AVIRAL CLASSES
CREATING SCHOLARS
FOR JEE (Main & advance), PMTs & FOUNDATION

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FINAL TEST SERIES JEE -2017

BRANCH OFFICE-1 : HO : World Trade Tower, 99- Rajpur Road. Dehradun (U.K.)
BRANCH OFFICE-2 : GMS Road, Ballupur Chowk, Dehradun (U.K.)
PH: 0135-6557701 / 02 / 03, **MOB.:** 9045062312 / 14 / 15 / 16 / 17 / 18 / 20. **web: www.aviral.co.in**

1. Answer (3)

$$g = (\tan\theta)x - \left(\frac{g}{2u\cos^2\theta}\right)x^2$$

$$= \tan\theta\left[x - \frac{g}{2u\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}.x^2\right]$$

$$= \tan\theta\left[x - \frac{g}{2u^2.\sin\theta\cos\theta}.x^2\right]$$

$$= \tan\theta\left[x - \frac{x^2}{R}\right] = x\tan\theta\left[1 - \frac{x}{R}\right]$$

Comparing with $y = x - x^2 \Rightarrow$ $R = 1$ m

2. Answer (4)

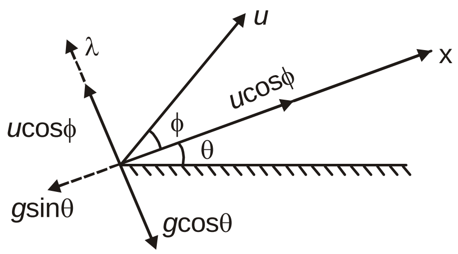
$$\vec{v} = \hat{i} + 2\hat{j} \Rightarrow v = \sqrt{5}\ \text{ms}^{-1}, \vec{a} = 2\hat{i} - 3\hat{j} \Rightarrow a = \sqrt{13}\ \text{ms}^{-2}$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{v}}{av} = \frac{2-6}{\sqrt{5}\sqrt{13}} = \frac{-4}{\sqrt{5}\sqrt{13}}$$

$$\Rightarrow \sin\theta = \sqrt{1 - \frac{16}{5\times 13}} = \frac{7}{\sqrt{65}}$$

Now $a\sin\theta = \dfrac{v^2}{R} \Rightarrow R = \dfrac{5\sqrt{5}}{7}$m

3. Answer (2)



Time of flight $T = \dfrac{2u\sin\phi}{g\cos\theta}$ ...(1)

Along the plane

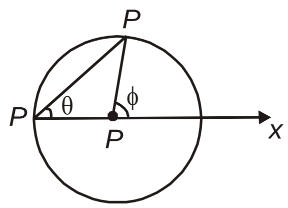
$$v_x = u_x + ax.t$$

$$0 = u\cos\phi - g\sin\theta\times\frac{2u\sin\phi}{g\cos\theta}$$

$$\Rightarrow \frac{1}{2} = \tan\theta\times\tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \theta + \phi = \frac{\pi}{6} + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

4. Answer (4)



Given: $\dfrac{d\theta}{dt} = \omega = 1$ rad/s

Now $\phi = 2\theta \Rightarrow \dfrac{d\phi}{dt} = \omega' = 2\omega = 2$ rad/s

Then, $a = \omega'^2.R = 4 \times 1$ cm/s^2 = 4 cm/s^2

5. Answer (4)

$$\left(\frac{\alpha E}{mv}\right) = \left[M^0L^0T^0\right] \Rightarrow \alpha = \left[\frac{MLT^{-1}}{ML^2T^{-2}}\right] = \left[L^{-1}T\right]$$

Now $\left[MLT^{-2}\right] = \dfrac{\beta}{\alpha} \Rightarrow \beta = \left[MLT^{-2}\right]\left[L^{-1}T\right] = \left[ML^0T^{-1}\right]$

$$\Rightarrow \left[\sqrt{\beta}\right] = \left[M^{\frac{1}{2}}L^0T^{-\frac{1}{2}}\right]$$

6. Answer (4)

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = 4\pi^2\frac{l}{T^2} \Rightarrow$$

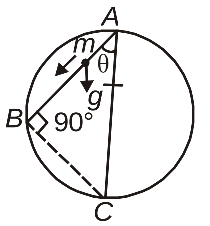
$$\frac{\Delta g}{g}\times 100 = \frac{\Delta l}{l}\times 100 + 2\frac{\Delta T}{T}\times 100$$

$$= \frac{\Delta l}{l} \times 100 + 2\frac{\Delta t}{t} \times 100$$

$$= \frac{0.1}{100.3} \times 100 + 2\frac{0.5}{10.5} \times 100$$

$$= \frac{100}{1003} + 2 \times \frac{500}{105} = 9.6\%$$

7. Answer (3)



$$AB = l = \frac{1}{2} g\cos\theta . t^2 \Rightarrow t = \sqrt{\frac{2l}{g\cos\theta}}$$

Now ΔABC, $\frac{l}{2R} = \cos\theta$

$$\Rightarrow t = \sqrt{\frac{2 \times 2R\cos\theta}{g\cos\theta}} = 2\sqrt{\frac{R}{g}}$$

8. Answer (4)

For A, $W = \int dW = \int xdx + ydy = \int_{x_1}^{x_2} xdx + \int_{y_1}^{y_2} ydy$

$$\frac{1}{2}\left[\left(x_2^2 - x_1^2\right)\right]\left[\left(y_2^2 - y_1^2\right)\right]$$

For B, $W = \int xdy + ydx = \int_{x_1,y_1}^{x_2,y_2} d(xy) = [xy]_{x_1,y_1}^{x_2,y_2}$

$$= \left(x_2y_2 - x_1y_1\right)$$

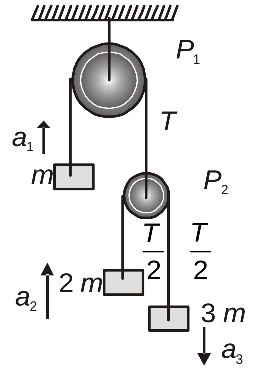
For C, $W = \int xy^2dx + yx^2dx = \frac{1}{2}\int d\left(x^2y^2\right)$

$$= \left[x^2y^2\right]_{x_1,y_1}^{x_2y_2}$$

For D, $W = \int y^2dx + \int x^2dy$

= Work done is path independent

9. Answer (3)



For A, $T - mg = ma_1$ ...(1)

For B, $\frac{T}{2} - 2mg = 2ma_2$ ...(2)

For C, $3mg - \frac{T}{2} = 3ma_3$ ...(3)

For P_2, $a_1 = \frac{a_3 - a_2}{2}$ ...(4)

From (4) $2a_1 = a_3 - a_2 \Rightarrow a_3 = 2a_1 + a_2$ ...(5)

From (1) and (2) $\frac{T}{2} - \frac{mg}{2} = \frac{ma_1}{2}$

$$\frac{T}{2} - 2mg = 2ma_2$$

$$- \quad + \quad -$$

$$\frac{3}{2}mg = m\left[\frac{a_1}{2} - 2a_2\right] \Rightarrow 3g = a_1 - 4a_2 \quad ...(6)$$

From (2)&(3) $3mg - \frac{T}{2} = 3ma_3$

$$\frac{T}{2} - 2mg = 2ma_2 \Rightarrow g = 3a_3 + 2a_2 \quad ...(7)$$

$$mg = m\left[3a_3 + 2a_2\right]$$

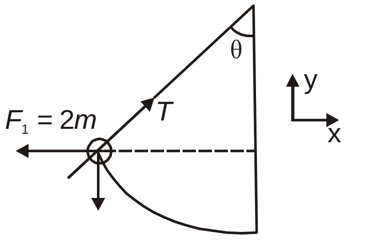
From (5) & (7)

$$g = 3\left(2a_1 + a_2\right) + 2a_2 = 6a_1 + 5a_2 \quad ...(8)$$

From (6) & (8)

$$a_2 = -\frac{17}{29}g$$

10. Answer (1)



FBD of pendulum bob. w.r.t. box.

Work energy theorem

2 *ml* sinθ – 13 *ml* (1 – cosθ) = 0

2 *l* sinθ = 13 *l* (1 – cosθ)

$$2\times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 13\times 2\ \sin^2\frac{\theta}{2}$$

$$\Rightarrow \cot\frac{\theta}{2} = \frac{13}{2} \qquad \Rightarrow \theta = 2\cot = \left(\frac{13}{2}\right)$$

11. Answer (2)

For *A*, Work energy theorem

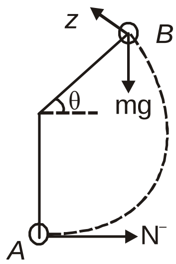
$$Fx = \frac{1}{2}kx^2 - \mu mgx = \Delta k = 0$$

$$F = \mu mgx = \frac{kx}{2} \quad ...(1)$$

For B to slide $\quad kx = (2\mu)(2mg) = 4\mu mg \quad ...(2)$

Thus $\quad F_{\text{min}} = \mu mg + 2\mu mg = 3\mu mg$

12. Answer (3)



Energy conservation of *A* and *B*

$$v'^2 = v^2 - 2gl(1+\sin\theta) \quad ...(1)$$

For circular motion at B

$$mg\sin\theta = \frac{mv'^2}{l} \quad ...(2)$$

$$\Rightarrow \quad gl\sin\theta = v^2 - 2gl(1+\sin\theta)$$

$$gl\sin\theta = 4gl - 2gl(1+\sin\theta)$$

$$\sin\theta = 4 - 2(1+\sin\theta) \Rightarrow \sin\theta = 4 - 2 - 2\sin\theta$$

$$3\sin\theta = 2 \Rightarrow \sin\theta = \frac{2}{3}$$

Then $\quad \cos\theta = \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}}$

Tengential acceleration = $g\cos\theta = \frac{g\sqrt{5}}{3}$

13. Answer (3)

$$R = \frac{R_1R_2}{R_1+R_2} = \frac{5\times 10}{5+10} = \frac{50}{15} = \frac{10}{3}\ \text{K}\Omega$$

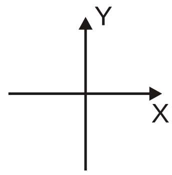
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \qquad \frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\frac{\Delta R}{R} = \frac{10}{3}\left(\frac{0.2}{25} + \frac{0.1}{100}\right)$$

$$\Rightarrow \frac{\Delta R}{R}\times 100 = \frac{10}{3}\left(\frac{2}{250} + \frac{1}{1000}\right) = 3\%$$

⇒ r = 3.3 kΩ ± 3%

14. Answer (4)



Given

$$\overrightarrow{v_A} = 5\hat{i} \quad ...(1)$$

$$\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A} = 6\cos\theta\hat{i} + 6\sin\theta\hat{j}$$

$$= 6\times\frac{1}{2}\hat{i} + 6\frac{\sqrt{3}}{2}\hat{j}$$

$$\overrightarrow{v_B} - \overrightarrow{v_A} = 3\hat{i} + 3\sqrt{3}\hat{j} \quad ...(2)$$

$$\overrightarrow{v_C} - \overrightarrow{v_B} = -10\hat{i} \quad ...(3)$$

From (2) & (3)

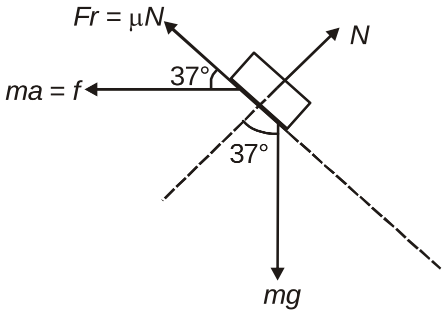
$$\overrightarrow{v_B} - 5\hat{j} = 3\hat{i} + 3\sqrt{3}\hat{j} \Rightarrow \overrightarrow{v_B} = 8\hat{i} + 3\sqrt{3}\hat{j} \quad ...(4)$$

From (3) & (4)

$$\overrightarrow{v_C} = \overrightarrow{v_B} - 10\hat{i} = 8\hat{i} + 3\sqrt{3}\hat{j} - 10\hat{i} = -2\hat{i} + 3\sqrt{3}\hat{j}$$

$$v_C = \sqrt{4+27} = \sqrt{31}\ \text{ms}^{-1}$$

15. Answer (3)



$a = \dfrac{F}{M+m}$ ...(1)

$N = ma\sin 37° + mg\cos 37°$ ...(2)

$mg\sin 37° = \mu N + ma\cos 37°$ ...(3)

From (2) $N = ma\times\dfrac{3}{5} + mg\times\dfrac{4}{5} = \dfrac{1}{5}[3ma + 4mg]$

$\Rightarrow \quad mg\times\dfrac{3}{5} = \mu.\dfrac{1}{5}[3ma+4mg] + \dfrac{4}{5}ma$

$\dfrac{3}{5}mg = \dfrac{1}{10}[3ma+4mg] + \dfrac{4}{5}ma$

$= \dfrac{3}{10}ma + \dfrac{4}{10}mg + \dfrac{4}{5}ma$

$\dfrac{6}{10}mq = \dfrac{11}{10}ma + \dfrac{4}{10}mq$

$\Rightarrow \quad \dfrac{2}{10}mg.\dfrac{11}{10}ma \Rightarrow a = \dfrac{2}{11}g$

Then $F_{mi} = (M+m)a = 8\times\dfrac{2}{11}g = \dfrac{6}{11}g$

16. Answer (3)

$\vec{v} = \hat{i} - \hat{j}\ \text{ms}^{-1} \Rightarrow v = \sqrt{2}\ \text{ms}^{-1}$

$\vec{a} = 3\hat{i} + 2\hat{j}\ \text{ms}^{-2} \Rightarrow a = \sqrt{9+4} = \sqrt{13}\ \text{ms}^{-2}$

$\cos\theta = \dfrac{\vec{a}.\vec{v}}{av} = \dfrac{3-2}{\sqrt{13}\sqrt{2}} = \dfrac{1}{\sqrt{26}}$,

$a_T = a\cos\theta = \sqrt{13}\times\dfrac{1}{\sqrt{26}} = \dfrac{1}{\sqrt{2}}\ \text{ms}^{-2}$

$F_T = maT = \dfrac{5}{\sqrt{2}}\ \text{N} \Rightarrow P = F_T{}^{v} = \dfrac{5}{\sqrt{2}}\times\sqrt{2}$ = 5 watt

17. Answer (3)

$P = \vec{F}.\vec{v} = Fv = mav = mv\dfrac{d\theta}{dt}$

$v\dfrac{dv}{dt} = \dfrac{P}{m} \Rightarrow \dfrac{v^2}{2} = \dfrac{P}{m}.t \Rightarrow v = \sqrt{\dfrac{2P}{m}}.t^{1/2} \quad \Rightarrow v \propto \sqrt{T}$

$\Rightarrow v \propto \dfrac{1}{\sqrt{P}}$

$\dfrac{ds}{dt} = \sqrt{\dfrac{2P}{m}}.t^{1/2} \Rightarrow s \propto t^{\frac{3}{2}}, v \propto \sqrt{P}, v \propto \sqrt{t}$

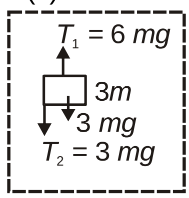
18. Answer (2)

In equilibrium

T_1 = 6 mg ...(1)

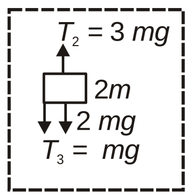
T_2 = 3 mg ...(2)

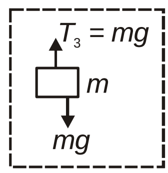
T_3 = mg ...(3)



When string (1) is cut.

T_1 = 0; T_2 & T_3 unchanged





Thus,

Acceleration of $m_1 = g\downarrow$

Acceleration of $2m_1 = \dfrac{3mg - 2mg}{2m} = \dfrac{g}{2}\uparrow$

Acceleration of $3m_1$ = 0

19. Answer (3)

$\text{KE}_{\text{blocks}} = 2\times\dfrac{1}{2}mv^2 = mv^2$ ...(1)

$\text{KE}_{\text{spring}} = 2\int dk = 2\int\dfrac{1}{2}(dm)(v_x)^2$

$= \int\left(\dfrac{m}{l}dx\right)\left(\dfrac{v}{l/2}.x\right)^2 = \int_{x=0}^{x=l/2}\dfrac{m}{l}.dx\times\left(\dfrac{2v}{l}.x\right)^2$

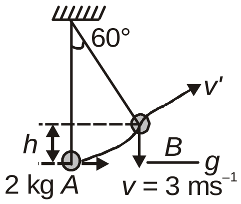
$= \int\dfrac{m}{l}.dx\times\dfrac{4v^2}{l^2}x^2$

$= \dfrac{4mv^2}{l^3}\times\left(\dfrac{x^3}{3}\right)_0^{l/2} = \dfrac{4mv^2}{l^3}\times\dfrac{1}{3}\times\dfrac{l^3}{8}$

$= \dfrac{mv^2}{6}$

TKE = $mv^2 + \dfrac{mv^2}{6} = \dfrac{7}{6}mv^2$

20. Answer (3)



$$v'^2 = v^2 - 2g(l - l\cos 60^\circ)$$

$$= v^2 - 2gl \times \frac{1}{2}$$

$$= v^2 - gl = 9 - 10 \times 0.5$$

$$v'^2 = 4 \Rightarrow v' = 2 \text{ ms}^{-1}$$

Now at B $K = \frac{1}{2}mv'^2$

$$\frac{dk}{dt} = \frac{1}{2} \times m \times 2v'\frac{dv'}{dt}$$

$$mv'\frac{dv'}{dt} = 2 \times 2 \times g\sin 60^\circ$$

$$= 2 \times 2 \times 10 \times \frac{\sqrt{3}}{2} = 20\sqrt{3} \text{ Joule / s}$$

21. Answer (4)

Equation of path $y = 2 + x$

Now $W = \int dW = \int Fx\, dx + Fydy$

$$\Rightarrow W = \int x^2(x+2)dx + 2(y-2)dy$$

$$= \int (x^3 + 2x^2)dx + \int (2y - 4)dy$$

$$= \left[\frac{2}{3}x^3 + \frac{x^4}{4}\right]_0^2 + \left[2.\frac{y^2}{2} - 4y\right]_2^4$$

$$= \left[\frac{2}{3} \times 8 + 4\right] + \left[y^2 - 4y\right]_2^4$$

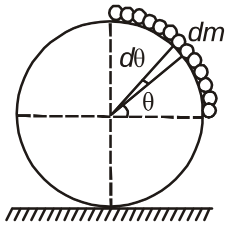
$$= \left[\frac{16}{3} + 4\right] + \left[(16-4) - 4(4-2)\right]$$

$$= \frac{28}{3} + (12 - 8) = \frac{28}{3} + 4 = \frac{28+12}{3} = \frac{40}{3} \text{ J}$$

22. Answer (2)

KE can be increased by internal as well as external forces.

23. Answer (4)



$$U_i = \int dUi = \int dm.gh = \int \frac{m}{l}.Rd\theta \times g \times R\sin\theta$$

$$= \frac{mgR^2}{l}.[-\cos\theta]_0^{\pi/2} = \frac{4mgl}{\pi^2}$$

$$U_f = -\frac{mgl}{2}$$

Loss = $U_i - U_f = mgl\left[\frac{1}{2} + \frac{4}{\pi^2}\right]$

$$\frac{1}{2}mv^2 = mgl\left[\frac{1}{2} + \frac{4}{\pi^2}\right] \Rightarrow v = \sqrt{2gl\left[\frac{1}{2} + \frac{4}{\pi^2}\right]}$$

24. Answer (2)

For no slipping between the blocks

$$F_{\max} = \frac{\mu mig}{m_2}(m_1 + m_2) = \frac{0.5 \times 4 \times 10}{6} \times 10$$

$$= \frac{100}{3}\text{N}$$

Since applied force F = 30 N which is less than $\frac{100}{3}$ N

Hence between block a_{rep} = 0.

Thus a common = $\frac{30}{10}$=$3ms^{-2}$

Then F_r = 6 × 3 = 18 N

25. Answer (3)

Using work-energy theorem

$$3mgR - mgR = \frac{1}{2}mv^2 \Rightarrow 2mgR = \frac{1}{2}mv^2$$

$$v = 2\sqrt{gR}$$

$$\Rightarrow N = \frac{mv^2}{R} = \frac{m}{R} \times 4gR = 4mg$$

26. Answer (4)

27. Answer (4)

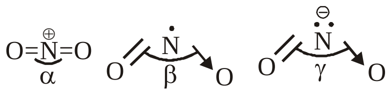
28. Answer (3)

29. Answer (2)

30. Answer (1)

[CHEMISTRY]

31. Answer (2)
32. Answer (2)



;$\alpha > \beta > \gamma$

33. Answer (3)
34. Answer (1)

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1\times10^{-6}\times20} = 3.3\times10^{-29}\text{ m}$$

35. Answer (4)

$2NO(g) + O_2(g) \longrightarrow 2NO_2(g)$

60 g 32 (g) 46 × 2 g

32 g of O_2 requires 60 g NO

Hence 3.2 g of O_2 requires 6 g NO

∴ NO is limiting reagent

Now 60 g NO produces 92 g NO_2

Hence 4.2 g NO produces $\frac{92}{60}\times 4.2\text{ g } NO_2 = 6.44\text{ g}$

36. Answer (1)
37. Answer (3)

Number of moles of NaCl $= \frac{5.85}{58.5} = 0.1\text{ mole}$

Number of moles of NaCl in 1 mL of solution

$= \frac{0.1}{1000} = 10^{-4}$ mole

∴ Total number of ions

$= 6.022 \times 10^{23} \times 2 \times 10^{-4} = 1.2 \times 10^{20}$

38. Answer (1)

Number of moles

$$= \frac{10^{22}}{6.022\times10^{23}} = \frac{10\times10^{-2}}{6.022} = 1.66\times10^{-2}\text{ mole}$$

Weight = $1.66 \times 10^{-2} \times 249.5 = 4.144$ g

39. Answer (1)
40. Answer (2)

$Sc(21) \rightarrow [Ar]_{18}3d^14s^2$

Last electron enters 3*d* orbital

Hence, $n = 3$

$l = 2$

$m_l = -2, -1, 0, 1, 2$

$m_s = \pm\frac{1}{2}$

41. Answer (3)

$29 \longrightarrow [Ar]_{18}3d^{10}4s^1$ which belongs to *d*-block.

42. Answer (4)
43. Answer (4)
44. Answer (4)

$$\frac{1}{\lambda} = 1.1\times10^7\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \nu = 3\times10^8\times1.1\times10^7\left[\frac{1}{9} - \frac{1}{n^2}\right]$$

Putting the values of *n* from the options only 5 satisfies the equation.

Hence answer is (4).

45. Answer (2)

Number of moles of H_3PO_4 formed = 0.85 × 0.9 × 4 × number of moles of P_4 used

$$\frac{w_{H_3PO_4}}{98} = 0.85\times0.9\times4\times\frac{62}{124}$$

$$\Rightarrow w_{H_3PO_4} = 0.85\times0.9\times4\times\frac{62}{124}\times98$$

= 149.94 g

46. Answer (1)

Equivalent weight of $K_2Cr_2O_7 = \frac{294}{6} = 49$

Equivalent weight of $KMnO_4 = \frac{158}{5} = 31.6$

meq. of $KMnO_4$ + meq. of $K_2Cr_2O_7$ = meq. of I_2 = meq. of hypo.

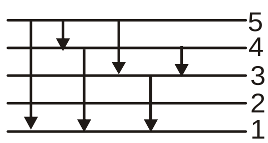
Let the mass of $K_2Cr_2O_7$ = x g;

∴ Mass of $KMnO_4$ = (0.5 – x) g

$$\frac{x}{49} + \frac{0.5-x}{31.6} = 150\times0.1\times10^{-3} \Rightarrow x = 0.0732$$

∴ % of $K_2Cr_2O_7 = \frac{0.0732}{0.5}\times100 = 14.64$

47. Answer (4)



Total radiations are = 6.

48. Answer (3)
    (i) F $\rightarrow 1s^2 2s^2 2p^5$
    (ii) O $\rightarrow 1s^2 2s^2 2p^4$
    (iii) N $\rightarrow 1s^2 2s^2 2p^3$
    (iv) S $\rightarrow 1s^2 2s^2 2p^6 3s^2 3p^4$
49. Answer (1)
    Ionisation enthalpy decreases down the group.
50. Answer (2)
    Fact.
51. Answer (1)
    Number of spherical nodes = n – *l* – 1.
52. Answer (3)
53. Answer (3)
    $CO_2(g) + C(solid) \longrightarrow 2CO(g)$
    1 – x   2x
    1 – x + 2x = 1.5 $\Rightarrow$ x = 0.5 L
    Hence volume of CO produced = 1 L
    Number of moles $= \frac{1}{22.4}$.
54. Answer (2)
55. Answer (2)
    1 mole S_8 gives 8 mole SO_3 = 8 × 80 = 640 g
56. Answer (4)
57. Answer (1)
58. Answer (4)
59. Answer (2)
60. Answer (1)

[MATHEMATICS]

61. Answer (2)
    Let sides be *a*, *b*, *c* where *c* is the hypotenuse.

$$a^2+b^2+c^2=294$$
$$2c^2=294$$
$$c^2=147 \Rightarrow \boxed{c=7\sqrt{3}}$$
$$\boxed{a^2+b^2=147}$$
$$a+b+7\sqrt{3}=12+8\sqrt{3}$$
$$a+b=12+\sqrt{3}$$
$$(a+b)^2=(12+\sqrt{3})^2$$
$$a^2+b^2+2ab=147+24\sqrt{3}$$
$$\boxed{ab=12\sqrt{3}}$$
$$\Delta=6\sqrt{3}$$
$$\left[\frac{\Delta}{3}\right]=\left[2\sqrt{3}\right]=3$$

62. Answer (3)

$$=\left(\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}\right)^2$$
$$=\left(\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}\right)^2$$
$$=\left(\frac{\sin.\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}\right)^2=\frac{1}{64}$$

63. Answer (2)
    $\tan^2 x = 0$
    $\Rightarrow x=0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi$
64. Answer (3)

$$(\log_7 x)^2+(\log_7 x)-2<0$$
$$(\log_7 x+2)(\log_7 x-1)<0$$
$$\log_7 x\in(-2,\ 1)$$
$$x\in\left(\frac{1}{49},\ 7\right)$$

65. Answer (4)

$$2a^2+2b^2+2ab-2a+2b+2\le 0$$
$$(a+b)^2+(a-1)^2+(b+1)^2\le 0$$
$$a=1,\ b=-1 \qquad \Rightarrow 7a+3b=4$$

66. Answer (3)

$\alpha+\beta=1-\sin\phi$

$\alpha\beta=-\frac{1}{2}\cos^2\phi$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(1-\sin\phi)^2+\cos^2\phi$

$=2-2\sin\phi$

$(\alpha^2+\beta^2)_{max}=4$

67. Answer (4)

$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=9$

$r-\alpha,\ r-\beta,\ r-\gamma,\ r-\delta\equiv\pm1,\ \pm3$

$\alpha,\ \beta,\ \gamma,\ \delta\equiv r+1,\ r-1,\ r+3,\ r-3$

$2\alpha+2\beta+2\gamma+2\delta-6r=2r$

68. Answer (4)

$\alpha+\beta+\gamma=1$

$\beta=\frac{1}{3}$

$\alpha\beta+\beta\gamma+\alpha\gamma=3p \quad \Rightarrow \beta(\alpha+\gamma)+\alpha\gamma=3p$

$3p=\frac{1}{3}\times\frac{2}{3}+\alpha\gamma$

$3p=\frac{2}{9}+\left(\frac{1}{3}-d\right)\left(\frac{1}{3}+d\right)=\frac{1}{3}-d^2$

$p=\frac{1}{9}-\frac{1}{3}d^2 \quad \Rightarrow p_{max}=\frac{1}{9}$

69. Answer (2)

70. Answer (1)

$|x-1|+|x^2+x+1|=|x(x+2)|$

$x<-2$

$1-x+x^2+x+1=x^2+2x$ No solution

$-2\le x<0$

$1-x+x^2+x+1=-x^2-2x$ No solution

$0\le x<1$

$1-x+x^2+x+1=x^2+2x$ No solution

$x\ge1$

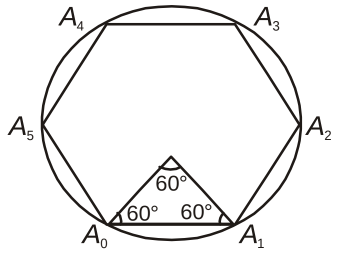
$x-1+x^2+x+1=x^2+2x \Rightarrow x\in[1,\infty)$

71. Answer (1)

By observation

$x=0,\ x=\frac{3\pi}{2},\ x=3\pi$

72. Answer (3)



$A_0A_1=1,\ A_0A_2=\sqrt{3},\ A_0A_4=\sqrt{3},$

73. Answer (2)

$3^x-2^{2y}=77$

$\left(3^{\frac{x}{2}}-2^y\right)\left(3^{\frac{x}{2}}+2^y\right)=77$

$3^{\frac{x}{2}}-2^y=7 \quad \Rightarrow \quad 3^{\frac{x}{2}}+2^y=11$

$\Rightarrow 3^{\frac{x}{2}}=9 \quad \Rightarrow x=4 \quad \Rightarrow y=1$

74. Answer (2)

$\frac{x}{x+6}-\frac{1}{x}\le0$

$\frac{x^2-x-6}{x(x+6)}\le0 \quad \Rightarrow \frac{(x-3)(x+2)}{x(x+6)}\le0$

$x\in(-6,-2]\cup(0,3]$

75. Answer (4)

$\left(x^2+\frac{1}{x^2}\right)^2=121 \quad \Rightarrow x^2+\frac{1}{x^2}=11$

$\Rightarrow \left(x-\frac{1}{x}\right)^2=9 \quad \Rightarrow x-\frac{1}{x}=\pm3$

$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$

$x^3-\frac{1}{x^3}=27+3\times3=36$

76. Answer (3)

$\log_{\sqrt{2}}(16\sin^2x+1)|_{min}=0$

Hence $(-\infty, 1]$

77. Answer (4)

$|x(x+5)|+|x(x-1)|=|6x|$

$x<-5$

$x^2+5x+x^2-x=-6x$

$2x^2+10x=0$

$x=0,-5$ No solution

$-5 \le x \le 0$

$-x^2 - 5x + x^2 - x = -6x$

$x \in [-5, 0]$

$0 < x \le 1$

$x^2 + 5x - x^2 + x = 6x$

$x \in (0, 1]$

$x > 1$

$x^2 + 5x + x^2 - x = 6x$

$2x^2 - 2x = 0$ No solution

Hence $x \in [-5, 1]$

78. Answer (1)

$0 < e^x < 1 \Rightarrow x \in (-\infty, 0)$

$0 < \ln|x| < 1 \Rightarrow 1 < |x| < \ell \Rightarrow x \in (-\ell, -1) \cup (1, \ell)$

Combined $\Rightarrow x \in (-\ell, -1)$

$[-\ell - 1 - 0.7] = -5$

79. Answer (4)

$f(x) = \{x\} + \{x+1\} + \{x+2\} + ... + \{x+99\}$

$= 100\{x\}$

$f(\sqrt{2}) = 100\{\sqrt{2}\} = 100(\sqrt{2}-1)$

$[f(\sqrt{2})] = [41.4...] = 41$

80. Answer (4)

$-3 \le |[x]| \le 2$

$\Rightarrow |[x]| \le 2$

$\Rightarrow -2 \le [x] \le 2$

$\Rightarrow x \in [-2, 3)$

81. f (x) = $\sin^8 + \cos^8$

$= (\sin^4 x + \cos^4 x)^2 - 2\sin^4 x \cos^4 x$

$= \left(1 - \frac{\sin^2 2x}{2}\right)^2 = \frac{1}{8}\sin^4 2x$

$= \frac{1}{8}[4 - \sin 2x]^2 - 1$

f (min) $= \frac{9}{8} - 1 = \frac{1}{8}$

f (max) $= \frac{1}{8}[4-0]^2 - 1$

$= 2 - 1$

$= 1$

82. Answer (2)

$x = \cot\frac{11\pi}{8} = \tan\frac{\pi}{8} = \sqrt{2} - 1$

$\Rightarrow x^2 + 2x - 1 = 0$

$A = x^4 + 4x^3 + 2x^2 - 4x + 7 = x^2(x^2 + 2x - 1) +$

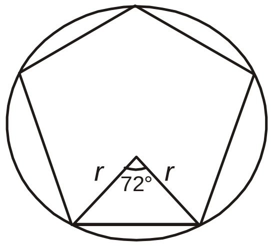
$2x(x^2 + 2x - 1) - (x^2 + 2x - 1) + 6 = 6$

$B = \frac{1 - \cos 8\theta}{\tan^2 4\theta} + \frac{1 + \cos 8\theta}{\cot^2 4\theta} = \frac{2\sin^2 4\theta}{\tan^2 4\theta} + \frac{2\cos^2 4\theta}{\cot^2 4\theta}$

$= 2\cos^2 4\theta + 2\sin^2 4\theta = 2$

$AB = 12 \Rightarrow C = 1 + 2 = 3$

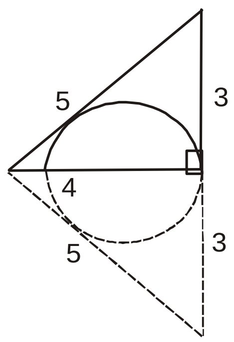
83. Answer (2)



Area = $5 \times \frac{1}{2} r^2 \sin 72°$

Hence ratio = $\frac{25}{4}$

84. Answer (4)



Radius of incircle = $\frac{\Delta}{s}$

$= \frac{\frac{1}{2} \times 4 \times 6}{\frac{1}{2} \times (5 + 5 + 6)} = \frac{3}{2}$

$20r - 23 = 7$

85. Answer (3)

$$f(-x) = -\left([a]^2 - 5[a] + 4\right)x^3 + \left(6\{a\}^2 - 5\{a\} + 1\right)x$$
$$-\tan x \,\text{sgn}(x)$$

$f(x) = f(-x)\ \forall x$

Hence, $[a]^2 - 5[a] + 4 = 0$ and $6\{a\}^2 - 5\{a\} + 1 = 0$

$[a] = 4, 1;$ $\{a\} = \frac{1}{2}, \frac{1}{3}$

$$= \left(4 + \frac{1}{2}\right) + \left(4 + \frac{1}{3}\right) + \left(1 + \frac{1}{2}\right) + \left(1 + \frac{1}{3}\right) = \frac{35}{3}$$

86. Answer (2)

87. Answer (1)

$ax^2 + bx + c = 0$ and $a + b + c = 0$

Hence 1 root is 1.

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

also has 1 root 1.

88. Answer (1)

$f(1) + f(2) = a + b + c + 4a + 2b + c = 0$

$5a + 3b + 2c = 0$ and $a - b + c = 0$

$5a + 2c + 3(a + c) = 0$

$8a + 5c = 0 \Rightarrow \frac{c}{a} = -\frac{8}{5}$

Using product of roots other root is $\frac{8}{5}$.

89. Answer (3)

In a ΔABC

$$\tan\frac{A}{2}.\tan\frac{B}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2} = 1$$

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$$

All being positive

Also $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$

but all may not positive.

90. Answer (3)

Statement 2 is not valid for negative numbers

$\log_2 x = 2, \frac{1}{2}$

$\Rightarrow x = 4, \sqrt{2}$

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