



(PART - II)

CAPACITOR

EXERCISE - I

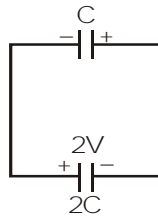
1. $k = \frac{f}{x} = \frac{5000}{0.2} = 2,5000 \text{ N/m}$

$$\frac{U_{\text{SPR}}}{U_{\text{CAP}}} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}CV^2} = \frac{25000 \times 0.2 \times 0.2}{10 \times 10^{-6} \times 10^8} = 1$$

2. $P = \frac{\Delta U}{\Delta t} = \frac{\frac{1}{2}CV^2}{\Delta t} = \frac{\frac{1}{2} \times 40 \times 10^{-6} \times 9 \times 10^6}{2 \times 10^{-3}} = 90 \text{ kW}$

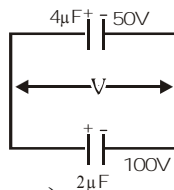
3. $V_0(C+CV) = CV + (2C)(2V)$
 $V_0 = V$ (Final pot. diff.)

$$\therefore U_{\text{final}} = \frac{1}{2}(C+2C)V^2 = \frac{3CV^2}{2}$$



4. $(4+2)V = (4 \times 50) + (2 \times 100)$

$$V = \frac{400}{6} = \frac{200}{3} \text{ V}$$



$$U_{\text{initial}} = \left(\frac{1}{2} \times 4 \times (50)^2 + \frac{1}{2} \times 2 \times (100)^2 \right) \times 10^{-6}$$

$$= (5000 + 10000) \times 10^{-6} = 1.5 \times 10^{-2} \text{ J}$$

$$U_{\text{final}} = \frac{1}{2}(4+2) \times 10^{-6} \times \frac{200}{3} \times \frac{200}{3}$$

$$= 1.33 \times 10^{-2} \text{ J}$$

5. Before sharing $U_i = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

After sharing $U_f = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)}$

$$\Delta U = U_f - U_i = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)} - \frac{Q_1^2}{2C_1} - \frac{Q_2^2}{2C_2}$$

$$= -\frac{(Q_1C_2 - Q_2C_1)^2}{2C_1C_2(C_1+C_2)}$$

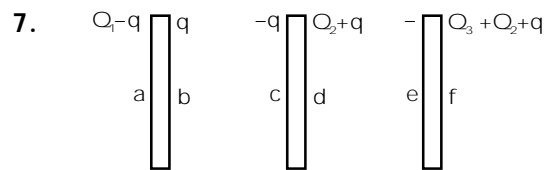
-ve sign indicates there is decrease in energy

But $Q_1C_2 - Q_2C_1 \neq 0 \Rightarrow Q_1C_2 \neq Q_2C_1$

$\Rightarrow Q_1 4\pi\epsilon_0 R_2 \neq Q_2 4\pi\epsilon_0 R_1 \Rightarrow Q_1 R_2 \neq Q_2 R_1$

6. $C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}} \left(t = \frac{d}{2}, K = \infty \right)$

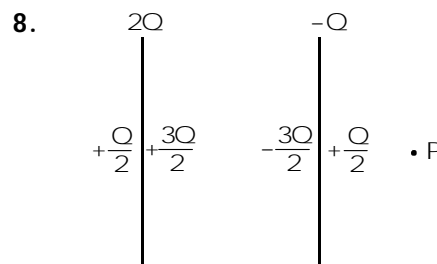
$$= \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2\epsilon_0 A}{d} = 2C_0$$



Here $Q_1 - q = Q_2 + Q_3 + q \Rightarrow q = \frac{Q_1 - (Q_2 + Q_3)}{2}$

Charge on a = Charge on f

$$\Rightarrow Q_1 - q = \frac{\Sigma Q}{2} = \frac{Q_1 + Q_2 + Q_3}{2}$$



Force on either plate

$$= \frac{(3Q/2)^2}{2A\epsilon_0} = \frac{9Q^2}{8A\epsilon_0}$$

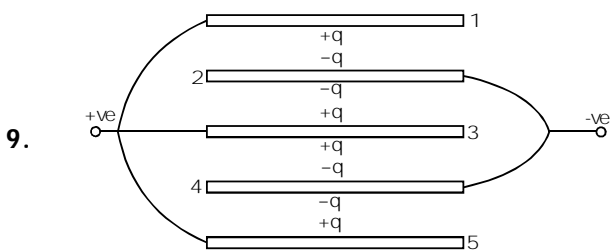
Force on point 'P' due to capacitor = 0

Potential diff. between the plates

$$= \frac{3Q}{2C}$$

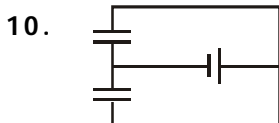
Energy stored in electric field between the plates

$$= \frac{1}{2}C \times \left(\frac{3Q}{2C} \right)^2 = \frac{9Q^2}{8C}$$



Therefore

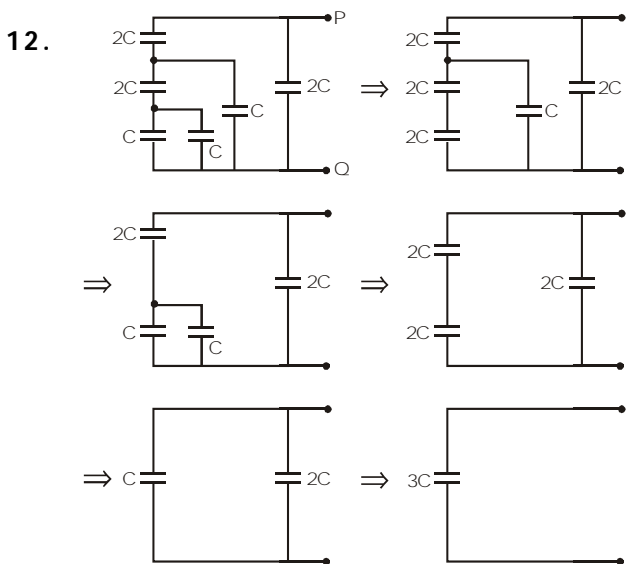
$$q_2 = -2q, q_3 = +2q, q_4 = -2q \text{ and } q_5 = +q$$



$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2} \left(\frac{8.85 \times 10^{-12} \times 0.1}{0.885 \times 10^{-3}} \times 2 \right) \times 10^2 = 10^{-1} \mu\text{J}$$

11. Each capacitor has potential difference 'V' and energy $\frac{1}{2}CV^2$. After reconnecting total energy remains constant and total voltage becomes NV.



$$5(V_A - V_B) = 15(V_B - V_C) \Rightarrow 5(2000 - V_B) = 15(V_B - 0)$$

$$\Rightarrow 2000 - V_B = 3V_B \Rightarrow V_B = 500V$$

14. $C_{\text{eff}} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + \dots$

$$= \frac{C}{1 - 1/2} = 2C = 2\mu\text{F}$$

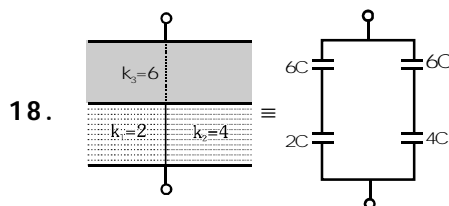
15. For 'n' plates; effective C will be (n-1)C.

16. $CV + 2CV = KCV' + 2CV' \Rightarrow V' = \frac{3V}{K+2}$

17. $C = \frac{\epsilon_0 A}{d} = 9\text{pF}$

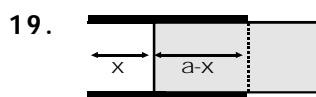
$$C' = \frac{\epsilon_0 A}{d - t_1 + \frac{t_1}{K_2} - t_2 + \frac{t_2}{K_2}} = \frac{\epsilon_0 A}{d - \frac{d}{3} + \frac{d}{9} - \frac{2d}{3} + \frac{d}{9}}$$

$$= \frac{9}{2} \frac{\epsilon_0 A}{d} = \frac{81}{2} \text{pF} = 40.5\text{pF}$$



where $C = \frac{\epsilon_0 A}{d}$

$$C_{\text{eq}} = \frac{6C \times 2C}{8C} + \frac{6C \times 4C}{10C} = 3.9C$$



$$C = \frac{\epsilon_0 ax}{d} + \frac{K \epsilon_0 (a-x)a}{d}$$

$$C = \frac{K \epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)}{d} x \text{ where } x = vt$$

∴ C- t graph is linear with negative slope.

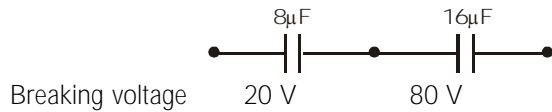
20. $\frac{1}{2} CV^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

21. $C = 4\pi \epsilon_0 a$



$$C' = \frac{4\pi\epsilon_0 ab}{b-a} = \frac{4\pi\epsilon_0 a}{1-\frac{a}{b}} = \frac{4\pi\epsilon_0 a}{1-\left(\frac{n-1}{n}\right)} = n(4\pi\epsilon_0 a)C$$

22.

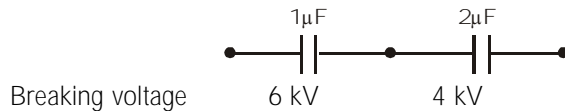


Breaking voltage 20 V 80 V

Safe Voltage 20 V 10 V

∴ Charge on each capacitor = 20 × 8 = 160 μC

23.



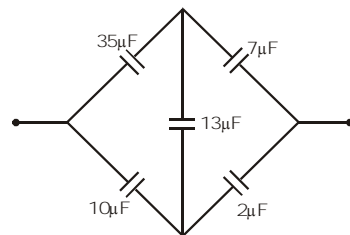
Breaking voltage 6 kV 4 kV

Safe Voltage 6 kV 3 kV

∴ Total voltage = 9 kV

24. Capacitance between 1 and 3 and between 2 and 4 are symmetrical.

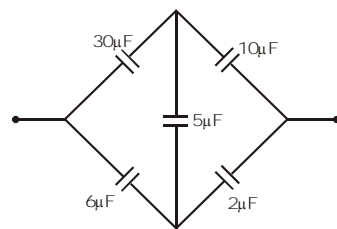
25.



The system is a balanced Wheatstone bridge.

$$\therefore C_{\text{eff}} = \left(\frac{35 \times 7}{35 + 7} + \frac{10 \times 2}{10 + 2} \right) = \frac{15}{2} \mu\text{F}$$

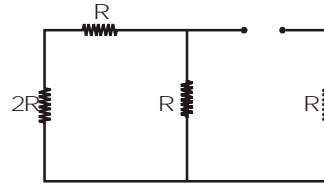
26.



The system is a balanced Wheatstone bridge.

$$\therefore C_{\text{eff}} = \left(\frac{10 \times 30}{10 + 30} + \frac{6 \times 2}{6 + 2} \right) = 9 \mu\text{F}$$

27.



To find the time constant of a RC circuit, Short circuit the battery

$$R_{\text{eff}} = \frac{7R}{4} \quad \therefore \tau = \frac{7RC}{4}$$

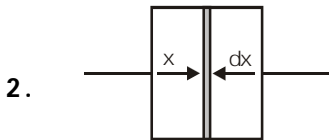
28. There is no closed path for flow of current. Hence no current flows. Hence heat developed is zero.

29. $V_A = 3 \left(\frac{q}{C} \right) = 3 \times 2.5 = 7.5 \text{ volt}$



EXERCISE -II

1. $E = \frac{V_0}{d} \Rightarrow E_F < E_D$ Also $\sigma_A > \sigma_B$



$$\int \frac{1}{dC} = \int \frac{dy}{K \epsilon_0 A} = \int_0^d \frac{dy}{\lambda \epsilon_0 A \sec\left(\frac{\pi y}{2d}\right)}$$

$$\Rightarrow C = \frac{\lambda \epsilon_0 A \pi}{2d}$$

3. Both A and B are always in parallel.

4. $V = V_0 \cdot e^{-t/RC}$

$$\left| \frac{dV}{dt} \right| = \frac{V_0}{RC} e^{-t/RC} = \text{slope}$$

At $t = 0$, for $R = R_A$; slope is least in curve-3.

5. $q = q_0 e^{-t/\tau} \therefore i = \frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} = i_0 e^{-t/\tau}$

$$\therefore q_0 = i_0 \tau$$

Initial stored energy

$$= \frac{1}{2} CV^2 = \frac{1}{2} (CV)V$$

$$= \frac{1}{2} (i_0 \tau) (i_0 R) = \frac{1}{2} i_0^2 R \tau$$

6. As B is in parallel with C and the potential develops slowly. Hence during charging more heat is produced in A than in B. In steady state, same current passes through A and B.

$$\therefore V_{\text{capacitor}} = \frac{E}{2} \therefore E_{\text{capacitor}} = \frac{1}{2} C \left(\frac{E}{2} \right)^2 = \frac{CE^2}{8}$$

7. $q = q_0 e^{-t/\tau}$

$$\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} = \frac{CV_0}{RC} e^{-t/\tau} = \frac{V_0}{R} e^{-t/\tau}$$

At $t=0$; $i_1 = \frac{V_0}{R_1}$; $i_2 = \frac{V_0}{R_2}$

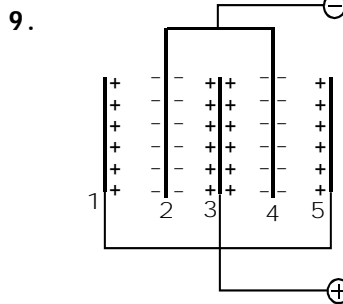
$$\therefore R_1 = R_2 \therefore i_1 = i_2$$

As τ is less for C_1 and hence it loses charges faster than C_2 .

8. $C_{\text{eff}} = 1/4 \mu\text{F}$

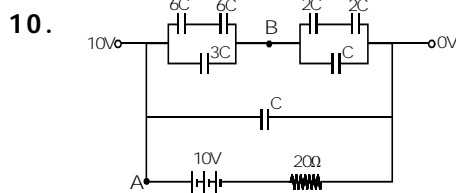
$$\therefore \text{Total no. of rows of capacitor} = \frac{C_{\text{net}}}{C_{\text{eff}}} = \frac{3}{1/4} = 12$$

$$\therefore \text{Total no. of capacitors needed} = 12 \times 4 = 48$$



Charge on plate $\neq 1 = \frac{\epsilon_0 AV}{d}$

Charge on plate $\neq 4 = - \frac{2 \epsilon_0 AV}{d}$



$$(V_A - V_B) 6C = (V_B - 0) 2C \Rightarrow V_B = 7.5 \text{ V}$$

$$\therefore V_A - V_B = 10 - 7.5 = 2.5 \text{ V}$$

11. Force on plate

$$= \frac{\sigma^2 A}{2 \epsilon_0} = \frac{Q^2}{2A \epsilon_0} = Kx = mg$$

$$\therefore Q = \sqrt{2mgA \epsilon_0}$$

12. $C_{\text{eff}} = C_{\text{EF}} = \frac{\epsilon_0 A}{d} \therefore E_{\text{net}} = \frac{1}{2} CV^2 = \frac{\epsilon_0 AV^2}{2d}$

13. $i = 10e^{-t/RC} \Rightarrow 2.5 = 10 e^{-2/RC}$

$$\Rightarrow RC = \tau = \frac{1}{\ln 2} \text{ \& } C = \frac{1}{10 \ln 2}$$

For capacitor

$$\frac{V_0}{R} = 10 \Rightarrow V_0 = 10R = 100 \text{ volt}$$

Total heat developed = Total initial energy stored



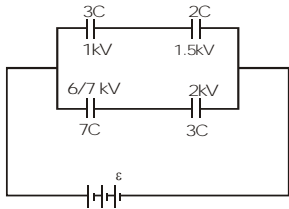
in capacitor = $\frac{1}{2} CV^2 = \frac{500}{2 \ln 2}$

Thermal power in resistor

$P = i^2 R = 100 R e^{-2t/RC}$

\therefore Time-constant = $\frac{RC}{2} = \frac{1}{2 \ln 2}$

14.



Safe voltages in each arm are mentioned.

$\therefore (1+1.5) < (6/7 + 2) \therefore E_{\text{safe}} = 1+1.5 = 2.5 \text{ kV}$

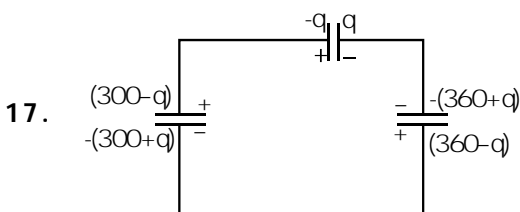
15. Time constant

= $CR_{\text{eff}} = (100 \times 10^{-6}) \left(\frac{10^3}{2} \right) \text{ s} = 50 \text{ m/s}$

16. $i_1 = \frac{V}{R} e^{-t/RC_1}, i_2 = \frac{V}{R} e^{-t/RC_2}$

$\therefore \frac{i_1}{i_2} = e^{t/R \left(\frac{1}{C_2} - \frac{1}{C_1} \right)} = e^{+\frac{t}{2RC_2}}$

$\Rightarrow i_1/i_2$ increases with time, t.



$\Rightarrow \frac{300-q}{2} - \frac{q}{1.5} + \frac{360-q}{3} = 0 \Rightarrow q = 180$

$\therefore q_{1.5\mu\text{F}} = 180 \mu\text{C}, q_{3\mu\text{F}} = 540 \mu\text{C}, q_{2\mu\text{F}} = 480 \mu\text{C}$

18. $i = \frac{i_0}{2} = i_0 e^{-t/RC} \Rightarrow \frac{1}{2} = e^{-t/4RC}$

$\Rightarrow RC=2 \Rightarrow (2+r) \frac{1}{2} = 2 \Rightarrow r = 2\Omega$

19. At t=0, $V_C = 0 \Rightarrow i_{R_3} = 0$

$Q_{\text{max}} = C \left[\frac{\epsilon}{\frac{R_1 R_2}{R_1 + R_2} + R_3} \right] = \frac{10C}{1+1} = 5 \times 1 = 5\mu\text{C}$

$\therefore (I_{R_3})_{\text{max}} = \frac{V_C}{R_3} = \frac{5}{1} = 5 \text{ A}$

Since R_1 and R_2 are in parallel hence current ratio of R_1 and R_2 will remain same.

20. $q = q_0 e^{-t/RC} \Rightarrow I = \frac{q_0}{RC} e^{-t/RC}$

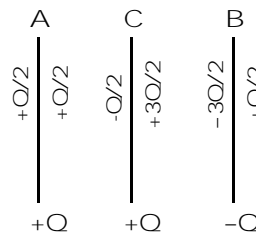
$\Rightarrow \ln I = \ln \left(\frac{q_0}{RC} \right) - \frac{t}{RC} = \ln \left(\frac{V_0}{R} \right) - \frac{t}{RC}$

As I_{max} does not change $\therefore R = \text{constant}$

$\left| \frac{d(\ln I)}{dt} \right| = \left| 0 - \frac{1}{RC} \right| \Rightarrow \left[\frac{d(\ln I)}{dt} \right]_1 > \left[\frac{d(\ln I)}{dt} \right]_2$

$\therefore C_2 > C_1 \Rightarrow C$ is increased

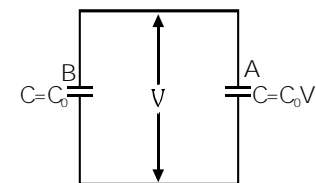
21.



Initial $V'_{AB} = \frac{Q}{C} = \frac{Qd}{\epsilon_0 A}$

Final $V_{AB} = \frac{Q/2}{\left(\frac{2\epsilon_0 A}{d} \right)} + \frac{(3Q/2)}{\left(\frac{2\epsilon_0 A}{d} \right)} = \frac{Qd}{\epsilon_0 A} = V_{AB}$

22.



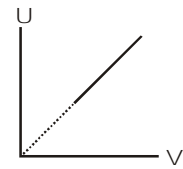
$(C_0 + C_0 V) V = 30 C_0$
 $\Rightarrow V^2 + V - 30 = 0 \Rightarrow V = 5 \text{ volt}$

$\therefore V_A = V_B = 5 \text{ volt}$

$Q_A = 5^2 C_0 = 25 C_0; Q_B = 5 C_0$

23.

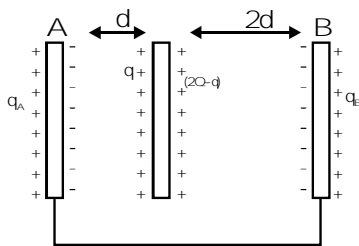
$C = \frac{\epsilon_0 ax}{d} + \frac{K \epsilon_0 (a-x)a}{d}$
 $= \frac{K \epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)vt}{d}$



$V = \frac{Q}{C}$ and $U = \frac{QV}{2} \therefore \frac{U}{V} = \frac{Q}{2}$



24.



$$\Delta V = \frac{qd}{\epsilon_0 A} = \frac{(2Q-q)(2d)}{\epsilon_0 A} \Rightarrow q = \frac{4Q}{3}$$

Total charge on inner faces of A and B = $-2Q$

Rest charge will equally appear on their outer faces

$$= \frac{Q - (-2Q)}{2} = \frac{3Q}{2}$$

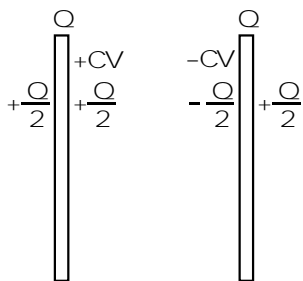
Final charge on plate A

$$= \frac{3Q}{2} - \frac{4Q}{3} = \frac{Q}{6}$$

\therefore Charge flown through wire

$$= Q - \frac{Q}{6} = \frac{5Q}{6}$$

25. Final charge distribution



Therefore potential difference across the capacitor

$$= \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

26. $Q = \frac{C}{2}E$

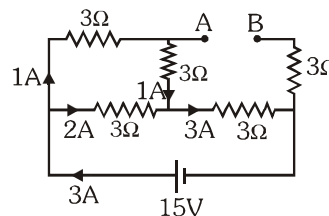
$$Q' = \frac{KCC}{KC+C}E = \frac{KC}{K+1}E$$

$$\therefore Q' - Q = \frac{KCE}{K+1} - \frac{CE}{2} = \frac{(K-1)CE}{2(K+1)}$$

This charge is supplied by battery.

27. $C_{eq} = \frac{KC}{K+1}, C'_{eq} = \frac{C}{2} \Rightarrow \frac{Q'_2}{Q_2} = \frac{K+1}{2K}$

28. At $t = \infty$, capacitor gets open circuited



$$\therefore I = \frac{15}{5} = 3A \Rightarrow V_A - 3 \times 1 - 3 \times 3 = V_B$$

$$\Rightarrow V_A - V_B = 12V$$

29. At $t=0$, $V_{capacitors} = 0$

$$\Rightarrow I_2 = I_3 = 0 \text{ and } I_1 = \frac{6}{2} = 3A$$

$$\text{At } t \rightarrow \infty, I_1 = I_3 = \frac{6}{2+8} = 0.6A, I_2 = 0$$

30. In steady state

$$I_{upper \text{ arm}} = I_{lower \text{ arm}} = \frac{120}{6} = 20A$$

For the right most loop

$$3I - 3I + \frac{q}{C_2} = 0 \Rightarrow q = 0$$

For the left most loop

$$20 \times 1 + \frac{q}{C_1} - 20 \times 2 = 0$$

$$\Rightarrow q = (40-20)C_1 = 20C_1 = 40 \mu C$$

31. Charge on $3\mu F$ capacitor

$$= 6 \times 7 = 42 \mu C$$

$$\therefore V_{3\mu F} = \frac{42}{3} = 14 \text{ volt}$$

$$\therefore V_{3.9\mu F} = 14 + 6 = 20 \text{ volt}$$

$$\text{Charge on } 3.9 \mu F \text{ capacitor} = 20 \times 3.9 = 78 \mu C$$

$$3.9 \mu F$$

$$\therefore \text{Total charge} = 78 + 42 = 120 \mu C$$



$$\therefore V_{12\mu F} = \frac{120}{12} = 10V$$

$$\therefore \epsilon = 20 + 10 = 30 V$$

32. Energy = $\frac{Q^2}{2C} = \frac{Q^2 d}{2 \epsilon_0 A}$

As d decreases, E decreases

33. $Q = CV = \frac{\epsilon_0 AV}{d}$

$$E = \frac{V'}{d} = \frac{V/K}{d} = \frac{V}{Kd}$$

$$W = \frac{1}{2} Q^2 \left(\frac{1}{C} - \frac{1}{C'} \right) = \frac{CV^2}{2} \left(1 - \frac{1}{K} \right)$$

34. $\epsilon = \frac{Q_0}{C_1} \therefore Q_1 = Q_0; Q_2 = \left(\frac{Q_0}{C_1} \right) C_2$

$$V_1 = V_2 = \epsilon = \frac{Q_0}{C_1}; U_1 = \frac{1}{2} C_1 \left(\frac{Q_0}{C_1} \right)^2 = \frac{Q_0^2}{2C_1}$$

$$U_2 = \frac{1}{2} C_2 \left(\frac{Q_0}{C_1} \right)^2 = \frac{Q_0^2 C_2}{2C_1^2}$$

35. S-open ; $V_{inner} = V_{outer}$
S-closed ; $V_{inner} = 0$

$$\Rightarrow \frac{KQ}{3R} + \frac{Kq}{R} = 0 \Rightarrow q = -Q/3$$

$$C_{initial} = 4\pi \epsilon_0 (3R)$$

$$C_{final} = 4\pi \epsilon_0 (3R) + \frac{4\pi \epsilon_0 (3R)(R)}{(3R-R)}$$

$$\therefore C_{final} > C_{initial}$$

36. $W_{ext} = -\Delta U = U_i - U_f$

$$= \frac{1}{2} \times 2\mu F \times 400 - \frac{1}{2} \times 1\mu F \times 400 = 200\mu J$$

37. $U_{initial} = \frac{1}{2} CV^2; U_{final} = \frac{1}{2} CV^2 \therefore \Delta U = 0$

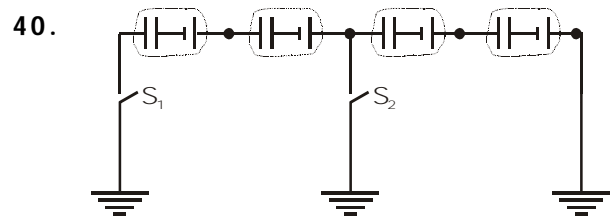
$$\therefore \text{Heat} = \text{work done by battery} = [CV - (-CV)]V = 2CV^2$$

38. $eV = \frac{1}{2} m (v_2^2 - v_1^2)$

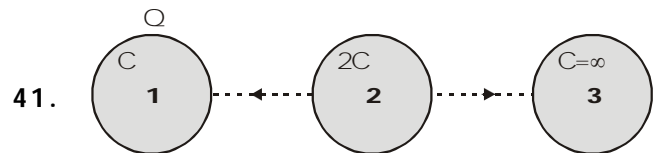
$$\Rightarrow 1.6 \times 10^{-19} \times 20 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (v^2 - 0)$$

$$\Rightarrow v = 2.65 \times 10^6 \text{ m/s}$$

39. V decreases continuously from left to right except in conductor where it is constant.



Potential difference across each capacitor and cell combination is zero.



Initial charge on 1 = Q when C_1 & C_2 touches

$$\Rightarrow \frac{Q_1}{C} = \frac{C}{2C} = \frac{1}{2} \Rightarrow Q_1 = \frac{Q}{3}, Q_2 = \frac{2Q}{3}$$

Now when Q_2 & Q_3 is touched

$$\Rightarrow \frac{Q_2}{Q_3} = \frac{C_2}{C_3} = \frac{2C}{\infty} = 0 \Rightarrow Q_2 = 0$$

Again when Q_1 & Q_2 is touched

$$Q_2 = 2 \frac{(Q/3)}{3} \Rightarrow Q_1 = \frac{(Q/3)}{3} = \frac{Q}{9}$$

Similarly we can say after N times it becomes

$$Q_1 = \frac{Q}{3^N}$$

42. $\Delta Q = 2CV - (-CV) = 3CV$

$$W_B = \Delta Q(2V) = 6CV^2$$

$$\Delta U = U_f - U_i = \frac{1}{2} C(2V)^2 - \frac{1}{2} CV^2 = \frac{3CV^2}{2}$$



$$\therefore \text{Heat} = W_B - \Delta U = \frac{9CV^2}{2}$$

$$U_f = \frac{1}{2}C(2V)^2 = 2CV^2$$

$$\therefore \frac{\text{Heat}}{U_f} = \frac{9}{4} = 2.25$$

EXERCISE -III

Fill in the blanks

- Net charge on capacitor is zero. Hence total flux through a closed surface enclosing the capacitor is zero.
- C_{Maximum} = All capacitors are in parallel
 $= 3C = 18\mu\text{F}$
 C_{Minimum} = All capacitor are in series
 $= C/3 = 2\mu\text{F}$
- $V_1 = V_2 \Rightarrow \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$
- Charge holding capacity increases, hence capacity increases.
- Air capacitor and dielectric capacitors are in series.

$$\therefore C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{2\epsilon_0 A}{d}\right) \times \left(\frac{2K\epsilon_0 A}{d}\right)}{\left(\frac{2\epsilon_0 A}{d}\right)(1+K)} = \frac{2KC}{1+K}$$

Match the column

- Initial charge $q_1 = \frac{CE}{2}$
 Final charge $q_2 = CE$
 Initial stored energy

$$U_1 = \frac{1}{2}C(E/2)^2 + \frac{1}{2}C(E/2)^2 = \frac{CE^2}{4}$$

$$\text{Final stored shergy } (U_2 = \frac{CE^2}{2})$$

Charge supplied by battery

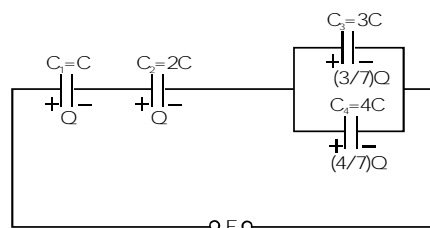
$$\Delta Q = q_2 - q_1 = CE - \frac{CE}{2} = \frac{CE}{2}$$

$$\text{Work done by battery } W_B = \Delta QE = \frac{CE^2}{2}$$

Heat developed in the system

$$H = W_B - \Delta U = \frac{CE^2}{2} - \left(\frac{CE^2}{2} - \frac{CE^2}{4}\right) = \frac{CE^2}{4}$$

-





At $C_1 = V_1 = \frac{Q}{C}$ and $U_1 = \frac{Q^2}{2C}$

At $C_2 = V_2 = \frac{Q}{2C}$ and $U_2 = \frac{Q^2}{4C}$

At $C_3 = V_3 = \frac{Q}{7C}$ and $U_3 = \frac{3Q^2}{98C}$

At $C_4 = V_4 = \frac{Q}{7C}$ and $U_4 = \frac{4Q^2}{98C}$

Therefore $V_{\max} = V_1$ and $V_{\min} = V_3 = V_4$
and $U_{\max} = U_1$ and $U_{\min} = U_3$

Comprehension - 1

1. In steady state

$$I_{\text{circuit}} = \frac{V}{R_1 + R_2} = \frac{18}{3 + 6} = 2A$$

$$V_{R_2} = V_{C_2} = IR_2 = 2 \times 6 = 12 V$$

$$Q_{C_2} = C_2 V_{C_2} = 12 \times 4 = 48 \mu C$$

2. $Q_{\text{initial}} = Q_{C_1} + Q_{C_2} = IR_1 C_1 + IR_2 C_2$
 $= 3 \times 2 \times 2 + 3 \times 4 \times 4 = 12 + 48 = 60 \mu C$
 $Q_{\text{final}} = V(C_1 + C_2) = 18(2+4) = 108 \mu C$
 $\therefore \Delta Q = 108 - 60 = 48 \mu C$ (through S_1)

3. $U_{\text{initial}} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$
 $= \frac{1}{2} \times 2 \times 6^2 + \frac{1}{2} \times 4 \times 12^2 = 324 \mu J$

$$U_{\text{final}} = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} (2+4) 18^2 = 972 \mu J$$

$$\Delta Q = Q_f - Q_i = 48 \mu C$$

$$W_{\text{Battery}} = \Delta Q \cdot V = 48 \times 18 = 864 \mu J$$

$$\therefore \text{Heat} = W_B - \Delta U = 864 - (972 - 324) = 216 \mu J$$

Comprehension-2

1. Time Constant)

$$\tau = R_1 C = 8 \times 6 = 48 \mu s$$

2. $V_{t=2\tau} = V_0(1 - e^{-t/\tau}) = 12(1 - e^{-2\tau/\tau})$
 $= 12 \left(1 - \frac{1}{7.4} \right) = 10.4 V$

3. $(V_{R_1})_{t=2\tau} = V_0 - V_{\text{capacitor}} = 12 - 10.4 = 1.6 V$

4. $V_{R_2} = V_0 = 12V$

Comprehension - 3

1. $V_b = \epsilon_0 (1 - e^{-t/RC})$

$$\Rightarrow 110 = 120 (1 - e^{-t/RC})$$

$$\Rightarrow e^{-t/RC} = 1/12$$

$$\Rightarrow t/RC = \ln/12 = 2.5$$

$$\Rightarrow t = RC \times 2.5 = 10^6 \times 10^{-6} \times 2.5 = 5/2 \text{ sec}$$

2. $\tau_0 = 10^{-6} \times 10 = 10 \mu s$

3. Flash duration = $3\tau_0 = 30 \mu s$

4. Energy in flash

$$= \frac{1}{2} CV^2 = \frac{1}{2} \times 1 \times 10^{-6} \times 110 \times 110 = 6.1 \text{ mJ}$$

Comprehension-4

1. $q_{1\max} = q_{2\max}$
 C_1 and C_2 may be different and hence E_1 and E_2 may be different.

2. $\tau_2 > \tau_1 \Rightarrow R_2 C_2 > R_1 C_1 \Rightarrow \frac{R_1}{R_2} < \frac{C_2}{C_1}$

Comprehension-5

1. $\frac{C_A}{C_B} = \frac{\epsilon_0 A / d}{K \epsilon_0 A / d} = 1 : K$

2. $\frac{V_A}{V_B} = \frac{Q / C_A}{Q / C_B} = \frac{C_B}{C_A} = K : 1$

3. $(V_A)_{\text{initial}} = \frac{V}{2}; (V_A)_{\text{final}} = \frac{E (KC)}{C (K+1)} = \frac{KE}{K+1}$

$$\therefore \frac{(V_A)_{\text{initial}}}{(V_A)_{\text{final}}} = \frac{K+1}{2K}$$

4. $(V_B)_{\text{initial}} = \frac{V}{2}; (V_B)_{\text{final}} = \frac{Q}{C_B} = \frac{E(KC)}{(K+1) KC} = \frac{E}{K+1}$

$$\therefore \frac{(V_B)_{\text{initial}}}{(V_B)_{\text{final}}} = (K+1) : 2$$

5. $(U_A)_{\text{final}} = \frac{Q^2}{2C_A}; (U_B)_{\text{final}} = \frac{Q^2}{2C_B} \therefore \left(\frac{U_A}{U_B} \right)_{\text{final}} = K:1$



EXERCISE -IV(A)

1. $CV = \frac{qt}{t} \Rightarrow 400 \times 10^{-6} \times 100 = 100 t$
 $\Rightarrow t = 400 \text{ s}$

2. Equivalent capacity between A and B

$$C = \frac{9}{3} + 3 = 6\mu\text{F}$$

(i) Stored charge

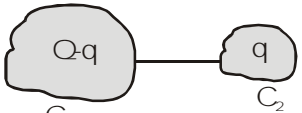
$$Q = CV = 6 \times 10^{-6} \times 4 = 24 \mu\text{C}$$

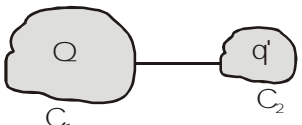
(ii) Stored energy

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 16 = 48 \mu\text{J}$$

3. Electric field

$$E = \frac{V_A - V_B}{d} = \frac{(10,000 - 0)}{(2 \times 10^{-3})} = 5 \times 10^6 \text{ V/m}$$

4.  $\frac{Q - q}{C_1} = \frac{q}{C_2} \dots(i)$

 $\frac{Q}{C_1} = \frac{q'}{C_2} \dots(ii)$

Eq. (i) \div (ii) : $q' = \frac{Qq}{Q - q}$

5. Common potential

$$V_{cm} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{2 \times 200 + 3 \times 400}{2 + 3} = 320\text{V}$$

Charge on $C_1 Q_1 = C_1 V_{cm} = 2 \times 320 \mu\text{C} = 640 \mu\text{C}$

Charge on $C_2 Q_2 = C_2 V_{cm} = 3 \times 320 \mu\text{C} = 960 \mu\text{C}$

6. (i) On connecting with the second capacitor the charge distributes equally

$$\therefore V_{cm} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{0.1 \times 10}{0.1 + 0.1} = 5\text{V}$$

Total stored energy

$$U_f = \frac{1}{2} C_1 V_{CM}^2 + \frac{1}{2} C_2 V_{CM}^2$$

$$= \frac{1}{2} \times 0.1 \times 10^{-6} \times (5)^2 + \frac{1}{2} \times 0.1 \times 10^{-6} \times (5)^2$$

$$= 2.5 \mu\text{J}$$

(ii) Initial stored energy in first capacitor

$$U_i = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 0.1 \times 10^{-6} \times 10^2 = 5.0 \mu\text{J}$$

$$\Rightarrow \frac{U_f}{U_i} = \frac{2.5}{5.0} = \frac{1}{2}$$

7. $\therefore C = \frac{\epsilon_0 A}{d}$; $q = \left(\frac{\epsilon_0 A}{d}\right)V$

Slope = $\frac{\epsilon_0 A}{d}$ $\therefore C_2 > C_1 > C_3$

8. By using KCL

$$C_1 (V_A - V_0) + C_2 (V_B - V_0) + C_3 (V_C - V_0) = 0 \Rightarrow V_0$$

$$= \frac{C_1 V_A + C_2 V_B + C_3 V_C}{C_1 + C_2 + C_3}$$

9. $x = \frac{2x}{2+x} + 1$

(Let $C_{eq} = x$)

$$x = \frac{2x + 2 + x}{2 + x}$$

$$\Rightarrow x(2 + x) = 3x + 2$$

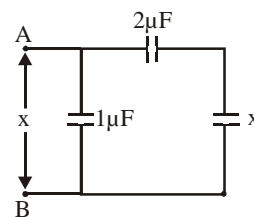
$$\Rightarrow 2x + x^2 = 3x + 2 \Rightarrow x^2 - x - 2 = 0$$

Use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 + 8}}{2} = \frac{1 \pm 3}{2} = 2$$

and -1

$$x = 2, C_{eq} = 2\mu\text{F}$$



10. $\frac{C_A}{C_B} = \frac{\left(\frac{K_1 \epsilon_0 A}{d/4}\right)}{\left(\frac{K_2 \epsilon_0 A}{3d/4}\right)} = \frac{3K_1}{K_2} = 3 \times 3 = 9$

Net capacity

$$C = \frac{C_A C_B}{C_A + C_B} = \frac{(9C_B)(C_B)}{9C_B + C_B} = \frac{9}{10} C_B$$

$$= \frac{9}{10} \left[\frac{K_2 \epsilon_0 A}{(3d/4)} \right] = \frac{6K_2 \epsilon_0 A}{5d} = \frac{1.2K_2 \epsilon_0 A}{d}$$

11. $\therefore E = \frac{V}{d}$

$$\therefore d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3} \text{ m}$$

Now $C = \frac{\epsilon_0 \epsilon_r A}{d}$

$$\Rightarrow A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{88.5 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 10} = 10^{-3} \text{ m}^2$$



12. $C_x = \frac{\epsilon_0 A}{d}$, $C_y = \frac{5\epsilon_0 A}{d} \Rightarrow C_y = 5C_x$
 (i) C_x and C_y are in series, so charge on each

$$q = C_x V_x = C_y V_y \Rightarrow \frac{V_x}{V_y} = 5$$

$$\therefore V_x + V_y = 12 \quad \therefore 6V_y = 12$$

$$\Rightarrow V_y = \frac{12}{6} = 2 \text{ volt and } V_x = 10 \text{ volt}$$

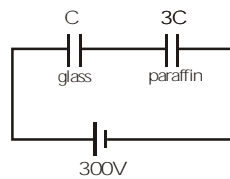
- (ii) Energy stored in capacitor

$$U = \frac{q^2}{2C} \Rightarrow \frac{U_x}{U_y} = \frac{\frac{q^2}{2C_x}}{\frac{q^2}{2C_y}} = \frac{C_y}{C_x} = 5$$

13. $CV_1 = 3CV_2 \dots(i)$

$$V_1 + V_2 = 300 \dots(ii)$$

$$\Rightarrow V_1 = 75V; V_2 = 225 V$$



(i) $\therefore E_1 = \frac{V_1}{d_1} = \frac{75 \times 100}{0.5} = 1.5 \times 10^4 \text{ V/m}$

$$E_2 = \frac{V_2}{d_2} = \frac{225 \times 100}{0.5} = 4.5 \times 10^4 \text{ V/m}$$

(ii) $V_1 = 75 \text{ V}; V_2 = 225 \text{ V}$

(iii) $Q = \left(\frac{C_1 C_2}{C_1 + C_2} \right) V = \frac{3}{4} C V = \frac{3}{4} \left(\frac{2 \epsilon_0 A}{d} \right) 300$

$$\Rightarrow \frac{Q}{A} = \frac{6 \times 300 \times 8.89 \times 10^{-12}}{4 \times 0.5 \times 10^{-2}} = 8 \times 10^{-7} \text{ C/m}^2$$

14. When S_{W1} is closed and S_{W2} is open then capacitor B is charged upto 10V.

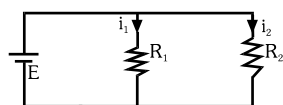
Now S_{W1} is open and S_{W2} is closed then

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{3 \times 10 + 2 \times 0}{3 + 2} = 6V$$

$$Q_A = 2 \times 10^{-6} V_{\text{cm}} = 12 \mu\text{C}$$

$$Q_B = 3 \times 10^{-6} V_{\text{cm}} = 18 \mu\text{C}$$

15. (i) At $t = 0$, capacitor has zero resistance, i.e., R_1 and R_2 are in parallel. The simple circuit is shown in figure



$$i_1 = \frac{E}{R_1} \quad \text{and} \quad i_2 = \frac{E}{R_2}$$

- (ii) At steady state ($t = \infty$), capacitor has infinite resistance.

$$\text{Hence, } i_1 = \frac{E}{R_1}, i_2 = 0$$

- (iii) Final potential difference across capacitor is E.

\therefore Final energy stored

$$U = \frac{1}{2} CE^2$$

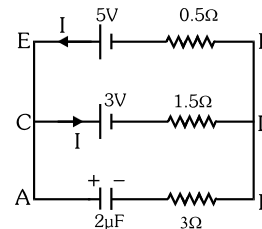
- (iv) When switch is opened, capacitor will discharge through two resistance as R_1 and R_2 (both in series).

$$\text{Hence, } \tau_c = C (R_1 + R_2)$$

- (v) When switch is closed, capacitor will charged through resistance R_2 .

$$\text{So } \tau = R_2 C$$

16. (a) In steady state no current in capacitor's branch.



$$\text{So curren } I = \frac{2}{0.5 + 1.5} = 1A$$

voltage across capacitor

$$V_C = 3 + 1.5 \times 1 = 4.5 \text{ V}$$

$$\Rightarrow Q = CV_C = 2 \times 10^{-6} \times 4.5$$

$$= 9 \times 10^{-6} \text{ C}$$

17. For the circuit ACDA and the cell :

$$6 - I_1(5) - 6 = 0 \Rightarrow I_1 = 0, \quad \therefore I = 0$$

$$\text{For the loop BCD } V_{2\mu\text{F}} = 6V$$

$$\text{For the loop ABD : } V_{7\mu\text{F}} = 6V$$

$$\therefore Q_{7\mu\text{F}} = 6 \times 7 = 42 \mu\text{C}$$

18. Total heat dissipated

$$H = \frac{1}{2} CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times 200 \times 200 = 0.1 \text{ J}$$

$$H_1 = \text{Heat developed across } R_1 = \int I^2 R_1 dt$$

$$H_2 = \text{Heat developed across } R_2 = \int I^2 R_2 dt$$



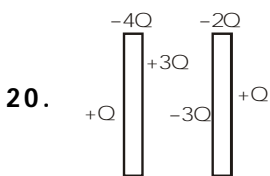
$$\Rightarrow H_1 = \frac{(H_1 + H_2)R_1}{(R_1 + R_2)} = \frac{H R_1}{(R_1 + R_2)}$$

$$= \frac{0.1 \times 500}{(500 + 330)} = 60 \text{ mJ}$$

19. $R_{\text{eff}} = \frac{2 \times 3}{3 + 2} + 2.8 = 4 \Omega$

$$I = \frac{V}{R_{\text{eff}}} = \frac{6}{4} = 1.5 \text{ A}$$

$$\therefore I_{2\Omega} = I \left(\frac{3}{2 + 3} \right) = \frac{1.5 \times 3}{5} = 0.9 \text{ A}$$



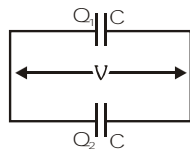
Initial effective charge = 3Q

$$CV + CV = Q_1 + Q_2$$

$$= 3Q + 0$$

$$= 3Q$$

$$\therefore V = \frac{3Q}{2C}$$



21. $\frac{1}{C_{\text{arm}}} = \frac{1}{C} \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) = \frac{1}{C} \left(\frac{1}{1 - \frac{1}{2}} \right) = \frac{2}{C}$

$$\therefore C_{\text{effective}} = 2C_{\text{arm}} = \frac{2C}{2} = C$$

22. $E_{\text{final}} = \frac{1}{2} \frac{Q^2}{C} = \frac{Q^2 d}{2 \epsilon_0 A}$; $E_{\text{initial}} = 0$

$$\therefore \text{Heat} = - (E_{\text{initial}} - E_{\text{final}}) = \frac{Q^2 d}{2 \epsilon_0 A}$$

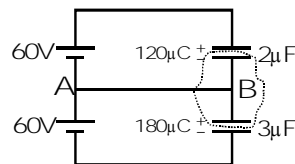
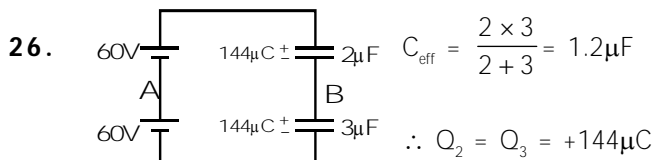
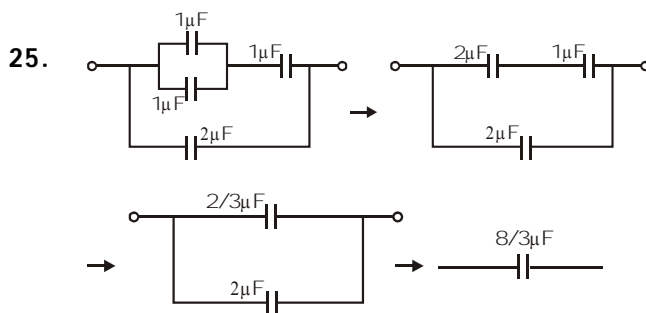
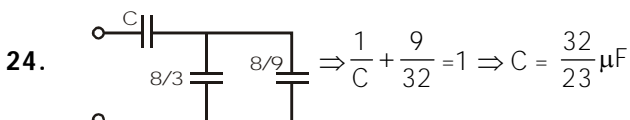
23. $V_{2 \text{ initial}} = \frac{20}{2} = 10 \text{ V}$

$$V_{5 \text{ initial}} = \frac{50}{5} = 10 \text{ V}$$

There is no potential difference.

Hence no charge flows.

Heat produce is zero.



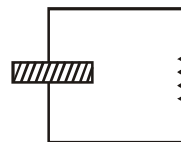
Q_{total} on the middle plates (

$$= + 180 + (-120) = + 60 \mu\text{C}$$

This charge flows from A to B.

EXERCISE -IV(B)

1. $\frac{q}{C} - iR = 0 \Rightarrow \frac{q}{C} + \frac{dq}{dt} R = 0 \Rightarrow q = q_0 e^{-t/RC}$



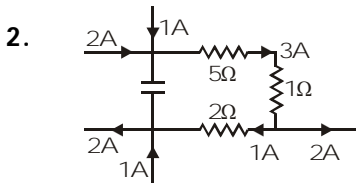
equivalent circuit $\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC}$

Where $R = \frac{L}{SA}$, $C = \frac{k \epsilon_0 A}{4}$

$$\therefore RC = \frac{k \epsilon_0}{S} = \frac{5 \times 8.85 \times 10^{-12}}{7.4 \times 10^{-12}} = \frac{5 \times 8.85}{7.4}$$

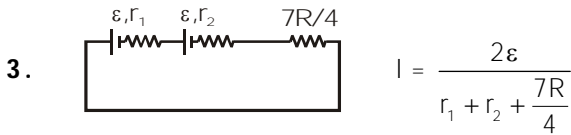
$$\therefore i = \frac{q_0}{RC} e^{-t/RC} = \frac{8.85 \times 10^{-3}}{\left(\frac{5 \times 8.85}{7.4} \right)} e^{-12/6}$$

$$= \frac{7.4}{5} \times \frac{1}{7.4} \text{ mA} = 0.2 \text{ mA}$$



$$V_c = (5 + 1) \times 3 + 2 \times 1 = 20V$$

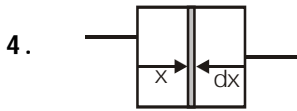
$$U_{cap} = \frac{1}{2} CV^2 = \frac{1}{2} \times 4 \times 20^2 = 0.8 \text{ mJ}$$



$$I = \frac{2\varepsilon}{r_1 + r_2 + \frac{7R}{4}}$$

Pot. diff. across (ε, r_1) cell : $\varepsilon - Ir_1 = 0$

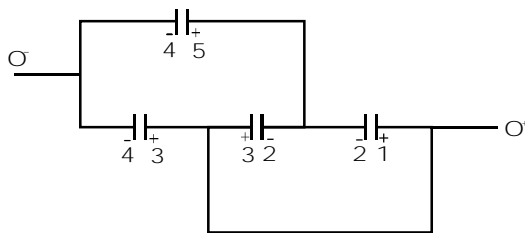
$$\Rightarrow \varepsilon = Ir_1 \Rightarrow \varepsilon = \frac{2\varepsilon r_1}{r_1 + r_2 + \frac{7R}{4}} \Rightarrow \frac{4(r_1 - r_2)}{7} = R$$



$$\int \frac{1}{dC} = \int \frac{dx}{KS \varepsilon_0} = \int_0^d \frac{dx}{KS \varepsilon_0 \left(1 + \sin \frac{\pi x}{d}\right)}$$

$$\Rightarrow C = \frac{K_1 S \varepsilon_0 \pi}{2d} \left[\int_0^d \frac{dx}{\left(1 + \sin \frac{\pi x}{d}\right)} = \frac{2d}{\pi} \right]$$

$$5. \quad C_{eq} = \frac{2C}{3} + C = \frac{5C}{3}$$



$$Q_3 = \frac{4}{3} \varepsilon_0 \frac{AV_0}{d} \quad \& \quad Q_5 = \frac{2}{3} \varepsilon_0 \frac{AV_0}{d}$$

$$6. \quad Q_{total} = C_1 V = \left[C_1 + \frac{C_2 C_3}{C_2 + C_3} \right] V_0$$

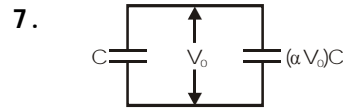
$$\Rightarrow V_0 = \frac{C_1 (C_2 + C_3) V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

\therefore Charge on C_1 ,

$$q_1 = C_1 V_0 = \frac{C_1^2 V (C_2 + C_3)}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

Charge on C_2 and C_3

$$q_2 = q_3 = \left(\frac{C_2 C_3}{C_2 + C_3} \right) V_0 = \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$



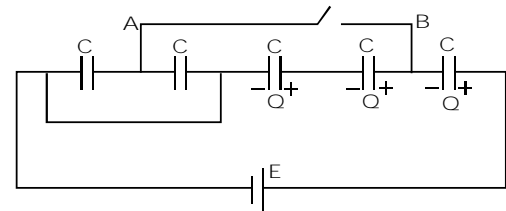
Total charge remains constant

$$156 C = (\alpha V_0) C V_0 + C V_0$$

$$\Rightarrow V_0^2 + V_0 - 156 = 0 \quad (\alpha = 1)$$

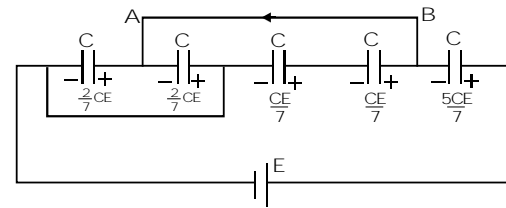
$$\Rightarrow (V_0 + 13)(V_0 - 12) = 0 \Rightarrow V_0 = 12 \text{ volt}$$

8. Initial condition



$$Q = \frac{CE}{3}$$

Final condition



$$\text{Charge flown from B to A} = \frac{4}{7} CE$$

9. Extra weight needed

$$= \left(\frac{6}{\varepsilon_0} \right)^2 \times \frac{\varepsilon_0 A}{2} = E^2 \times \frac{\varepsilon_0 A}{2} = \left(\frac{V}{d} \right)^2 \frac{\varepsilon_0 A}{2}$$

$$\Rightarrow mg = \left(\frac{5000}{5 \times 10^{-3}} \right)^2 \times \frac{8.85 \times 10^{-12} \times 100}{2 \times 100 \times 100}$$

$$\Rightarrow m = 4.52 \times 10^{-3} \text{ kg}$$

11. $Q_{pq} = 2C_2 = 6C_1 = Q_{bp}$

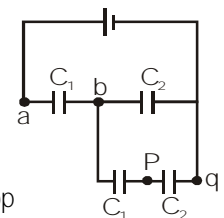
$$\therefore V_{bp} = \frac{6C_1}{C_1} = 6V$$

$$\therefore V_{bq} = 6 + 2 = 8V$$

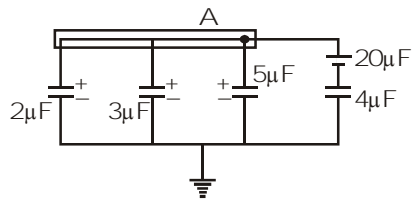
Total charge flown into right loop

$$= C_2 V_{bq} + C_1 V_{bp} = 3C_1 \times 8 + C_1 \times 6 = 30C_1$$

$$\therefore V_{ab} = \frac{Q_{total}}{C_{ab}} = \frac{30C_1}{C_1} = 30 \text{ volt}$$



12. Applying junction law at A :



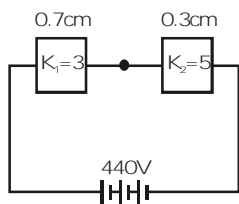
$$2(V_A - 5) + 3(V_A - 20) + 5(V_A - 10) + 4(V_A - 20) = 0$$

$$\Rightarrow V_A = \frac{100}{7} = 14.28 \text{ volt}$$

$$\therefore Q_{2\mu F} = 28.56 \mu C, Q_{3\mu F} = 42.84 \mu C,$$

$$Q_{5\mu F} = 71.40 \mu C, Q_{4\mu F} = 22.88 \mu C$$

13. $V_1 C_1 = V_2 C_2$ and $V_1 + V_2 = 440$



$$\Rightarrow V_2 = \frac{V_1 C_1}{C_2} \Rightarrow V_1 + \frac{V_1 C_1}{C_2} = 440$$

$$\Rightarrow V_1 = \frac{440 C_2}{C_1 + C_2} = \frac{440}{\frac{C_1}{C_2} + 1}$$

$$\Rightarrow V_1 = \frac{440}{\left(\frac{K_1 / d_1}{K_2 / d_2}\right) + 1} = \frac{440}{\frac{9}{35} + 1} = \frac{440 + 35}{44} = 350 \text{ V}$$

$$\therefore E_1 = \frac{V_1}{d} = \frac{350 \times 100}{0.7} = 5 \times 10^4 \text{ V/m}$$

$$E_2 = \frac{V_2}{d} = \frac{90}{0.3} \times 100 = 3 \times 10^4 \text{ V/m}$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2} C_1 V_1^2}{\frac{1}{2} C_2 V_2^2} = \frac{35}{9}$$

14. Work done by battery
 $= \Delta QV = (3CV)V = 3CV^2$

Energy stored in capacitors $= \frac{1}{2} (3C)V^2$

(i) \therefore Heat developed $= W_B - \Delta U = \frac{1}{2} (3C)V^2$

(ii) Work done by external agent $= - (K-1)$

(iii) Final voltage after 'dielectric is removed' $= V'$

$$3CV' = (K+2)CV \Rightarrow V' = V \left(\frac{K+2}{3} \right)$$

$$W_{\text{agent}} = U_i - U_f$$

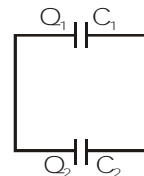
$$= \frac{1}{2} (3C)V^2 \left(\frac{K+2}{3} \right)^2 - \frac{1}{2} (K+2)CV^2$$

$$= \frac{(K+2)(K-1)CV^2}{6}$$

15. $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \dots (i)$

$Q_1 + Q_2 = 2Q \dots (ii)$

$C_1 = \frac{\epsilon_0 A}{d-x}$ and $C_2 = \frac{\epsilon_0 A}{d+x}$



$\Rightarrow Q_2 = \frac{Q(d-x)}{d}$ and $Q_1 = \frac{Q(d+x)}{d}$

$\Rightarrow \frac{dQ_2}{dt} = - \frac{Q}{2d} \left(\frac{dx}{dt} \right)$ & $\frac{dQ_1}{dt} = \frac{Q}{2d} \left(\frac{dx}{dt} \right)$

$\therefore I = \frac{dQ_1}{dt} - \frac{dQ_2}{dt} = \frac{Q}{d} \left(\frac{dx}{dt} \right) = \frac{200}{0.1} \times 0.001 = 2 \mu A$

16. $U_1 = \frac{Q^2}{2C_1}, C_1 = 4\pi \epsilon_0 \left[\frac{ab}{b-a} + b \right]$

$U_2 = \frac{Q^2}{2C_2}, C_2 = 4\pi \epsilon_0 b$

$\therefore \Delta U = U_1 - U_2 = 9 \text{ J}$

17. $C_{\text{initial}} = \frac{2C \times C}{2C + C} = \frac{2C}{3}; C_{\text{final}} = C$

(i) $\therefore \Delta Q = \Delta C \times V$

$$= \left(C - \frac{2C}{3} \right) V = \frac{CV}{3} = \frac{2 \times 30}{3} = 20 \mu C$$

(ii) $H = W_B - \Delta U = \Delta QV - \left(\frac{1}{2} CV^2 - \frac{1}{2} \frac{2CV^2}{3} \right)$

$= 600 - (900 - 600) = 300 \mu J = 0.3 \text{ mJ}$

(iii) Energy supplied by the battery

$= \Delta QV = 600 \mu J = 0.6 \text{ mJ}$

(iv) Initial charge on each capacitor



$$= \frac{2C}{3} V = 40 \mu\text{C}$$

Final charge on right capacitor = $60 \mu\text{C}$

Final charge on left capacitors = 0

\therefore Total charge from through switch, $S = 60 \mu\text{C}$

EXERCISE-V-A

1. $C_{\text{eff(Parallel)}} = nC$

If connected across V volts then energy stored

$$= \frac{1}{2} (nC) V^2$$

2. Capacitance of an isolated sphere is

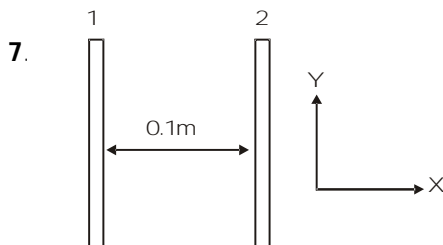
$$C = (4\pi\epsilon_0)(\text{Radius})$$

$$C = \frac{1}{9 \times 10^9} \times 1 = 0.11 \times 10^{-9} = 1.1 \times 10^{-10} \text{ F}$$

4. Work done = $\frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{(8 \times 10^{-18})^2}{100 \times 10^{-6}}$
 $= \frac{1}{2} \times \frac{64 \times 10^{-36}}{10^{-4}} = 32 \times 10^{-32} \text{ J}$

5. $\frac{1}{2} CV^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

6. Two plates stacked together form a single capacitor of capacitance C . n plates stacked together form $(n-1)$ number of capacitors of effective capacitance $(n-1)C$.



Applying law of conservation of energy

We get $\frac{1}{2} mv^2 = eV$

[Here, v = speed of electron, $V = V_2 - V_1$ = potential difference]

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 20}{9 \times 10^{-31}}}$$

On solving, we get $v = 2.65 \times 10^6 \text{ m/s}$

8. Energy stored in a capacitor when it is charged by a potential difference of V_0 volt = $\frac{1}{2} QV_0$

Total work done by battery in sending a charge of Q through emf $V_0 = QV_0$

hence $\frac{\text{energy stored in capacitor}}{\text{work done by battery}} = \frac{\frac{1}{2} QV_0}{QV_0} = \frac{1}{2}$

9. Net work done by the system in the process is zero, as in removing the dielectric, work done is equal and opposite to the work done is re-inserting the dielectric.

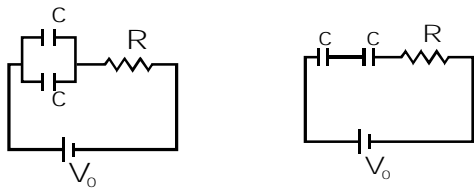
10. $C = \frac{\delta A}{d} = 9 \text{ Pf}$; $C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}$
 $\Rightarrow \frac{\left(\frac{3\epsilon_0 AK_1}{d}\right) \left(\frac{3\epsilon_0 AK_2}{2d}\right)}{\frac{3\epsilon_0 AK_1}{d} + \frac{3\epsilon_0 AK_2}{2d}} \Rightarrow \frac{d}{2} \text{ F} = 40.5 \text{ pF}$

11. $U = \frac{1}{2} CV^2$; $\frac{U_0}{2} = \frac{1}{2} CV_0^2 e^{-2t_1/RC}$
 $\frac{1}{2} = e^{-2t_1/RC}$ ($U_0 = \frac{1}{2} CV_0^2$)
 $\frac{2t_1}{RC} = \ln 2$
 $t_1 = \frac{RC \ln 2}{2}$... (i) and $\frac{q_0}{4} = q_0 e^{-t_2/RC}$
 $\frac{t_2}{RC} = 2 \ln 2$; $t_2 = 2RC \ln 2$... (ii)

from equation (i) and (ii) $\frac{t_1}{t_2} = \frac{1}{4}$

12. $V = V_0 (1 - e^{-t/RC}) \Rightarrow 120 = 200 (1 - e^{-\frac{5}{RC}})$
 $\Rightarrow R = 2.7 \times 10^6 \Omega$

13. Parallel Series



$$\frac{V_0}{2} = V_0 \left(1 - e^{-\frac{t_p}{R \times 2C}}\right) \dots(i)$$

$$\frac{V_0}{2} = V_0 \left(1 - e^{-\frac{t_s}{R \times \frac{C}{2}}}\right) \dots(2)$$

from (i) and (ii) $e^{-\frac{t_p}{2RC}} = e^{-\frac{2t_s}{RC}}$

$$t_s = \frac{t_p}{4} = \frac{10}{4} = 2.5 \text{ sec}$$

14. $t = 0.37\%$ of V_0
 $= 0.37 \times 25 = 9.25$ volt
 where is in between 100 and 150 sec.

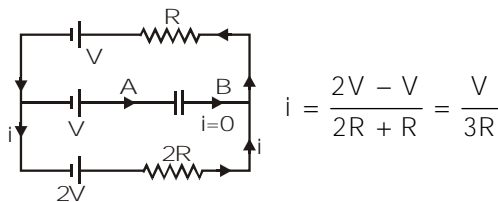
15. Common voltage = $\frac{C_1 V_1 - C_2 V_2}{C_1 + C_2}$

(positive plate of one capacitor is connected with negative plate of second capacitor)

$$\Rightarrow 120 C_1 = 200 C_2 \Rightarrow 3C_1 = 5C_2$$

EXERCISE -V-B

1. In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,



Potential difference between A and B :

$$V_A - V + V + iR = V_B$$

$$\therefore V_B - V_A = iR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$$

2. Charging current $I = \frac{E}{R} e^{-\frac{t}{RC}}$

Taking log both sides

$$\log I = \log\left(\frac{E}{R}\right) - \frac{t}{RC}$$

When R is doubled, slope of curve increase. Also at $t=0$, the current will be less. Graph Q represents the best.

3. Given : $V_C = 3V_R = 3(V - V_C)$
 Here, V is the applied potential.

$$\therefore V_C = \frac{3}{4}V \Rightarrow V(1 - e^{-t/RC}) = \frac{3}{4}V \therefore e^{-t/RC} = \frac{1}{4}$$

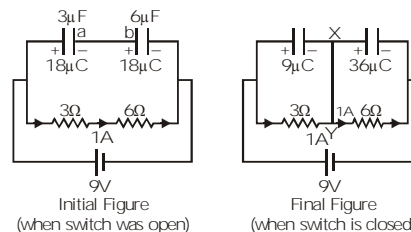
Here $\tau_c = CR = 10s$

Substituting this value of τ_c in equation and solving

We get : $t = 13.86 s$

4. $\tau = CR$
 $\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18 \mu s$
 $\tau_2 = \left(\frac{C_1 C_2}{C_1 + C_2}\right) \left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9} \mu s$

5. From Y to X charge flows to plates a and b.
 $(q_a + q_b)_i = 0, (q_a + q_b)_f = 27 \mu C$



$\therefore 27 \mu C$ charge flows from Y to X.

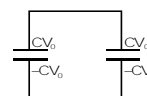
6. Time constant = RC

Where $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\left(\frac{2d}{3} + Vt\right)}{\epsilon_0} + \frac{\left(\frac{d}{3} - Vt\right)}{2\epsilon_0}$

$$\Rightarrow C = \frac{6\epsilon_0}{5d + 3Vt}$$

MCQ's

1. Before S_3 is pressed

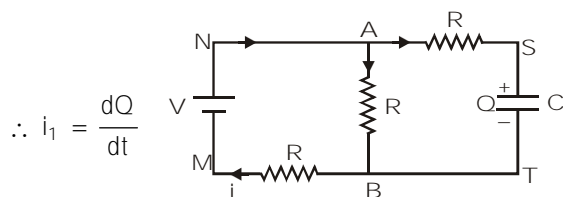


After S_3 is pressed

Subjective



1. Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t .



$$\therefore i_1 = \frac{dQ}{dt}$$

(i) Applying Kirchhoff's second law in loop MNABM

$$V = (i - i_1)R + iR \Rightarrow V = 2iR - i_1R \quad \dots(i)$$

Similarly, applying Kirchhoff's second law in loop MNSTM

we have $V = i_1R + \frac{Q}{C} + iR \quad \dots(ii)$

Eliminating i from equations (i) and (ii), we get

$$V = 3i_1R + \frac{2Q}{C} \Rightarrow 3i_1R = V - \frac{2Q}{C}$$

$$\Rightarrow i_1 = \frac{1}{3R} \left(V - \frac{2Q}{C} \right) \Rightarrow \frac{dQ}{dt} = \frac{1}{3R} \left(V - \frac{2Q}{C} \right)$$

$$\Rightarrow \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R} \Rightarrow \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2} (1 - e^{-2t/3RC})$$

(ii) $i_1 = \frac{dQ}{dt} = \frac{V}{3R} e^{-2t/3RC}$

From equation (i)

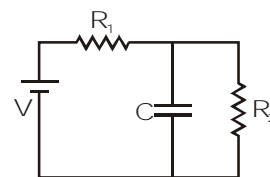
$$i = \frac{V + i_1R}{2R} = \frac{V + \frac{V}{3} e^{-2t/3RC}}{2R}$$

\therefore Current through AB

$$i_2 = i - i_1 = \frac{V + \frac{V}{3} e^{-2t/3RC}}{2R} - \frac{V}{3R} e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R} e^{-2t/3RC} \Rightarrow i_2 = \frac{V}{2R} \text{ as } t \rightarrow \infty$$

2. Q_0 is the steady state charge stored in the capacitor.

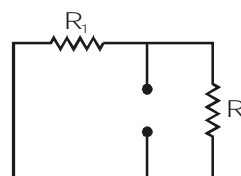


$$Q_0 = C [\text{PD across capacitor in steady state}]$$

$$= C [\text{steady state current through } R_2] (R_2)$$

$$= C \left(\frac{V}{R_1 + R_2} \right) R_2$$

$$\therefore Q_0 = \frac{CV R_2}{R_1 + R_2} \alpha \text{ is } \frac{1}{\tau_C} \Rightarrow \frac{1}{C R_{\text{net}}}$$



Here, R_{net} is equivalent resistance across capacitor after short circuiting the battery.

$$R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2}$$

(As R_1 and R_2 are in parallel)

$$\alpha = \frac{1}{C \left(\frac{R_1 R_2}{R_1 + R_2} \right)} = \frac{R_1 + R_2}{C R_1 R_2}$$