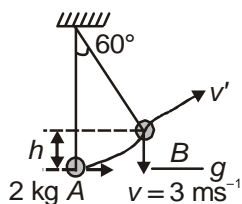
$$= \int \frac{m}{l} \cdot dx \times \frac{4v^2}{l^2} x^2$$

$$= \frac{4mv^2}{l^3} \times \left(\frac{x^3}{3} \right)_0^{l/2} = \frac{4mv^2}{l^3} \times \frac{1}{3} \times \frac{l^3}{8}$$

$$= \frac{mv^2}{6}$$

$$TKE = mv^2 + \frac{mv^2}{6} = \frac{7}{6} mv^2$$

20. Answer (3)



$$v'^2 = v^2 - 2g(l - l \cos 60^\circ)$$

$$= v^2 - 2gl \times \frac{1}{2}$$

$$= v^2 - gl = 9 - 10 \times 0.5$$

$$v'^2 = 4 \Rightarrow v' = 2 \text{ ms}^{-1}$$

Now at B $K = \frac{1}{2}mv'^2$

$$\frac{dk}{dt} = \frac{1}{2} \times m \times 2v' \frac{dv'}{dt}$$

$$mv' \frac{dv'}{dt} = 2 \times 2 \times g \sin 60^\circ$$

$$= 2 \times 2 \times 10 \times \frac{\sqrt{3}}{2} = 20\sqrt{3} \text{ Joule / s}$$

21. Answer (4)

Equation of path $y = 2 + x$

Now $W = \int dW = \int F_x dx + F_y dy$

$$\Rightarrow W = \int x^2(x+2) dx + 2(y-2) dy$$

$$= \int (x^3 + 2x^2) dx + \int (2y - 4) dy$$

$$= \left[\frac{2}{3}x^3 + \frac{x^4}{4} \right]_0^2 + \left[2 \cdot \frac{y^2}{2} - 4y \right]_2^4$$

$$= \left[\frac{2}{3} \times 8 + 4 \right] + [y^2 - 4y]_2^4$$

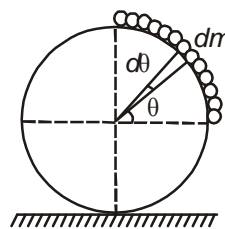
$$= \left[\frac{16}{3} + 4 \right] + [(16 - 4) - 4(4 - 2)]$$

$$= \frac{28}{3} + (12 - 8) = \frac{28}{3} + 4 = \frac{28 + 12}{3} = \frac{40}{3} \text{ J}$$

22. Answer (2)

KE can be increased by internal as well as external forces.

23. Answer (4)



$$U_i = \int dU_i = \int dm \cdot gh = \int \frac{m}{l} \cdot R d\theta \times g \times R \sin \theta$$

$$= \frac{mgR^2}{l} \cdot [-\cos \theta]_0^{\pi/2} = \frac{4mgR}{\pi^2}$$

$$U_f = -\frac{mgR}{2}$$

$$\text{Loss} = U_i - U_f = mgR \left[\frac{1}{2} + \frac{4}{\pi^2} \right]$$

$$\frac{1}{2}mv^2 = mgR \left[\frac{1}{2} + \frac{4}{\pi^2} \right] \Rightarrow v = \sqrt{2gR \left[\frac{1}{2} + \frac{4}{\pi^2} \right]}$$

24. Answer (2)

For no slipping between the blocks

$$F_{\max} = \frac{\mu m_1 g}{m_2} (m_1 + m_2) = \frac{0.5 \times 4 \times 10}{6} \times 10 = \frac{100}{3} \text{ N}$$

Since applied force $F = 30 \text{ N}$ which is less than $\frac{100}{3} \text{ N}$

Hence between block $a_{\text{rep}} = 0$.

$$\text{Thus a common } a = \frac{30}{10} = 3 \text{ ms}^{-2}$$

$$\text{Then } F_f = 6 \times 3 = 18 \text{ N}$$

25. Answer (3)

Using work-energy theorem

$$3mgR - mgR = \frac{1}{2}mv^2 \Rightarrow 2mgR = \frac{1}{2}mv^2$$

$$v = 2\sqrt{gR}$$

$$\Rightarrow N = \frac{mv^2}{R} = \frac{m}{R} \times 4gR = 4mg$$

26. Answer (4)

27. Answer (4)

28. Answer (3)

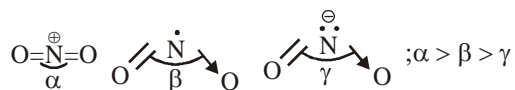
29. Answer (2)

30. Answer (1)

[CHEMISTRY]

31. Answer (2)

32. Answer (2)

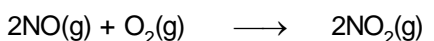


33. Answer (3)

34. Answer (1)

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1 \times 10^{-6} \times 20} = 3.3 \times 10^{-29} \text{ m}$$

35. Answer (4)



$$60 \text{ g} \quad 32 \text{ (g)} \quad \quad \quad 46 \times 2 \text{ g}$$

32 g of O₂ requires 60 g NOHence 3.2 g of O₂ requires 6 g NO

∴ NO is limiting reagent

Now 60 g NO produces 92 g NO₂

$$\text{Hence } 4.2 \text{ g NO produces } \frac{92}{60} \times 4.2 \text{ g NO}_2 = 6.44 \text{ g}$$

36. Answer (1)

37. Answer (3)

$$\text{Number of moles of NaCl} = \frac{5.85}{58.5} = 0.1 \text{ mole}$$

Number of moles of NaCl in 1 mL of solution

$$= \frac{0.1}{1000} = 10^{-4} \text{ mole}$$

∴ Total number of ions

$$= 6.022 \times 10^{23} \times 2 \times 10^{-4} = 1.2 \times 10^{20}$$

38. Answer (1)

Number of moles

$$= \frac{10^{22}}{6.022 \times 10^{23}} = \frac{10 \times 10^{-2}}{6.022} = 1.66 \times 10^{-2} \text{ mole}$$

$$\text{Weight} = 1.66 \times 10^{-2} \times 249.5 = 4.144 \text{ g}$$

39. Answer (1)

40. Answer (2)



Last electron enters 3d orbital

Hence, $n = 3$

$$l = 2$$

$$m_l = -2, -1, 0, 1, 2$$

$$m_s = \pm \frac{1}{2}$$

41. Answer (3)

29 → [Ar]₁₈ 3d¹ 4s¹ which belongs to d-block.

42. Answer (4)

43. Answer (4)

44. Answer (4)

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left[\frac{1}{3^2} - \frac{1}{n^2} \right]$$

$$\Rightarrow v = 3 \times 10^8 \times 1.1 \times 10^7 \left[\frac{1}{9} - \frac{1}{n^2} \right]$$

Putting the values of n from the options only 5 satisfies the equation.

Hence answer is (4).

45. Answer (2)

Number of moles of H₃PO₄ formed = 0.85 × 0.9 × 4 × number of moles of P₄ used

$$\frac{W_{\text{H}_3\text{PO}_4}}{98} = 0.85 \times 0.9 \times 4 \times \frac{62}{124}$$

$$\begin{aligned} \Rightarrow W_{\text{H}_3\text{PO}_4} &= 0.85 \times 0.9 \times 4 \times \frac{62}{124} \times 98 \\ &= 149.94 \text{ g} \end{aligned}$$

46. Answer (1)

$$\text{Equivalent weight of } \text{K}_2\text{Cr}_2\text{O}_7 = \frac{294}{6} = 49$$

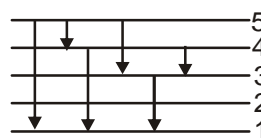
$$\text{Equivalent weight of } \text{KMnO}_4 = \frac{158}{5} = 31.6$$

meq. of KMnO₄ + meq. of K₂Cr₂O₇ = meq. of I₂ = meq. of hypo.Let the mass of K₂Cr₂O₇ = x g;∴ Mass of KMnO₄ = (0.5 - x) g

$$\frac{x}{49} + \frac{0.5 - x}{31.6} = 150 \times 0.1 \times 10^{-3} \Rightarrow x = 0.0732$$

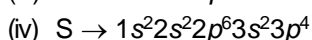
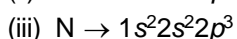
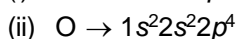
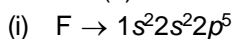
$$\therefore \% \text{ of } \text{K}_2\text{Cr}_2\text{O}_7 = \frac{0.0732}{0.5} \times 100 = 14.64$$

47. Answer (4)



Total radiations are = 6.

48. Answer (3)



49. Answer (1)

Ionisation enthalpy decreases down the group.

50. Answer (2)

Fact.

51. Answer (1)

Number of spherical nodes = $n - l - 1$.

52. Answer (3)

53. Answer (3)



$$1 - x + 2x = 1.5 \Rightarrow x = 0.5 \text{ L}$$

Hence volume of CO produced = 1 L

$$\text{Number of moles} = \frac{1}{22.4}$$

54. Answer (2)

55. Answer (2)

1 mole S_8 gives 8 mole $\text{SO}_3 = 8 \times 80 = 640 \text{ g}$

56. Answer (4)

57. Answer (1)

58. Answer (4)

59. Answer (2)

60. Answer (1)

[MATHEMATICS]

61. Answer (2)

Let sides be a, b, c where c is the hypotenuse.

$$a^2 + b^2 + c^2 = 294$$

$$2c^2 = 294$$

$$c^2 = 147 \Rightarrow \boxed{c = 7\sqrt{3}}$$

$$\boxed{a^2 + b^2 = 147}$$

$$a + b + 7\sqrt{3} = 12 + 8\sqrt{3}$$

$$a + b = 12 + \sqrt{3}$$

$$(a + b)^2 = (12 + \sqrt{3})^2$$

$$a^2 + b^2 + 2ab = 147 + 24\sqrt{3}$$

$$\boxed{ab = 12\sqrt{3}}$$

$$\Delta = 6\sqrt{3}$$

$$\left[\frac{\Delta}{3} \right] = [2\sqrt{3}] = 3$$

62. Answer (3)

$$= \left(\sin \frac{\pi}{14} \cdot \sin \frac{3\pi}{14} \cdot \sin \frac{5\pi}{14} \right)^2$$

$$= \left(\cos \frac{\pi}{7} \cdot \cos \frac{2\pi}{7} \cdot \cos \frac{4\pi}{7} \right)^2$$

$$= \left(\frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}} \right)^2 = \frac{1}{64}$$

63. Answer (2)

$$\tan^2 x = 0$$

$$\Rightarrow x = 0, \pi, 2\pi, 3\pi, 4\pi$$

64. Answer (3)

$$(\log_7 x)^2 + (\log_7 x) - 2 < 0$$

$$(\log_7 x + 2)(\log_7 x - 1) < 0$$

$$\log_7 x \in (-2, 1)$$

$$x \in \left(\frac{1}{49}, 7 \right)$$

65. Answer (4)

$$2a^2 + 2b^2 + 2ab - 2a + 2b + 2 \leq 0$$

$$(a + b)^2 + (a - 1)^2 + (b + 1)^2 \leq 0$$

$$a = 1, b = -1 \Rightarrow 7a + 3b = 4$$

66. Answer (3)

$$\alpha + \beta = 1 - \sin \phi$$

$$\alpha\beta = -\frac{1}{2} \cos^2 \phi$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (1 - \sin \phi)^2 + \cos^2 \phi = 2 - 2\sin \phi$$

$$(\alpha^2 + \beta^2)_{\max} = 4$$

67. Answer (4)

$$(x - \alpha)(x - \beta)(x - \gamma)(x - \delta) = 9$$

$$r - \alpha, r - \beta, r - \gamma, r - \delta \equiv \pm 1, \pm 3$$

$$\alpha, \beta, \gamma, \delta \equiv r + 1, r - 1, r + 3, r - 3$$

$$2\alpha + 2\beta + 2\gamma + 2\delta - 6r = 2r$$

68. Answer (4)

$$\alpha + \beta + \gamma = 1$$

$$\beta = \frac{1}{3}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = 3p \quad \Rightarrow \quad \beta(\alpha + \gamma) + \alpha\gamma = 3p$$

$$3p = \frac{1}{3} \times \frac{2}{3} + \alpha\gamma$$

$$3p = \frac{2}{9} + \left(\frac{1}{3} - d\right)\left(\frac{1}{3} + d\right) = \frac{1}{3} - d^2$$

$$p = \frac{1}{9} - \frac{1}{3}d^2 \quad \Rightarrow \quad p_{\max} = \frac{1}{9}$$

69. Answer (2)

70. Answer (1)

$$|x - 1| + |x^2 + x + 1| = |x(x + 2)|$$

$$x < -2$$

$$1 - x + x^2 + x + 1 = x^2 + 2x \quad \text{No solution}$$

$$-2 \leq x < 0$$

$$1 - x + x^2 + x + 1 = -x^2 - 2x \quad \text{No solution}$$

$$0 \leq x < 1$$

$$1 - x + x^2 + x + 1 = x^2 + 2x \quad \text{No solution}$$

$$x \geq 1$$

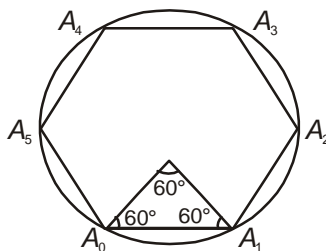
$$x - 1 + x^2 + x + 1 = x^2 + 2x \quad \Rightarrow \quad x \in [1, \infty)$$

71. Answer (1)

By observation

$$x = 0, x = \frac{3\pi}{2}, x = 3\pi$$

72. Answer (3)



$$A_0A_1 = 1, A_0A_2 = \sqrt{3}, A_0A_4 = \sqrt{3}$$

73. Answer (2)

$$3^x - 2^{2y} = 77$$

$$\left(3^{\frac{x}{2}} - 2^y\right)\left(3^{\frac{x}{2}} + 2^y\right) = 77$$

$$3^{\frac{x}{2}} - 2^y = 7 \quad \Rightarrow \quad 3^{\frac{x}{2}} + 2^y = 11$$

$$\Rightarrow 3^{\frac{x}{2}} = 9 \quad \Rightarrow x = 4 \quad \Rightarrow y = 1$$

74. Answer (2)

$$\frac{x}{x+6} - \frac{1}{x} \leq 0$$

$$\frac{x^2 - x - 6}{x(x+6)} \leq 0 \quad \Rightarrow \quad \frac{(x-3)(x+2)}{x(x+6)} \leq 0$$

$$x \in (-6, -2] \cup (0, 3]$$

75. Answer (4)

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 121 \quad \Rightarrow \quad x^2 + \frac{1}{x^2} = 11$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 9 \quad \Rightarrow \quad x - \frac{1}{x} = \pm 3$$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$x^3 - \frac{1}{x^3} = 27 + 3 \times 3 = 36$$

76. Answer (3)

$$\log_{\sqrt{2}}(16\sin^2 x + 1) \Big|_{\min} = 0$$

Hence $(-\infty, 1]$

77. Answer (4)

$$|x(x+5)| + |x(x-1)| = |6x|$$

$$x < -5$$

$$x^2 + 5x + x^2 - x = -6x$$

$$2x^2 + 10x = 0$$

$$x = 0, -5$$

No solution

$$-5 \leq x \leq 0$$

$$-x^2 - 5x + x^2 - x = -6x$$

$$x \in [-5, 0]$$

$$0 < x \leq 1$$

$$x^2 + 5x - x^2 + x = 6x$$

$$x \in (0, 1]$$

$$x > 1$$

$$x^2 + 5x + x^2 - x = 6x$$

$$2x^2 - 2x = 0$$

No solution

Hence $x \in [-5, 1]$

78. Answer (1)

$$0 < e^x < 1 \Rightarrow x \in (-\infty, 0)$$

$$0 < \ln|x| < 1 \Rightarrow 1 < |x| < e \Rightarrow x \in (-e, -1) \cup (1, e)$$

Combined $\Rightarrow x \in (-e, -1)$

$$[-e - 1 - 0.7] = -5$$

79. Answer (4)

$$f(x) = \{x\} + \{x+1\} + \{x+2\} + \dots + \{x+99\}$$

$$= 100\{x\}$$

$$f(\sqrt{2}) = 100\{\sqrt{2}\} = 100(\sqrt{2} - 1)$$

$$\lceil f(\sqrt{2}) \rceil = \lceil 41.4\dots \rceil = 42$$

80. Answer (4)

$$-3 \leq \lfloor x \rfloor \leq 2$$

$$\Rightarrow \lfloor x \rfloor \leq 2$$

$$\Rightarrow -2 \leq \lfloor x \rfloor \leq 2$$

$$\Rightarrow x \in [-2, 3)$$

81. $f(x) = \sin^8 + \cos^8$

$$= (\sin^4 x + \cos^4 x)^2 - 2\sin^4 x \cos^4 x$$

$$= \left(1 - \frac{\sin^2 2x}{2}\right)^2 - \frac{1}{8} \sin^4 2x$$

$$= \frac{1}{8} [4 - \sin 2x]^2 - 1$$

$$f(\min) = \frac{9}{8} - 1 = \frac{1}{8}$$

$$f(\max) = \frac{1}{8} [4 - 0]^2 - 1$$

$$= 2 - 1$$

$$= 1$$

82. Answer (2)

$$x = \cot \frac{11\pi}{8} = \tan \frac{\pi}{8} = \sqrt{2} - 1$$

$$\Rightarrow x^2 + 2x - 1 = 0$$

$$A = x^4 + 4x^3 + 2x^2 - 4x + 7 = x^2(x^2 + 2x - 1) +$$

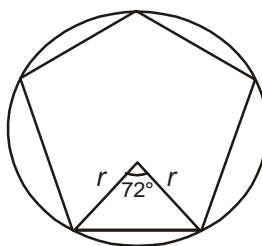
$$2x(x^2 + 2x - 1) - (x^2 + 2x - 1) + 6 = 6$$

$$B = \frac{1 - \cos 8\theta}{\tan^2 4\theta} + \frac{1 + \cos 8\theta}{\cot^2 4\theta} = \frac{2 \sin^2 4\theta}{\tan^2 4\theta} + \frac{2 \cos^2 4\theta}{\cot^2 4\theta}$$

$$= 2 \cos^2 4\theta + 2 \sin^2 4\theta = 2$$

$$AB = 12 \Rightarrow C = 1 + 2 = 3$$

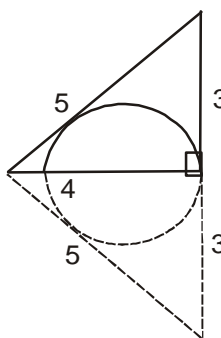
83. Answer (2)



$$\text{Area} = 5 \times \frac{1}{2} r^2 \sin 72^\circ$$

$$\text{Hence ratio} = \frac{25}{4}$$

84. Answer (4)



$$\text{Radius of incircle} = \frac{\Delta}{s}$$

$$= \frac{\frac{1}{2} \times 4 \times 3}{\frac{1}{2} \times (5 + 5 + 6)} = \frac{3}{2}$$

$$20r - 23 = 7$$

85. Answer (3)

$$f(-x) = -\left([a]^2 - 5[a] + 4\right)x^3 + \left(6\{a\}^2 - 5\{a\} + 1\right)x - \tan x \operatorname{sgn}(x)$$

$$f(x) = f(-x) \forall x$$

$$\text{Hence, } [a]^2 - 5[a] + 4 = 0 \text{ and } 6\{a\}^2 - 5\{a\} + 1 = 0$$

$$[a] = 4, 1; \quad \{a\} = \frac{1}{2}, \frac{1}{3}$$

$$= \left(4 + \frac{1}{2}\right) + \left(4 + \frac{1}{3}\right) + \left(1 + \frac{1}{2}\right) + \left(1 + \frac{1}{3}\right) = \frac{35}{3}$$

86. Answer (2)

87. Answer (1)

$$ax^2 + bx + c = 0 \text{ and } a + b + c = 0$$

Hence 1 root is 1.

$$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$$

also has 1 root 1.

88. Answer (1)

$$f(1) + f(2) = a + b + c + 4a + 2b + c = 0$$

$$5a + 3b + 2c = 0 \text{ and } a - b + c = 0$$

$$5a + 2c + 3(a + c) = 0$$

$$8a + 5c = 0 \Rightarrow \frac{c}{a} = -\frac{8}{5}$$

Using product of roots other root is $\frac{8}{5}$.

89. Answer (3)

In a $\triangle ABC$

$$\tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \tan \frac{A}{2} = 1$$

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2}$$

All being positive

$$\text{Also } \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

but all may not be positive.

90. Answer (3)

Statement 2 is not valid for negative numbers

$$\log_2 x = 2, \frac{1}{2}$$

$$\Rightarrow x = 4, \sqrt{2}$$

