(PART - II)

CAPACITOR

EXERCISE –I

1. $k = \frac{f}{x} = \frac{5000}{0.2} = 2{,}5000\ \text{N/m}$

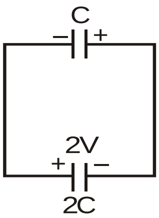
$$\frac{U_{SPR}}{U_{CAP}} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}CV^2} = \frac{25000 \times 0.2 \times 0.2}{10 \times 10^{-6} \times 10^{8}} = 1$$

2. $$P = \frac{\Delta U}{\Delta t} = \frac{\frac{1}{2}CV^2}{\Delta t} = \frac{\frac{1}{2} \times 40 \times 10^{-6} \times 9 \times 10^{6}}{2 \times 10^{-3}} = 90\text{kW}$$

3. V_0 (C+CV) = CV + (2C) (2V)

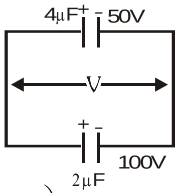
V_0 = V (Final pot. diff.)

$\therefore\ U_{final} = \frac{1}{2}(C + 2C)\ V^2 = \frac{3CV^2}{2}$



4. (4+2) V = (4 × 50) + (2 × 100)

$V = \frac{400}{6} = \frac{200}{3}\ V$



$U_{initial} = \left(\frac{1}{2} \times 4 \times (50)^2 + \frac{1}{2} \times 2 \times (100)^2\right) \times 10^{-6}$

$= (5000 + 10000) \times 10^{-6} = 1.5 \times 10^{-2}$ J

$U_{final} = \frac{1}{2}(4+2) \times 10^{-6} \times \frac{200}{3} \times \frac{200}{3}$

$= 1.33 \times 10^{-2}$ J

5. Before sharing $\quad U_i = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

After sharing $\quad U_f = \frac{(Q_1 + Q_2)^2}{2(C_1 + C_2)}$

$$\Delta U = U_f - U_i = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)} - \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

$$= -\frac{(Q_1C_2 - Q_2C_1)^2}{2C_1C_2(C_1+C_2)}$$

–ve sign indicates there is decrease in energy

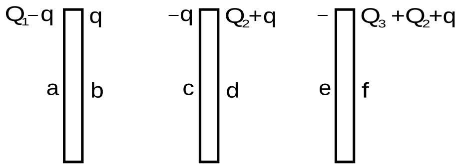
But $Q_1C_2 - Q_2C_1 \neq 0 \qquad \Rightarrow Q_1C_2 \neq Q_2C_1$

$\Rightarrow Q_1 4\pi\varepsilon_0 R_2 \neq Q_2 4\pi\varepsilon_0 R_1 \qquad \Rightarrow Q_1R_2 \neq Q_2R_1$

6. $C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}} \left(t = \frac{d}{2}, K = \infty\right)$

$$= \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2\epsilon_0 A}{d} = 2C_0$$

7.

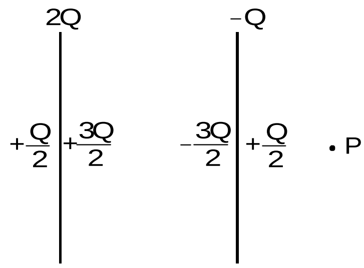


Here $Q_1 - q = Q_2 + Q_3 + q \Rightarrow q = \frac{Q_1 - (Q_2 + Q_3)}{2}$

Charge on a= Charge on f

$\Rightarrow Q_1 - q = \frac{\Sigma Q}{2} = \frac{Q_1 + Q_2 + Q_3}{2}$

8.



Force on either plate

$$= \frac{(3Q/2)^2}{2A\epsilon_0} = \frac{9Q^2}{8A\epsilon_0}$$

Force on point 'P' due to capacitor = 0

Potential diff. between the plates

$= \frac{3Q}{2C}$

Energy stored in electric field between the plates

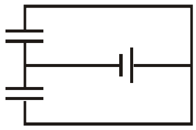
$= \frac{1}{2}C \times \left(\frac{3Q}{2C}\right)^2 = \frac{9Q^2}{8C}$

9.

Therefore

$q_2 = -2q,\ q_3 = +2q,$
$q_4 = -2q$ and $q_5 = +q$

10.

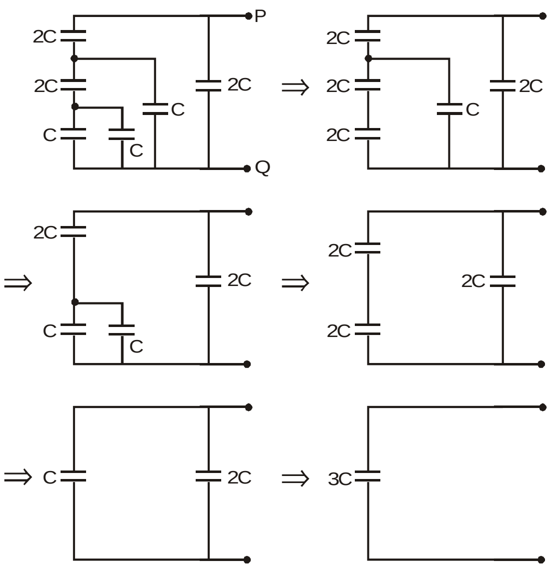


$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}\left(\frac{8.85\times10^{-12}\times0.1}{0.885\times10^{-3}}\times2\right)\times10^2 = 10^{-1}\mu J$$

11. Each capacitor has potential difference 'V' and energy $\frac{1}{2}CV^2$. After reconnecting total energy remains constant and total voltage becomes NV.

12.



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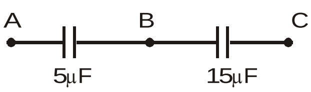


$5(V_A - V_B) = 15\,(V_B - V_C) \Rightarrow 5(2000 - V_B) = 15(V_B - 0)$
$\Rightarrow 2000 - V_B = 3V_B \Rightarrow V_B = 500V$

14. $C_{eff} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + \ldots.$

$$= \frac{C}{1 - 1/2} = 2C = 2\mu F$$

15. For 'n' plates; effective C will be (n–1)C.

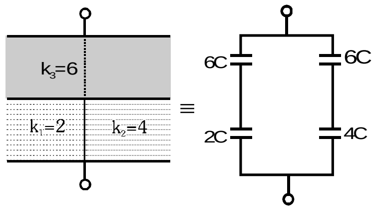
16. $CV + 2CV = KCV' + 2CV' \Rightarrow V' = \frac{3V}{K+2}$

17. $C = \frac{\in_0 A}{d} = 9pF$

$$C' = \frac{\in_0 A}{d - t_1 + \frac{t_1}{K_2} - t_2 + \frac{t_2}{K_2}} = \frac{\in_0 A}{d - \frac{d}{3} + \frac{d}{9} - \frac{2d}{3} + \frac{d}{9}}$$

$$= \frac{9}{2}\frac{\in_0 A}{d} = \frac{81}{2}pF = 40.5pF$$

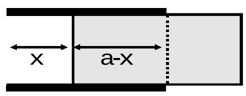
18.



where $C = \frac{\in_0 A}{d}$

$$C_{eq} = \frac{6C\times2C}{8C} + \frac{6C\times4C}{10C} = 3.9\ C$$

19.



$$C = \frac{\in_0 ax}{d} + \frac{K\in_0(a-x)a}{d}$$

$C = \frac{K\in_0 a^2}{d} - \frac{\in_0 a(K-1)}{d}x$ where x= vt

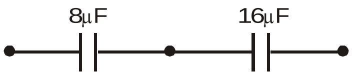
$\therefore$ C– t graph is linear with negative slope.

20. $\frac{1}{2}CV^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

21. $C = 4\pi \in_0 a$

$$C' = \frac{4\pi \in_0 ab}{b-a} = \frac{4\pi \in_0 a}{1-\frac{a}{b}} = \frac{4\pi \in_0 a}{1-\left(\frac{n-1}{n}\right)} = n\left(4\pi \in_0 a\right)C$$

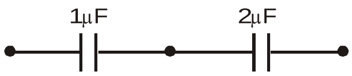
22.



Breaking voltage	20 V	80 V
Safe Voltage	20 V	10 V

$\therefore$ Charge on each capacitor = 20 x 8 = 160 μC

23.

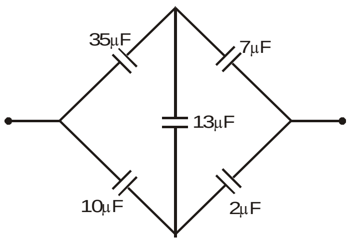


Breaking voltage	6 kV	4 kV
Safe Voltage	6 kV	3 kV

$\therefore$ Total voltage = 9 kV

24. Capacitance between 1 and 3 and between 2 and 4 are symmetrical.

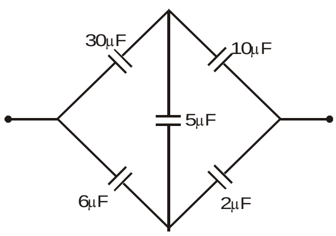
25.



The system is a balanced Wheatstone bridge.

$$\therefore C_{eff} = \left(\frac{35\times 7}{35+7} + \frac{10\times 2}{10+2}\right) = \frac{15}{2}\mu F$$

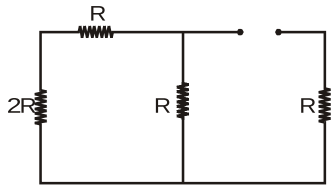
26.



The system is a balanced Wheatstone bridge.

$$\therefore C_{eff} = \left(\frac{10\times 30}{10+30} + \frac{6\times 2}{6+2}\right) = 9\mu F$$

27.



To find the time constant of a RC circuit, Short circuit the battery

$R_{eff} = \frac{7R}{4}$ $\quad \therefore \tau = \frac{7RC}{4}$

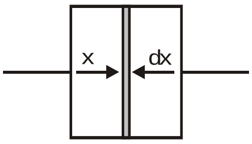
28. There is no closed path for flow of current. Hence no current flows. Hence heat developed is zero.

29. $V_A = 3\left(\frac{q}{C}\right) = 3 \times 2.5 = 7.5$ volt

EXERCISE –II

1. $E = \frac{V_0}{d} \Rightarrow E_F < E_D$ Also $\sigma_A > \sigma_B$

2.



$$\int \frac{1}{dC} = \int \frac{dy}{K \in_0 A} = \int_0^d \frac{dy}{\lambda \in_0 A \sec\left(\frac{\pi y}{2d}\right)}$$

$$\Rightarrow C = \frac{\lambda \in_0 A\pi}{2d}$$

3. Both A and B are always in parallel.

4. $V = V_0 . e^{-t/RC}$

$\left|\frac{dV}{dt}\right| = \frac{V_0}{RC} e^{-t/RC}$ = slope

At t = 0, for R = R_A;slope is least in curve–3.

5. $q = q_0 e^{-t/\tau}$ $\therefore i = \frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} = i_0 e^{-t/\tau}$

$\therefore q_0 = i_0 \tau$

Initial stored energy

$= \frac{1}{2} CV^2 = \frac{1}{2}(CV)V$

$= \frac{1}{2}(i_0\tau)(i_0R) = \frac{1}{2} i_0^2 R\tau$

6. As B is in parallel with C and the potential develops slowly. Hence during charging more heat is produced in A than in B. In steady state, same current passes through A and B.

$\therefore V_{capacitor} = \frac{E}{2}$ $\therefore E_{capacitor} = \frac{1}{2} C\left(\frac{E}{2}\right)^2 = \frac{CE^2}{8}$

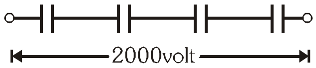
7. $q = q_0 e^{-t/\tau}$

$\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} = \frac{CV_0}{RC} e^{-t/\tau} = \frac{V_0}{R} e^{-t/\tau}$

At t=0; $i_1 = \frac{V_0}{R_1}$; $i_2 = \frac{V_0}{R_2}$

$\because R_1 = R_2$ $\therefore i_1 = i_2$

As τ is less for C_1 and hence it looses charges faster than C_2.

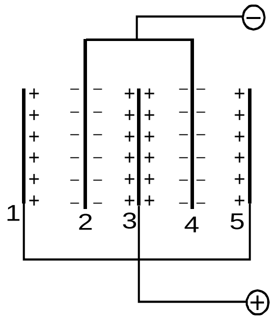
8. 

$C_{eff} = 1/4\ \mu F$

$\therefore$ Total no. of rows of capacitor $= \frac{C_{net}}{C_{eff}} = \frac{3}{1/4} = 12$

$\therefore$ Total no. of capacitors needed = 12 x 4 = 48

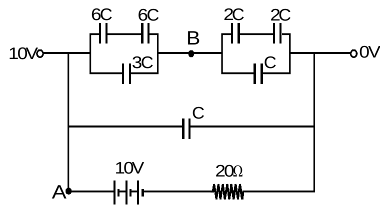
9.



Charge on plate $\neq$ 1 = $\frac{\in_0 AV}{d}$

Charge on plate $\neq$ 4 = $-\frac{2 \in_0 AV}{d}$

10.



$(V_A - V_B)\, 6C = (V_B - 0)\, 2C \Rightarrow V_B = 7.5$ V

$\therefore V_A - V_B = 10 - 7.5 = 2.5$ V

11. Force on plate

$= \frac{\sigma^2 A}{2 \in_0} = \frac{Q^2}{2A \in_0} = Kx = mg$

$\therefore Q = \sqrt{2mgA \in_0}$

12. $C_{eff} = C_{EF} = \frac{\in_0 A}{d}$ $\therefore E_{net} = \frac{1}{2} CV^2 = \frac{\in_0 AV^2}{2d}$

13. $i = 10e^{-t/RC} \Rightarrow 2.5 = 10\ e^{-2/RC}$

$\Rightarrow RC = \tau = \frac{1}{\ell n2}$ & $C = \frac{1}{10 \ell n2}$

For capacitor

$\frac{V_0}{R} = 10 \Rightarrow V_0 = 10R = 100$ volt

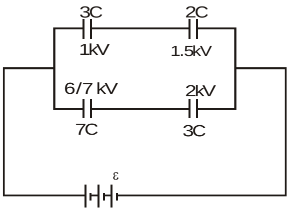
Total heat developed = Total initial energy stored in capacitor = $\frac{1}{2}CV^2 = \frac{500}{\ell n2}$

Thermal power in resistor

P = i^2R = $100\,Re^{-2t/RC}$

$\therefore$ Time–constant = $\frac{RC}{2} = \frac{1}{2\ell n2}$

14.



Safe voltages in each arm are mentioned.

$\because$ (1+1.5) < (6/7 +2) $\therefore$ E_{safe} = 1+1.5 = 2.5 kV

15. Time constant

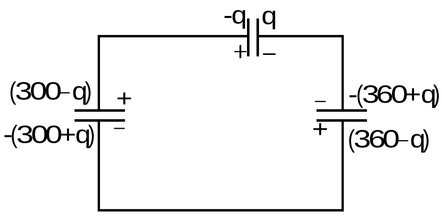
$= CR_{eff} = \left(100 \times 10^{-6}\right)\left(\frac{10^3}{2}\right)s$ = 50 m/s

16. $i_1 = \frac{V}{R}e^{-t/RC_1}$, $i_2 = \frac{V}{R}e^{-t/RC_2}$

$\therefore \frac{i_1}{i_2} = e^{t/R\left(\frac{1}{C_2}-\frac{1}{C_1}\right)} = e^{+\frac{t}{2RC_2}}$

$\Rightarrow i_1/i_2$ increases with time, t.

17.



$\Rightarrow \frac{300-q}{2} - \frac{q}{1.5} + \frac{360-q}{3} = 0 \Rightarrow q = 180$

$\therefore q_{1.5\mu F} = 180\ \mu C,\ q_{3\mu F} = 540\ \mu C,\ q_{2\mu F} = 480\mu C$

18. $i = \frac{i_0}{2} = i_0 e^{-t/RC} \Rightarrow \frac{1}{2} = e^{-\ell n4/RC}$

$\Rightarrow RC=2 \Rightarrow (2+r)\frac{1}{2} = 2 \Rightarrow r = 2\Omega$

19. At t=0, $V_C = 0 \Rightarrow i_{R_3} = 0$

$Q_{max} = C\left[\frac{\varepsilon}{\frac{R_1R_2}{R_1+R_2}+R_3}\right] = \frac{10C}{1+1} = 5\times 1 = 5\mu C$

$\therefore \left(I_{R_3}\right)_{max} = \frac{V_C}{R_3} = \frac{5}{1} = 5A$

Since R_1 and R_2 are in parallel hence current ratio of R_1 and R_2 will remain same.

20. $q = q_0 e^{-t/RC} \Rightarrow I = \frac{q_0}{RC}e^{-t/RC}$

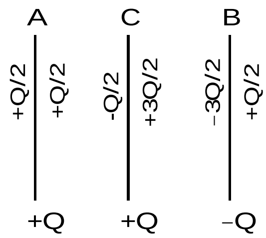
$\Rightarrow \ell nI = \ell n\left(\frac{q_0}{RC}\right) - \frac{t}{RC} = \ell n\left(\frac{V_0}{R}\right) - \frac{t}{RC}$

As I_{max} does not change $\therefore$ R = constant

$\left|\frac{d(\ell nI)}{dt}\right| = \left|0 - \frac{1}{RC}\right| \Rightarrow \left[\frac{d(\ell nI)}{dt}\right]_1 > \left[\frac{d(\ell nI)}{dt}\right]_2$

$\therefore C_2 > C_1 \Rightarrow C$ is increased

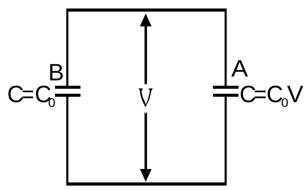
21.



Initial $V'_{AB} = \frac{Q}{C} = \frac{Qd}{\epsilon_0 A}$

Final $V_{AB} = \frac{Q/2}{\left(\frac{2\epsilon_0 A}{d}\right)} + \frac{(3Q/2)}{\left(\frac{2\epsilon_0 A}{d}\right)} = \frac{Qd}{\epsilon_0 A} = V_{AB}$

22.



$(C_0 + C_0V)\ V = 30\ C_0$

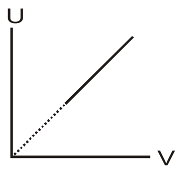
$\Rightarrow V^2 + V - 30 = 0 \Rightarrow V = 5$ volt

$\therefore V_A = V_B = 5$ volt

$Q_A = 5^2C_0 = 25\ C_0;\ Q_B = 5C_0$

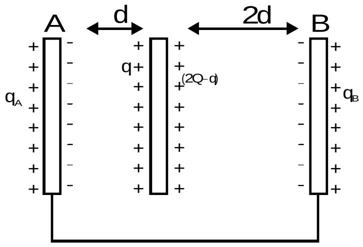
23. $C = \frac{\epsilon_0 ax}{d} + \frac{K\epsilon_0(a-x)a}{d}$

$= \frac{K\epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)vt}{d}$



$V = \frac{Q}{C}$ and $U = \frac{QV}{2}$ $\therefore \frac{U}{V} = \frac{Q}{2}$

24.



$$\Delta V = \frac{qd}{\epsilon_0 A} = \frac{(2Q-q)(2d)}{\epsilon_0 A} \Rightarrow q = \frac{4Q}{3}$$

Total charge on inner faces of A and B = –2Q

Rest charge will equally appear on their outer faces

$$= \frac{Q-(-2Q)}{2} = \frac{3Q}{2}$$

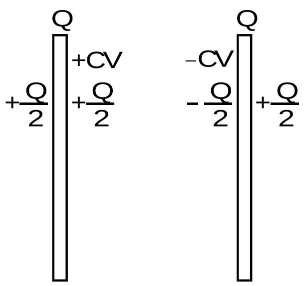
Final charge on plate A

$$= \frac{3Q}{2} - \frac{4Q}{3} = \frac{Q}{6}$$

$\therefore$ Charge flown through wire

$$= Q - \frac{Q}{6} = \frac{5Q}{6}$$

25. Final charge distribution



Therefore potential difference across the capacitor

$$= \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

26. $Q = \frac{C}{2}E$

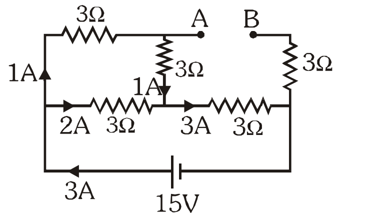
$$Q' = \frac{KCC}{KC+C}E = \frac{KC}{K+1}E$$

$$\therefore Q'-Q = \frac{KCE}{K+1} - \frac{CE}{2} = \frac{(K-1)CE}{2(K+1)}$$

This charge is supplied by battery.

27. $C_{eq} = \frac{KC}{K+1}, C'_{eq} = \frac{C}{2} \Rightarrow \frac{Q'_2}{Q_2} = \frac{K+1}{2K}$

28. At t= ∞, capacitor gets open circuited



$$\therefore I = \frac{15}{5} = 3A \Rightarrow V_A - 3 \times 1 - 3 \times 3 = V_B$$

$$\Rightarrow V_A - V_B = 12\ V$$

29. At t=0, $V_{capacitors} = 0$

$$\Rightarrow I_2 = I_3 = 0 \text{ and } I_1 = \frac{6}{2} = 3A$$

At t $\rightarrow \infty$, $I_1 = I_3 = \frac{6}{2+8} = 0.6$ A, $I_2 = 0$

30. In steady state

$$I_{upper\ arm} = I_{lower\ arm} = \frac{120}{6} = 20A$$

For the right most loop

$$3I - 3I + \frac{q}{C_2} = 0 \Rightarrow q = 0$$

For the left most loop

$$20 \times 1 + \frac{q}{C_1} - 20 \times 2 = 0$$

$\Rightarrow q = (40–20)\ C_1 = 20C_1 = 40\ \mu C$

31. Charge on 3μF capacitor

= 6 x 7 = 42 μC

$$\therefore V_{3\mu F} = \frac{42}{3} = 14 \text{ volt}$$

$\therefore V_{3.9\ \mu F}$ = 14+ 6 = 20 volt

Charge on 3.9 μF capacitor = 20 x 3.9 = 78 μC

3.9 μF

$\therefore$Total charge = 78 + 42 = 120 μC

$$\therefore V_{12\mu F} = \frac{120}{12} = 10V$$

$$\therefore \varepsilon = 20 + 10 = 30\ V$$

32. Energy $= \frac{Q^2}{2C} = \frac{Q^2 d}{2\epsilon_0 A}$

As d decreases, E decreases

33. $Q = CV = \frac{\epsilon_0 AV}{d}$

$$E = \frac{V'}{d} = \frac{V/K}{d} = \frac{V}{Kd}$$

$$W = \frac{1}{2}Q^2\left(\frac{1}{C} - \frac{1}{C'}\right) = \frac{CV^2}{2}\left(1 - \frac{1}{K}\right)$$

34. $\varepsilon = \frac{Q_0}{C_1}$ $\quad \therefore Q_1 = Q_0;\quad Q_2 = \left(\frac{Q_0}{C_1}\right)C_2$

$$V_1 = V_2 = \varepsilon = \frac{Q_0}{C_1};\quad U_1 = \frac{1}{2}C_1\left(\frac{Q_0}{C_1}\right)^2 = \frac{Q_0^2}{2C_1}$$

$$U_2 = \frac{1}{2}C_2\left(\frac{Q_0}{C_1}\right)^2 = \frac{Q_0^2 C_2}{2C_1^2}$$

35. S–open ; $V_{inner} = V_{outer}$

S –closed ; $V_{inner} = 0$

$$\Rightarrow \frac{KQ}{3R} + \frac{Kq}{R} = 0 \Rightarrow q = -Q/3$$

$$C_{initial} = 4\pi\epsilon_0(3R)$$

$$C_{final} = 4\pi\epsilon_0(3R) + \frac{4\pi\epsilon_0(3R)(R)}{(3R - R)}$$

$$\therefore C_{final} > C_{initial}$$

36. $W_{ext} = -\Delta U = U_i - U_f$

$$= \frac{1}{2}\times 2\mu F\times 400 - \frac{1}{2}\times 1\mu F \times 400 = 200\mu J$$

37. $U_{initial} = \frac{1}{2}CV^2;\qquad U_{final} = \frac{1}{2}CV^2 \quad \therefore \Delta U = 0$

$\therefore$ Heat = work done by battery

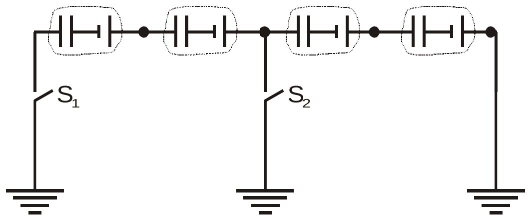
$= [CV-(-CV)]V = 2CV^2$

38. $eV = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$

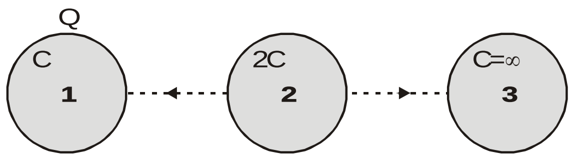
$\Rightarrow 1.6 \times 10^{-19} \times 20 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (v^2 - 0)$

$\Rightarrow v = 2.65 \times 10^6\ m/s$

39. V decreases continuously from left to right except in conductor where it is constant.

40. 

Potential difference across each capacitor and cell combination is zero.

41. 

Initial charge on 1 = Q when C_1 & C_2 touches

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{C}{2C} = \frac{1}{2} \Rightarrow Q_1 = \frac{Q}{3}, Q_2 = \frac{2Q}{3}$$

Now when Q_2 & Q_3 is touched

$$\Rightarrow \frac{Q_2}{Q_3} = \frac{C_2}{C_3} = \frac{2C}{\infty} = 0 \Rightarrow Q_2 = 0$$

Again when Q_1 & Q_2 is touched

$$Q_2 = 2\frac{(Q/3)}{3} \Rightarrow Q_1 = \frac{(Q/3)}{3} = \frac{Q}{9}$$

Similarly we can say after N times it becomes

$$Q_1 = \frac{Q}{3^N}$$

42. $\Delta Q = 2CV - (-CV) = 3CV$

$W_B = \Delta Q(2V) = 6CV^2$

$$\Delta U = U_f - U_i = \frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2 = \frac{3CV^2}{2}$$

$\therefore$ Heat = $W_B - \Delta U = \dfrac{9CV^2}{2}$

$U_f = \dfrac{1}{2}C(2V)^2 = 2CV^2$

$\therefore \dfrac{\text{Heat}}{U_f} = \dfrac{9}{4}$ = 2.25

EXERCISE –III

Fill in the blanks

1. Net charge on capacitor is zero. Hence total flux through a closed surface enclosing the capacitor is zero.

2. C_{Maximum} = All capacitors are in parallel

= 3C = 18μF

C_{Minimum} = All capacitor are in series

= C/3 = 2μF

3. $V_1 = V_2 \Rightarrow \dfrac{kQ_1}{R_1} = \dfrac{kQ_2}{R_2} \Rightarrow \dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

4. Charge holding capacity increases, hence capacity increases.

5. Air capacitor and dielectric capacitors are in series.

$$\therefore C = \frac{C_1C_2}{C_1+C_2} = \frac{\left(\frac{2\in_0 A}{d}\right)\times\left(\frac{2K\in_0 A}{d}\right)}{\left(\frac{2\in_0 A}{d}\right)(1+K)} = \frac{2KC}{1+K}$$

Match the column

1. Initial charge $q_1 = \dfrac{CE}{2}$

Final charge q_2 = CE

Initial stored energy

$U_1 = \dfrac{1}{2}C(E/2)^2 + \dfrac{1}{2}C(E/2)^2 = \dfrac{CE^2}{4}$

Final stored shergy ($U_2 = \dfrac{CE^2}{2}$

Charge supplied by battery

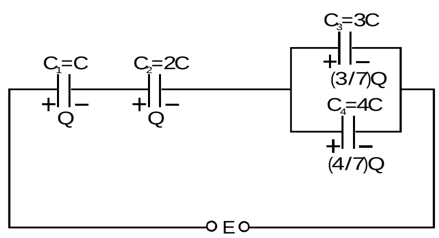
$\Delta Q = q_2 - q_1 = CE - \dfrac{CE}{2} = \dfrac{CE}{2}$

Work done by battery $W_B = \Delta QE = \dfrac{CE^2}{2}$

Heat developed in the system

$H = W_B - \Delta U = \dfrac{CE^2}{2} - \left(\dfrac{CE^2}{2} - \dfrac{CE^2}{4}\right) = \dfrac{CE^2}{4}$

2.



At C_1 = $V_1 = \dfrac{Q}{C}$ and $U_1 = \dfrac{Q^2}{2C}$

At C_2 = $V_2 = \dfrac{Q}{2C}$ and $U_2 = \dfrac{Q^2}{4C}$

At C_3 = $V_3 = \dfrac{Q}{7C}$ and $U_3 = \dfrac{3Q^2}{98C}$

At C_4 = $V_4 = \dfrac{Q}{7C}$ and $U_4 = \dfrac{4Q^2}{98C}$

Therefore $V_{max} = V_1$ and $V_{min} = V_3 = V_4$
and $U_{max} = U_1$ and $U_{min} = U_3$

Comperehension -1

1. In steady state

$I_{circuit} = \dfrac{V}{R_1+R_2} = \dfrac{18}{3+6} = 2A$

$V_{R_2} = V_{C_2} = IR_2 = 2 \times 6 = 12$ V

$Q_{C_2} = C_2V_{C_2} = 12 \times 4 = 48\ \mu C$

2. $Q_{initial} = Q_{C_1} + Q_{C_2} = IR_1C_1 + IR_2C_2$
$= 3 \times 2 \times 2 + 3 \times 4 \times 4 = 12 + 48 = 60\mu C$
$Q_{final} = V(C_1 + C_2) = 18(2+4) = 108\ \mu C$
$\therefore\ \Delta Q = 108 - 60 = 48\ \mu C$ (through S_1)

3. $U_{initial} = \dfrac{1}{2}C_1V_1^2 + \dfrac{1}{2}C_2V_2^2$

$= \dfrac{1}{2} \times 2 \times 6^2 + \dfrac{1}{2} \times 4 \times 12^2 = 324\ \mu J$

$U_{final} = \dfrac{1}{2}(C_1 + C_2)V^2 = \dfrac{1}{2}(2+4)18^2 = 972\mu J$
$\Delta Q = Q_f - Q_i = 48\ \mu C$
$W_{Battery} = \Delta Q.V = 48 \times 18 = 864\ \mu J$
$\therefore$ Heat $= W_B - \Delta U = 864 - (972 - 324) = 216\ \mu J$

Comprehension-2

1. Time Constant)
$\tau = R_1C = 8 \times 6 = 48\ \mu s$

2. $V_{t=2\tau} = V_0(1 - e^{-t/\tau}) = 12(1 - e^{-2\tau/\tau})$

$= 12\left(1 - \dfrac{1}{7.4}\right) = 10.4$ V

3. $(V_{R_1})_{t=2\tau} = V_0 - V_{capacitor} = 12 - 10.4 = 1.6$ V

4. $V_{R_2} = V_0 = 12V$

Comprehension -3

1. $V_b = \varepsilon_0 (1 - e^{-t/RC})$
$\Rightarrow 110 = 120 (1 - e^{-t/RC})$
$\Rightarrow e^{-t/RC} = 1/12$
$\Rightarrow t/RC = \ell n/12 = 2.5$
$\Rightarrow t = RC \times 2.5 = 10^6 \times 10^{-6} \times 2.5 = 5/2$ sec

2. $\tau_0 = 10^{-6} \times 10 = 10\ \mu s$

3. Flash duration= $3\tau_0 = 30\ \mu s$

4. Energy in flash

$= \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 1 \times 10^{-6} \times 110 \times 110 = 6.1$ mJ

Comprehension-4

1. $q_{1max} = q_{2max}$
C_1 and C_2 may be different and hence E_1 and E_2 may be different.

2. $\tau_2 > \tau_1 \Rightarrow R_2C_2 > R_1C_1 \Rightarrow \dfrac{R_1}{R_2} < \dfrac{C_2}{C_1}$

Comprehension-5

1. $\dfrac{C_A}{C_B} = \dfrac{\epsilon_0 A/d}{K\epsilon_0 A/d}$ =1 : K

2. $\dfrac{V_A}{V_B} = \dfrac{Q/C_A}{Q/C_B} = \dfrac{C_B}{C_A}$ = K : 1

3. $(V_A)_{initial} = \dfrac{V}{2}$; $(V_A)_{final} = \dfrac{E}{C}\dfrac{(KC)}{(K+1)} = \dfrac{KE}{K+1}$

$\therefore\ \dfrac{(V_A)_{Initial}}{(V_A)_{Final}} = \dfrac{K+1}{2K}$

4. $(V_B)_{Initial} = \dfrac{V}{2}$; $(V_B)_{Final} = \dfrac{Q}{C_B} = \dfrac{E(KC)}{(K+1)} \times \dfrac{1}{KC} = \dfrac{E}{K+1}$

$\therefore\ \dfrac{(V_B)_{Initial}}{(V_B)_{Final}}$ = (K+1) : 2

5. $(U_A)_{final} = \dfrac{Q^2}{2C_A}$; $(U_B)_{final} = \dfrac{Q^2}{2C_B}$ $\therefore \left(\dfrac{U_A}{U_B}\right)_{Final}$ = K:1

EXERCISE –IV(A)

1. $CV = \frac{qt}{t} \Rightarrow 400 \times 10^{-6} \times 100 = 100\ t$

$\Rightarrow t = 400$ s

2. Equivalent capacity between A and B

$C = \frac{9}{3} + 3 = 6\mu F$

(i) Stored charge

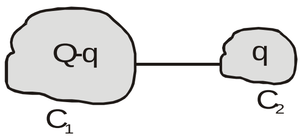
$Q = CV = 6 \times 10^{-6} \times 4 = 24\ \mu C$

(ii) Stored energy

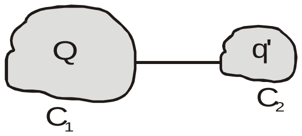
$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 16 = 48\ \mu J$

3. Electric field

$E = \frac{V_A - V_B}{d} = \frac{(10{,}000 - 0)}{(2 \times 10^{-3})} = 5 \times 10^6\ V/m$

4. 

$\frac{Q-q}{C_1} = \frac{q}{C_2}$ ...(i)



$\frac{Q}{C_1} = \frac{q'}{C_2}$ ...(ii)

Eq. (i) ÷ (ii) : $q' = \frac{Qq}{Q-q}$

5. Common potential

$V_{cm} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{2 \times 200 + 3 \times 400}{2+3} = 320V$

Charge on $C_1Q_1 = C_1V_{cm} = 2 \times 320\ \mu C = 640\ \mu C$

Charge on $C_2Q_2 = C_2V_{cm} = 3 \times 320\ \mu C = 960\ \mu C$

6. (i) On connecting with the second capacitor the charge distributes equally

$\therefore V_{CM} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{0.1 \times 10}{0.1 + 0.1} = 5V$

Total stored energy

$U_f = \frac{1}{2}C_1V_{CM}^2 + \frac{1}{2}C_2V_{CM}^2$

$= \frac{1}{2} \times 0.1 \times 10^{-6} \times (5)^2 + \frac{1}{2} \times 0.1 \times 10^{-6} \times (5)^2$

$= 2.5\ \mu J$

(ii) Initial stored energy in first capacitor

$U_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 0.1 \times 10^{-6} \times 10^2 = 5.0\ \mu J$

$\Rightarrow \frac{U_f}{U_i} = \frac{2.5}{5.0} = \frac{1}{2}$

7. $\because C = \frac{\epsilon_0 A}{d}$; $q = \left(\frac{\epsilon_0 A}{d}\right)V$

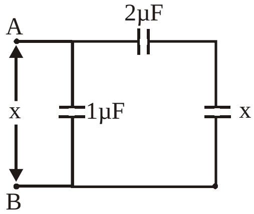
Slope $= \frac{\epsilon_0 A}{d}$ $\therefore C_2 > C_1 > C_3$

8. By using KCL

$C_1(V_A - V_0) + C_2(V_B - V_0) + C_3(V_C - V_0) = 0 \Rightarrow V_0$

$= \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}$

9. $x = \frac{2x}{2+x} + 1$



(Let $C_{eq} = x$)

$x = \frac{2x + 2 + x}{2 + x}$

$\Rightarrow x(2 + x) = 3x + 2$

$\Rightarrow 2x + x^2 = 3x + 2 \Rightarrow x^2 - x - 2 = 0$

Use

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = 2$

and –1

$x = 2,\ C_{eq} = 2\mu F$

10. $\frac{C_A}{C_B} = \frac{\left(\frac{K_1 \epsilon_0 A}{d/4}\right)}{\left(\frac{K_2 \epsilon_0 A}{3d/4}\right)} = \frac{3K_1}{K_2} = 3 \times 3 = 9$

Net capacity

$C = \frac{C_AC_B}{C_A + C_B} = \frac{(9C_B)(C_B)}{9C_B + C_B} = \frac{9}{10}C_B$

$= \frac{9}{10}\left[\frac{K_2 \epsilon_0 A}{(3d/4)}\right] = \frac{6K_2 \epsilon_0 A}{5d} = \frac{1.2K_2 \epsilon_0 A}{d}$

11. $\because E = \frac{V}{d}$

$\therefore d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}$ m

Now $C = \frac{\epsilon_0 \epsilon_r A}{d}$

$\Rightarrow A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{88.5 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 10} = 10^{-3}\ m^2$

12. $C_X = \frac{\varepsilon_0 A}{d}$, $C_Y = \frac{5\varepsilon_0 A}{d} \Rightarrow C_Y = 5C_X$

(i) C_X and C_Y are in series, so charge on each

$q = C_X V_X = C_Y V_Y \quad \Rightarrow \quad \frac{V_X}{V_Y} = 5$

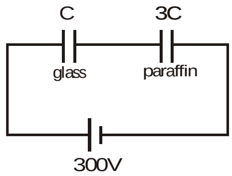
$\because V_X + V_Y = 12 \quad \therefore \quad 6V_Y = 12$

$\Rightarrow V_Y = \frac{12}{6} = 2$ volt and $V_X = 10$ volt

(ii) Energy stored in capacitor

$U = \frac{q^2}{2C} \Rightarrow \frac{U_X}{U_Y} = \left(\frac{q^2}{2C_X}\right)\left(\frac{2C_Y}{q^2}\right) = \frac{C_Y}{C_X} = 5$

13. $CV_1 = 3CV_2$ ....(i)



$V_1 + V_2 = 300$ ...(ii)

$\Rightarrow V_1 = 75V; \quad V_2 = 225$ V

(i) $\therefore E_1 = \frac{V_1}{d_1} = \frac{75 \times 100}{0.5} = 1.5 \times 10^4$ V/m

$E_2 = \frac{V_2}{d_2} = \frac{225 \times 100}{0.5} = 4.5 \times 10^4$ V/m

(ii) $V_1 = 75$ V; $\quad V_2 = 225$ V

(iii) $Q = \left(\frac{C_1 C_2}{C_1 + C_2}\right) V = \frac{3}{4} C, V = \frac{3}{4}\left(\frac{2 \in_0 A}{d}\right) 300$

$\Rightarrow \frac{Q}{A} = \frac{6 \times 300 \times 8.89 \times 10^{-12}}{4 \times 0.5 \times 10^{-2}} = 8 \times 10^{-7}$ C/m^2

14. When S_{W1} is closed and S_{W2} is open then capacitor B is charged upto 10V.

Now S_{W1} is open and S_{W2} is closed then

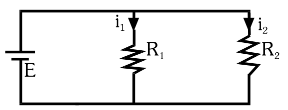
$V_{common} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{3 \times 10 + 2 \times 0}{3 + 2} = 6V$

$Q_A = 2 \times 10^{-6}\, V_{cm} = 12\ \mu C$

$Q_B = 3 \times 10^{-6}\, V_{cm} = 18\ \mu C$

15. (i) At t = 0, capacitor has zero resistance, i.e., R_1 and R_2 are in parallel.

The simple circuit is shown in figure



$i_1 = \frac{E}{R_1}$ and $i_2 = \frac{E}{R_2}$

(ii) At steady state ($t=\infty$), capacitor has infinite resistance.

Hence, $i_1 = \frac{E}{R_1}$, $i_2 = 0$

(iii) Final potential difference across capacitor is E.

$\therefore$ Final energy stored

$U = \frac{1}{2} CE^2$

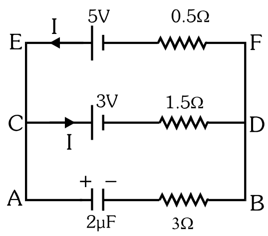
(iv) When switch is opened, capacitor will discharge through two resistance as R_1 and R_2 (both in series).

Hence, $\tau_c = C(R_1 + R_2)$

(v) When switch is closed, capacitor will charged through resistance R_2.

So $\tau = R_2 C$

16. (a) In steady state no current in capacitor's branch.



So curren $I = \frac{2}{0.5 + 1.5} = 1A$

voltage across capacitor

$V_C = 3 + 1.5 \times 1 = 4.5$ V

$\Rightarrow Q = CV_C = 2 \times 10^{-6} \times 4.5$

$= 9 \times 10^{-6}$ C

17. For the circuit ACDA and the cell :

$6 - I_1(5) - 6 = 0 \Rightarrow I_1 = 0, \quad \therefore I = 0$

For the loop BCD $V_{2\mu F} = 6V$

For the loop ABD : $V_{7\mu F} = 6V$

$\therefore Q_{7\mu F} = 6 \times 7 = 42\ \mu C$

18. Total heat dissipated

$H = \frac{1}{2} CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times 200 \times 200 = 0.1$ J

H_1 = Heat developed across $R_1 = \int I^2 R_1 dt$

H_2 = Heat developed across $R_2 = \int I^2 R_2 dt$

$$\Rightarrow H_1 = \frac{(H_1+H_2)R_1}{(R_1+R_2)} = \frac{H\,R_1}{(R_1+R_2)}$$

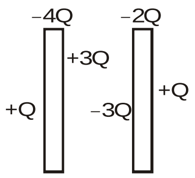
$$= \frac{0.1\times 500}{(500+330)} = 60 \text{ mJ}$$

19. $R_{eff} = \frac{2\times 3}{3+2} + 2.8 = 4\Omega$

$I = \frac{V}{R_{eff}} = \frac{6}{4} = 1.5$ A

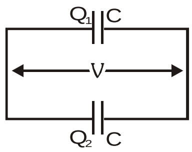
$\therefore I_{2\Omega} = I\left(\frac{3}{2+3}\right) = \frac{1.5\times 3}{5} = 0.9$ A

20.



Initial effective charge = 3Q

$CV + CV = Q_1 + Q_2$
$= 3Q + 0$
$= 3Q$



$\therefore V = \frac{3Q}{2C}$

21. $$\frac{1}{C_{arm}} = \frac{1}{C}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....\right) = \frac{1}{C\left(1-\frac{1}{2}\right)} = \frac{2}{C}$$

$\therefore C_{effective} = 2C_{arm} = \frac{2C}{2} = C$

22. $E_{final} = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2 d}{2\in_0 A}$; $E_{initial} = 0$

$\therefore$ Heat = – $(E_{initial} - E_{final}) = \frac{Q^2 d}{2\in_0 A}$

23. $V_{2\ initial} = \frac{20}{2} = 10V$

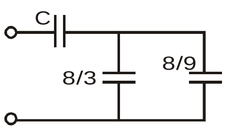
$V_{5\ initial} = \frac{50}{5} = 10V$

There is no potential difference.

Hence no charge flows.

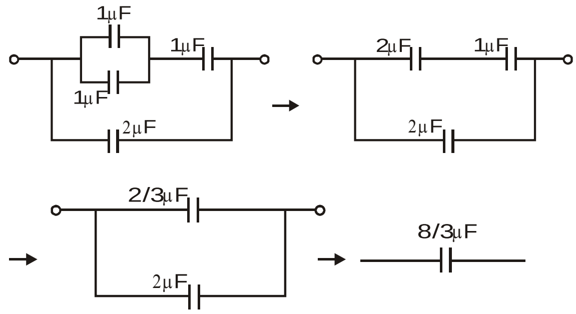
Heat produce is zero.

24.

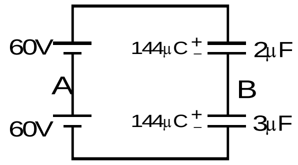


$\Rightarrow \frac{1}{C} + \frac{9}{32} = 1 \Rightarrow C = \frac{32}{23}\mu F$

25.

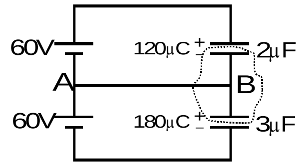


26.



$C_{eff} = \frac{2\times 3}{2+3} = 1.2\mu F$

$\therefore Q_2 = Q_3 = +144\mu C$



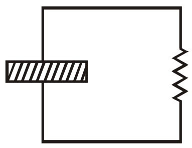
Q_{total} on the middle plates (

$= +180 + (-120) = +60\ \mu C$

This charge flows from A to B.

EXERCISE –IV(B)

1. $\frac{q}{C} - iR = 0 \Rightarrow \frac{q}{C} + \frac{dq}{dt}R = 0 \Rightarrow q = q_0 e^{-t/RC}$



equivalent circuit $\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{RC}e^{-t/RC}$

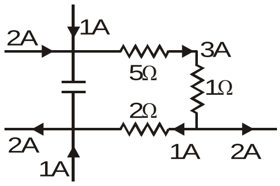
Where $R = \frac{L}{SA}$, $C = \frac{k\in_0 A}{4}$

$\therefore RC = \frac{k\in_0}{S} = \frac{5\times 8.85\times 10^{-12}}{7.4\times 10^{-12}} = \frac{5\times 8.85}{7.4}$

$\therefore i = \frac{q_0}{R_C}e^{-t/RC} = \frac{8.85\times 10^{-3}}{\left(\frac{5\times 8.85}{7.4}\right)}e^{-12/6}$

$= \frac{7.4}{5}\times\frac{1}{7.4}$ mA = 0.2mA

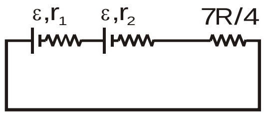
2.



$V_C = (5 + 1) \times 3 + 2 \times 1 = 20V$

$U_{cap} = \frac{1}{2}CV^2 = \frac{1}{2} \times 4 \times 20^2 = 0.8 \text{ mJ}$

3.

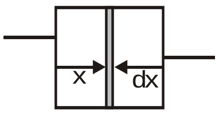


$I = \dfrac{2\varepsilon}{r_1 + r_2 + \frac{7R}{4}}$

Pot. diff. across (ε, r_1) cell : $\varepsilon - Ir_1 = 0$

$\Rightarrow \varepsilon = Ir_1 \Rightarrow \varepsilon = \dfrac{2\varepsilon r_1}{r_1 + r_2 + \frac{7R}{4}} \Rightarrow \dfrac{4(r_1 - r_2)}{7} = R$

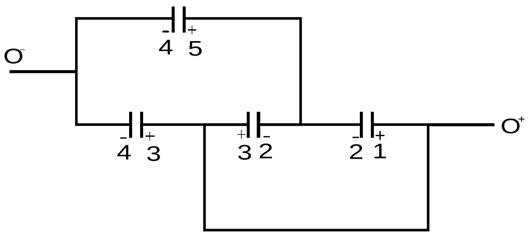
4.



$$\int \frac{1}{dC} = \int \frac{dx}{KS\epsilon_0} = \int_0^d \frac{dx}{KS\epsilon_0\left(1 + \sin\frac{\pi x}{d}\right)}$$

$$\Rightarrow C = \frac{K_1 S \epsilon_0 \pi}{2d}\left[\int_0^d \frac{dx}{\left(1 + \sin\frac{\pi x}{d}\right)} = \frac{2d}{\pi}\right]$$

5. $C_{eq} = \dfrac{2C}{3} + C = \dfrac{5C}{3}$



$Q_3 = \dfrac{4}{3}\epsilon_0\dfrac{AV_0}{d}$ & $Q_5 = \dfrac{2}{3}\dfrac{\epsilon_0 AV_0}{d}$

6. $Q_{total} = C_1V = \left[C_1 + \dfrac{C_2C_3}{C_2 + C_3}\right]V_0$

$\Rightarrow V_0 = \dfrac{C_1(C_2 + C_3)V}{C_1C_2 + C_2C_3 + C_3C_1}$

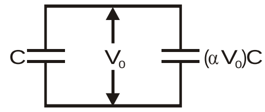
$\therefore$ Charge on C_1,

$q_1 = C_1V_0 = \dfrac{C_1^2V(C_2 + C_3)}{C_1C_2 + C_2C_3 + C_3C_1}$

Charge on C_2 and C_3

$q_2 = q_3 = \left(\dfrac{C_2C_3}{C_2 + C_3}\right)V_0 = \dfrac{C_1C_2C_3V}{C_1C_2 + C_2C_3 + C_3C_1}$

7.



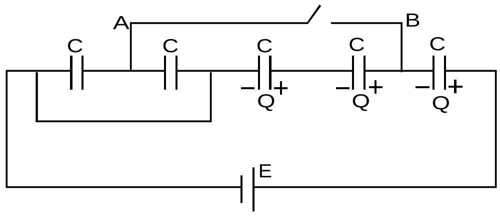
Total charge remains constant

$156\ C = (\alpha V_0)CV_0 + CV_0$

$\Rightarrow V_0^2 + V_0 - 156 = 0\ (\alpha = 1)$

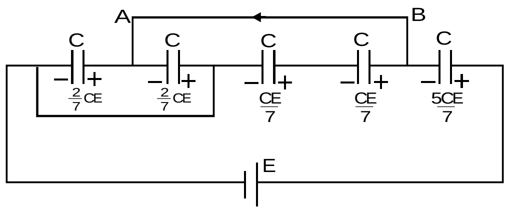
$\Rightarrow (V_0 + 13)(V_0 - 12) = 0 \Rightarrow V_0 = 12$ volt

8. **Initial condition**



$Q = \dfrac{CE}{3}$

Final condition



Charge flown from B to A = $\dfrac{4}{7}CE$

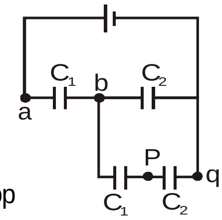
9. Extra weight needed

$= \left(\dfrac{6}{\epsilon_0}\right)^2 \times \dfrac{\epsilon_0 A}{2} = E^2 \times \dfrac{\epsilon_0 A}{2} = \left(\dfrac{V}{d}\right)^2 \dfrac{\epsilon_0 A}{2}$

$\Rightarrow mg = \left(\dfrac{5000}{5 \times 10^{-3}}\right)^2 \times \dfrac{8.85 \times 10^{-12} \times 100}{2 \times 100 \times 100}$

$\Rightarrow m = 4.52 \times 10^{-3}$ kg

11. $Q_{pq} = 2C_2 = 6C_1 = Q_{bp}$



$\therefore V_{bp} = \dfrac{6C_1}{C_1} = 6V$

$\therefore V_{bq} = 6 + 2 = 8V$

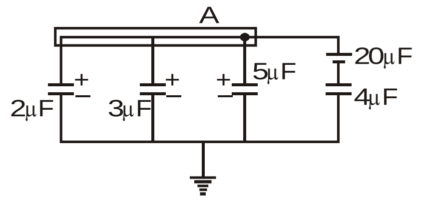
Total charge flown into right loop

$= C_2V_{bq} + C_1V_{bp}$

$= 3C_1 \times 8 + C_1 \times 6 = 30C_1$

$\therefore V_{ab} = \dfrac{Q_{total}}{C_{ab}} = \dfrac{30C_1}{C_1} = 30$ volt

12. Applying junction law at A :

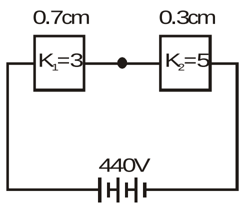


$2(V_A - 5) + 3(V_A - 20) + 5(V_A - 10) + 4(V_A - 20) = 0$

$\Rightarrow V_A = \frac{100}{7} = 14.28$ volt

$\therefore Q_{2\mu F} = 28.56\ \mu C,\ Q_{3\mu F} = 42.84\ \mu C,$
$Q_{5\mu F} = 71.40\ \mu C,\ Q_{4\mu F} = 22.88\ \mu C$

13. $V_1C_1 = V_2C_2$ and $V_1 + V_2 = 440$



$$\Rightarrow V_2 = \frac{V_1C_1}{C_2} \Rightarrow V_1 + \frac{V_1C_1}{C_2} = 440$$

$$\Rightarrow V_1 = \frac{440C_2}{C_1 + C_2} = \frac{440}{\frac{C_1}{C_2} + 1}$$

$$\Rightarrow V_1 = \frac{440}{\left(\frac{K_1/d_1}{K_2/d_2}\right) + 1} = \frac{440}{\frac{9}{35} + 1} = \frac{440 + 35}{44} = 350V$$

$$\therefore E_1 = \frac{V_1}{d} = \frac{350 \times 100}{0.7} = 5 \times 10^4\ V/m$$

$$E_2 = \frac{V_2}{d} = \frac{90}{0.3} \times 100 = 3 \times 10^4\ V/m$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}C_1V_1^2}{\frac{1}{2}C_2V_1^2} = \frac{35}{9}$$

14. Work done by battery
$= \Delta QV = (3CV)V = 3CV^2$

Energy stored in capacitors $= \frac{1}{2}(3C)V^2$

(i) $\therefore$ Heat developed $= W_B - \Delta U = \frac{1}{2}(3C)V^2$

(ii) Work done by external agent = – (K–1)

(iii) Final voltage after 'dielectric is removed = V'

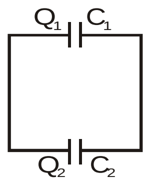
$$3CV' = (K+2)CV \Rightarrow V' = V\left(\frac{K+2}{3}\right)$$

$W_{agent} = U_i - U_f$

$$= \frac{1}{2}(3C)V^2\left(\frac{K+2}{3}\right)^2 - \frac{1}{2}(K+2)CV^2$$

$$= \frac{(K+2)(K-1)CV^2}{6}$$

15. $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ....(i)



$Q_1 + Q_2 = 2Q$ ....(ii)

$C_1 = \frac{\epsilon_0 A}{d - x}$ and $C_2 = \frac{\epsilon_0 A}{d + x}$

$$\Rightarrow Q_2 = \frac{Q(d-x)}{d} \text{ and } Q_1 = \frac{Q(d+x)}{d}$$

$$\Rightarrow \frac{dQ_2}{dt} = -\frac{Q}{2d}\left(\frac{dx}{dt}\right) \ \&\ \frac{dQ_1}{dt} = \frac{Q}{2d}\left(\frac{dx}{dt}\right)$$

$$\therefore I = \frac{dQ_1}{dt} - \frac{dQ_2}{dt} = \frac{Q}{d}\left(\frac{dx}{dt}\right) = \frac{200}{0.1} \times 0.001 = 2\mu A$$

16. $U_1 = \frac{Q^2}{2C_1}$, $C_1 = 4\pi\epsilon_0\left[\frac{ab}{b-a} + b\right]$

$U_2 = \frac{Q^2}{2C_2}$, $C_2 = 4\pi\epsilon_0 b$

$\therefore \Delta U = U_1 - U_2 = 9$ J

17. $C_{initial} = \frac{2C \times C}{2C + C} = \frac{2C}{3}$; $C_{final} = C$

(i) $\therefore \Delta Q = \Delta C \times V$

$$= \left(C - \frac{2C}{3}\right)V = \frac{CV}{3} = \frac{2 \times 30}{3} = 20\mu C$$

(ii) $H = W_B - \Delta U = \Delta QV - \left(\frac{1}{2}CV^2 - \frac{1}{2}\frac{2CV^2}{3}\right)$

$= 600 - (900 - 600) = 300\ \mu J = 0.3$ mJ

(iii) Energy supplied by the battery
$= \Delta QV = 600\ \mu J = 0.6$ mJ

(iv) Initial charge on each capacitor

$=\frac{2C}{3}V = 40\mu C$

Final charge on right capacitor = 60 μC

Final charge on left capacitors = 0

$\therefore$ Total charge from through switch, S = 60μC

EXERCISE-V-A

1. $C_{eff_{(Parallel)}} = nC$

If connected across V volts then energy stored

$= \frac{1}{2}(nC)V^2$

2. Capacitance of an isolated sphere is

C=(4pe_0)(Radius)

$C = \frac{1}{9\times10^9}\times1 = 0.11\times10^{-9} = 1.1\times10^{-10}F$

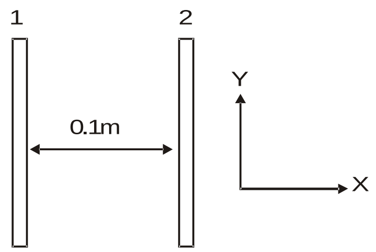
4. Work done = $\frac{1}{2}\frac{Q^2}{C}$ = $=\frac{1}{2}\frac{(8\times10^{-18})^2}{100\times10^{-6}}$

$=\frac{1}{2}\times\frac{64\times10^{-36}}{10^{-4}}$ = 32 x 10^{-32} J

5. $\frac{1}{2}CV^2$ = msΔT $\Rightarrow$ V = $\sqrt{\frac{2ms\Delta T}{C}}$

6. Two plates stacked together form a single capacitor of capacitance C. n plates stacked together form (n–1) number of capacitors of effective capacitance (n–1)C.

7.



Applying law of conservation of energy

We get $\frac{1}{2}mv^2 = eV$

[Here, v = speed of electron, V=V_2–V_1=potential difference]

$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times20}{9\times10^{-31}}}$

On solving, we get v=2.65x 10^6 m/s

8. Energy stored in a capacitor when it is charged by a potential difference of V_0 volt $=\frac{1}{2}QV_0$

Total work done by battery in sending a charge of Q through emf V_0=QV_0

hence $\frac{\text{energy stored in capacitor}}{\text{work done by battery}} = \frac{\frac{1}{2}QV_0}{QV_0} = \frac{1}{2}$

9. Net work done by the system in the process is zero, as in removing the dielectric, work done is equal and opposite to the work done is re-inserting the dielectric.

10. $C = \frac{\delta A}{d} = 9Pf$; $C_{eq} = \frac{C_1C_2}{C_1+C_2}$

$\Rightarrow \frac{\left(\frac{3\varepsilon_0AK_1}{d}\right)\left(\frac{3\varepsilon_0AK_2}{2d}\right)}{\frac{3\varepsilon_0AK_1}{d}+\frac{3\varepsilon_0AK_1}{2d}} \Rightarrow \frac{d1}{2}F = 40.5pF$

11. U = $\frac{1}{2}CV^2$; $\frac{U_0}{2} = \frac{1}{2}CV_0^2e^{-2t_1/RC}$

$\frac{1}{2} = e^{-2t_1/RC}$ (U_0 = $\frac{1}{2}CV_0^2$)

$\frac{2t_1}{RC} = \ell n2$

$t_1 = \frac{RC\ell n2}{2}$ ...(i) and $\frac{q_0}{4} = q_0e^{-t_2/RC}$

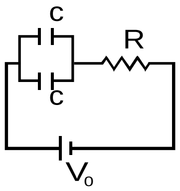
$\frac{t_2}{RC} = 2\ell n2$; t_2 = 2RC ℓn2...(ii)

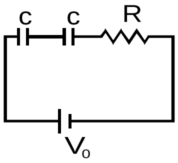
from equation (i) and (ii) $\frac{t_1}{t_2} = \frac{1}{4}$

12. $V = V_0\left(1-e^{-t/RC}\right) \Rightarrow$ 120 = 200 $\left(1-e^{-\frac{5}{RC}}\right)$

$\Rightarrow$ R = 2.7 x 10^6 Ω

13. Parallel Series





$$\frac{V_0}{2} = V_0\left(1 - e^{-\frac{t_p}{R \times 2C}}\right) \qquad ...(i)$$

$$\frac{V_0}{2} = V_0\left(1 - e^{-\frac{t_s}{R \times \frac{C}{2}}}\right) \qquad ...(2)$$

from (i) and (ii) $e^{-\frac{t_p}{2Rc}} = e^{-\frac{2t_s}{Rc}}$

$t_s = \frac{t_p}{4} = \frac{10}{4} = 2.5$ sec

14. t = 0.37% of V_0

= 0.37 x 25 = 9.25 volt

where is in between 100 and 150 sec.

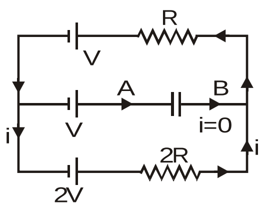
15. Common voltage = $\frac{C_1V_1 - C_2V_2}{C_1 + C_2}$

(positive plate of one capacitor is connected with negative plate of second capacitor)

$\Rightarrow 120\ C_1 = 200\ C_2 \Rightarrow 3C_1 = 5C_2$

EXERCISE –V-B

1. In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,



$$i = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

Potential difference between A and B :

$V_A - V + V + iR = V_B$

$\therefore V_B - V_A = iR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$

2. Charging current $I = \frac{E}{R}e^{-\frac{t}{RC}}$

Taking log both sides

$$\log I = \log\left(\frac{E}{R}\right) - \frac{t}{RC}$$

When R is doubled, slope of curve increase. Also at t=0, the current will be less. Graph Q represents the best.

3. Given : $V_C = 3V_R = 3(V - V_C)$

Here, V is the applied potential.

$\therefore V_C = \frac{3}{4}V \Rightarrow V(1-e^{-t/RC}) = \frac{3}{4}V \quad \therefore e^{-t/RC} = \frac{1}{4}$

Here $\tau_c = cR = 10s$

Substituting this value of τ_c in equation and solving

We get : t = 13.86 s

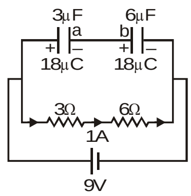
4. $\tau = CR$

$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\ \mu s$

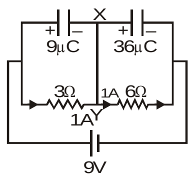
$$\tau_2 = \left(\frac{C_1C_2}{C_1 + C_2}\right)\left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\mu s$$

5. From Y to X charge flows to plates a and b.

$(q_a + q_b)_i = 0,\ (q_a + q_b)_f = 27\mu C$



Initial Figure
(when switch was open)



Final Figure
(when switch is closed)

$\therefore$ 27μC charge flows from Y to X.

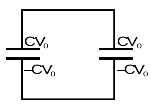
6. Time constant)= RC

$$\text{Where } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\left(\frac{2d}{3} + Vt\right)}{\epsilon_0} + \frac{\left(\frac{d}{3} - Vt\right)}{2\epsilon_0}$$

$$\Rightarrow C = \frac{6\epsilon_0}{5d + 3Vt}$$

MCQ's

1. Before S_3 is pressed

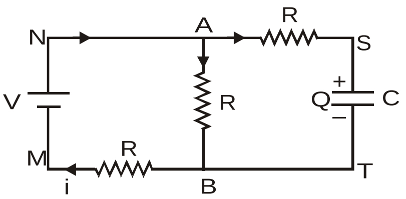


After S_3 is pressed

Subjective

1. Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t.

$\therefore\ i_1 = \frac{dQ}{dt}$



(i) Applying Kirchhoff's second law in loop MNABM

$V = (i - i_1)R + iR \Rightarrow V = 2iR - i_1R$ ...(i)

Simillarly, applying Kirchhoff's second law in loop MNSTM

we have $V = i_1R + \frac{Q}{C} + iR$ ....(ii)

Eliminating i from equations (i) and (ii), we get

$$V = 3i_1R + \frac{2Q}{C} \Rightarrow 3i_1R = V - \frac{2Q}{C}$$

$$\Rightarrow i_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right) \Rightarrow \frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R} \Rightarrow \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}(1 - e^{-2t/3RC})$$

(ii) $i_1 = \frac{dQ}{dt} = \frac{V}{3R}e^{-2t/3RC}$

From equation (i)

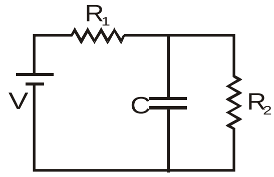
$$i = \frac{V + i_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

$\therefore$ Current through AB

$$i_2 = i - i_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC} \Rightarrow i_2 = \frac{V}{2R} \text{ as } t \to \infty$$

2. Q_0 is the steady state charge stored in the capacitor.

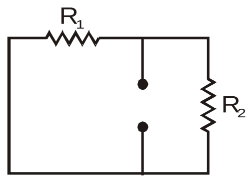


Q_0 = C [PD across capacitor in steady state]

=C[steady state current through R_2] (R_2)

$$= C\left(\frac{V}{R_1 + R_2}\right)R_2$$

$$\therefore Q_0 = \frac{CV\,R_2}{R_1 + R_2} \quad \alpha \text{ is } \frac{1}{\tau_C} \Rightarrow \frac{1}{C\,R_{net}}$$



Here, R_{net} is equivalent resistance across capacitor after short circuiting the battery.

$$R_{net} = \frac{R_1R_2}{R_1 + R_2}$$

(As R_1 and R_2 are in parallel)

$$\alpha = \frac{1}{C\left(\frac{R_1R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{C\,R_1R_2}$$